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THESIS

ANALYSIS OF FIRST PRICE SEALED BIDDING (FPSB) USING GAME THEORY

by

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December 1997

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USING GAME THEORY**

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Submitted in partial fulfillment of the
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I. INTRODUCTION

The Turkish Armed Forces (TAF) have been continually trying to enhance their war-fighting capabilities through either direct procurement of new systems or modernization of existing equipment. In either case, these efforts involve acquisition of systems and several support equipment as well as the acquisition of new equipment for existing systems' modernization. The Turkish Government is preparing to launch massive defense procurement programs to bolster its Army, Navy and Air Force and intends to spend \$31 billion for the contracts over the next 10 years. According to the Turkish Government and Industry sources this Figure may reach as much as \$150 billion through 2025. [Ref. 1]

The scarcity of resources mandates practicing more effective and more efficient ways of purchasing. The international military market differs day by day because of economic reasons and the change in global military needs. The recent example of this is the mergers of giant companies like Boeing and McDonnell Douglas.

To better equip the war-fighters with scarce resources, TAF plans to integrate smart acquisition programs. In order to achieve efficiency and effectiveness, TAF should be able to understand the strategic behavior of the prospective bidders.

This thesis will analyze the strategic behavior of the bidders in the First Price Sealed Bid (FPSB) procurement method. To understand the behavior of the bidders and to find opportunities to influence the future procurement practice of the Turkish Naval forces (TNF), this Thesis proposes a model of the FPSB. The model describes the behavior of the bidders and their actions in similar conditions to actual bidding conditions using the findings of the Game Theory.

A. AN OVERVIEW OF TURKISH NAVY PROCUREMENT ACTIVITY

The State Procurement Act of 1983 (the Act) has established general rules guiding the Government agencies' procurement procedures. Turkish Navy mainly uses four of the procurement methods outlined in this act and they are: [Ref. 2]

1. Sealed Bidding.
2. Sealed Bidding among Selected Contractors.
3. Open Bidding.
4. Negotiation.

1. Sealed Bidding

The Act bases the Sealed Bidding method as the main procurement method. This method is generally used to:

- a. provide offerors to submit their bids confidentially.
- b. provide full and open competition.

The Act established the rules of the FPSB and these rules apply to all the government agencies. According to the act, FPSB is the main method for buying goods and services.

The nature of the goods and services make them vital to national security. Therefore, when acquiring them, we must take into account some special considerations. First Price Sealed Bid method is one of the methods TNF has been using.

As the international market continually changes, FPSB method seems to remain fair and efficient whereas applicable. It provides a competitive environment for public procurement as well.

There are mainly two major bidding methods, first and second price bidding. In the first-price bidding, the bidders submit their bids, bids are opened simultaneously and the lowest bidder wins the award. In the second-price bidding, the lowest bidder wins the award but gets the second lowest bid-price. While the second-price sealed bidding has some useful properties, there is no regulation governing it in the public procurement practices of Turkey.

2. Sealed Bidding among Selected Contractors (SBASC)

The Act allows the use of SBASC for the acquisition of aircraft, Navy vessels, ammunition, electronic devices, other weapon systems and its spares and related defense industry services if there are at least three capable offerors.

If it is necessary to receive bids from less than three offerors, the soliciting agency needs to get approval from the Cabinet.

Publication of IFB is not mandatory for this type of procurement.

The goods and services that can be procured through the use of SBASC may be grouped in five categories:

- *Special Military Needs:* Aircraft, fighting ships, ammunition, electronic devices, and other weapon systems and their spares.
- *Technical Equipment for Civilian Purposes:* Aircraft, electronic devices and systems, equipment related to railroads and their spares.
- *Large and Special Buildings:* Construction of dams, power stations, irrigation plants, ports, airports and roads.
- *Special Art Works*

- *City Planning Works*

3. Open Bidding

This type of procurement is done by orally in front of the agency's procurement commission. This type of procurement is the same as auctioning either positive or negative.

4. Negotiation

Acquisition by negotiation is the method where buyer and seller try to reach an agreement through negotiation. An acquisition commission receives the proposals and negotiates with the offerors. Negotiations may be made with one or more offerors. However, the Act suggests taking at least three proposals to be able to provide competition in the acquisition. A government estimate is made and documented. The commission determines the winner and documents the rationale for the selection.

B. AREA OF RESEARCH

This thesis research will develop a computer simulation model to apply the "Nash Equilibrium Theory" into the bidding process.

C. RESEARCH QUESTIONS AND METHODOLOGY

1. Research Questions

The primary research question for this thesis is as follows:

“How do profit maximizing bidders choose their bids in a competitive environment?”

This thesis research examines the following subsidiary research questions:

1. What are the equilibrium strategies of the bidders?
2. Do the bidders have a dominant strategy in FPSB?
3. How does the number of bidders affect the outcome of the FPSB?
4. How does the cost distribution affect the outcome of the FPSB?
5. How can the procuring agencies use the findings of “Game Theory” in their acquisition practice?

2. Methodology

The thesis is conducted first by reviewing the literature for first price sealed bid activity, game theory, use of order statistics and some basic statistical procedures.

The second step in conducting the thesis was to develop a model of FPSB method and simulate the model with Visual Basic programming language.

The third step was to conduct experimentation by the simulation program under uniform and triangular statistical cost distributions. The experiments were conducted for different number of bidders.

The final step was to analyze the results of the experiments and conclude upon the analysis.

D. SCOPE OF THE RESEARCH AND ORGANIZATION OF THESIS

This thesis will:

1. Review the bidding methods used by TNF.
2. Review the game theory providing the background for constructing a model of the FPSB.
3. Develop a computer simulation to analyze the FPSB and experiment with the model.
4. Provide recommendations to use computer simulations of different bidding methods used by the TNF in its contracting practice.

This thesis has been organized in four chapters. Next chapters are as follows. Chapter II outlines the background information for developing the simulation model and mentions about the development of auctioning theory as well.

Chapter III first develops a model for simulation and then describes the experiments conducted as well as developing the bidding functions under uniform and triangular cost distributions. Then, the chapter analyzes the results of the experiments conducted with two, three, five, ten and fifteen bidders under different scenarios.

Conclusions and recommendations are provided in Chapter IV. This chapter addresses each of the primary and secondary research questions written in this chapter. In addition to this, it gives recommendations for areas for further research.

E. DEFINITIONS

Strategy -- refers to the definition, given by von Neuman and Morgenstern, founders of the Game theory, “a complete plan: a plan that specifies what choices [the player] will make in every possible situation.” [Ref. 3]

Bidder -- refers to a responsive and responsible prospective supplier, where:

Responsiveness of the supplier implies the offeror has the ability to comply with the specifications, quantities to be delivered, and terms and conditions of the invitation for bids (IFB) and

Responsibility of the supplier means offeror,

1. has adequate financial resources to perform the contract or the ability to obtain such resources,
2. is able to comply with the contracted delivery schedule,
3. has the necessary organization , experience, and technical skills,
4. has the necessary production, construction or technical equipment and facility to perform the contract obligations. [Ref. 4]

Buyer -- as the name implies, refers to a government agency soliciting an invitation for bids for a specific contract.

Bidding for contract game -- refers to the definition of a single state static game with incomplete information played by bidders.

II. BACKGROUND

This chapter of the thesis will explore the basic background information used to develop the simulation model for FPSB. The first section mentions about the elements of and conditions for sealed bidding. The second section will analyze the games to which auctioning theory applies. The final section of the chapter will provide information about the order statistics, which will be used to develop bidding functions for contract games.

A. SEALED BIDDING

Sealed bidding is a method of contracting that employs competitive bids, public opening of bids, and awards. The following steps are involved:

1. Preparation of Invitations for Bids

Invitations must describe the requirements of the Government clearly, accurately, and completely. Unnecessarily restrictive specifications or requirements that might unduly limit the number of bidders are prohibited. The invitation includes all documents (whether attached or incorporated by reference) furnished prospective bidders for the purpose of bidding.

2. Publicizing the Invitation for Bids

Invitations must be publicized through distribution to prospective bidders, posting in public places and such other means as may be appropriate. Publicizing must occur a sufficient time before public opening of bids to enable prospective bidders to prepare and submit bids.

3. Submission of Bids

Bidders must submit sealed bids to be opened at the time and place stated in the solicitation for the public opening of bids.

4. Evaluation of Bids

Bids are evaluated without discussions. Price and price related factors are the only evaluation factors.

5. Contract Award

After bids are publicly opened, an award will be made with reasonable promptness to that responsible bidder whose bid, conforming to the invitation for bids, will be most advantageous to the Government, considering only price and the price-related factors included in the invitation. [Ref. 5]

Sealed bidding can operate efficiently only when the following conditions are present:

1. There is a complete, detailed and realistic specification or purchase description.
2. There are two or more suppliers available, willing and able to compete effectively for the government's business.
3. Selection of the successful bidder can be made, without discussions of the bid, on the basis of price or price-related factors alone.
4. Enough time is available to prepare a complete statement of the government's needs and the terms under which it will do business and to carry out its administrative procedures. [Ref. 6]

B. THEORY OF FIRST PRICE SEALED BIDDING

This section will define the representation of the bidding for contract game, after studying non-cooperative games with incomplete information. Defining the equilibrium in non-cooperative games with incomplete information will provide the theoretical basis to determine the bidders' equilibrium strategies. The appearance of the equilibrium in the game will facilitate surveying the bidders' strategies in FPSB.

1. Normal-form Representation of Non-cooperative Games with Incomplete Information

This thesis surveys non-cooperative games of incomplete information based on the study of R. Gibson. [Ref. 7] To develop a normal form representation of the static game with incomplete information, also called Bayesian games; we have to consider non-cooperative games of complete information. We can represent the normal form of an n player game with complete information, as follows:

$$G = \{S_1, S_2, \dots, S_n; u_1, u_2, \dots, u_n\}$$

Where: S_i -- player i 's strategic space

u_i -- player i 's payoff function when the player selects strategy s_i

In the static non-cooperative game of complete information, the timing of moves is as follows: the players simultaneously choose an action from the feasible set of actions, A_i (i.e., player i chooses action a_i), and the payoff $u_i(u_1, \dots, u_n)$ is received.

The first step in developing the normal-form representation of the non-cooperative game with incomplete information is denoting the idea that each player has private information about his/her payoff. The players, however, are uncertain about other

players' payoff functions. Let player i 's possible payoff function be represented as $u_i(u_1, \dots, u_n, t_i)$, where t_i is called the player i 's type. t_i belongs to a set of possible types (i.e., type space). Each t_i corresponds to a different payoff function. J. Harsanyi first applied this notion for representing the payoff functions in games of incomplete information. [Ref. 8]

Given the definition of players' types, if players know their payoff functions it means they know their types. Likewise, saying the player i may be uncertain about the other players' payoff function means that player i is uncertain about the other players' types.

Players have beliefs about the other players' types. We denote the probability distribution of player i 's belief about the probability distribution of other players' types, given that player i 's type is t_i , as $p_i(t_{-i} | t_i)$, where t_{-i} represents the set of all possible types. In our analysis, the players' types (production costs as well) are identical and independent. In this case, $p_i(t_{-i} | t_i)$ does not depend on t_i , thus we can write player i 's belief as $p_i(t_{-i})$.

We can derive the normal form representation of the non-cooperative game with incomplete information by joining the normal form representation of the non-cooperative game with complete information and the concepts of type and distribution of beliefs.

The normal form representation of an n -player non-cooperative game with incomplete information specifies the players' action space, A_1, \dots, A_n , their type space, T_1, \dots, T_n , their beliefs, p_1, \dots, p_n , and their payoff functions, u_1, \dots, u_n . Player i 's type, privately known by player i , determines player i 's payoff function $u_i(a_1, \dots, a_n; t_i)$, and is a member of the set of possible types. We denote this game:

$$G = \{A_1, \dots, A_n; T_1, \dots, T_n; p_1, \dots, p_n; u_1, \dots, u_n\}$$

As stated by Harsanyi [Ref. 8], assume that the timing of the static Bayesian game is as follows:

- a. nature draws a type vector t_i where it is drawn from the set of all possible types of T_i ,
- b. nature reveals t_i to player i but not to any other player,
- c. the players simultaneously choose actions; and player i chooses action a_i .
- d. payoff u_i is received

Introducing the fictional move by nature, in steps a and b, produces a game with incomplete information that also satisfies the requirement for the games with imperfect information. Because nature only reveals player i 's type to player i , but not to others, other players do not know the complete history of the game when taking their actions. This is a condition of the game with imperfect information.

Two technical assumptions complete the discussion about the normal form representation of an n -player non-cooperative game with incomplete information. First, player i has private information about the type of some of other player(s), as well as his/her own. We can not exclude this condition explicitly, however, we can assume that the signal received about the other players' types may be false, and bidders do not consider this information when taking their actions.

The second point involves beliefs about the other players. It is known that the timing of the game is common knowledge, so is the $p(t)$ from which the nature draws

type vector t_i . When nature reveals t_i to player i , he/she can compute the belief $p_i(t_{-i} | t_i)$ using Bayes' rule of conditional probability. [Ref. 9]

$$p_i(t_{-i} | t_i) = p(t_{-i}, t_i) / p(t_{-i}) = p(t_{-i}, t_i) / \sum p(t_{-i}, t_i)$$

Furthermore, a player can compute the beliefs that the other players might hold. We assume that the type distribution is common knowledge and takes the form of either uniform or triangular distribution.

2. Definition of Bayesian Nash Equilibrium

To define the equilibrium in the static Bayesian game, we have to first define the strategic space of players. The players' strategy is a complete plan of action, specifying a worthwhile action in every circumstance in which the player might be engaged. In a static game with incomplete information, nature begins the game by drawing the players' type. Thus, a strategy for a player must specify a feasible action for each of players' possible type....

In static Bayesian games, unlike games with complete information, the strategic space is not given in the normal form representation of the game. In the games of incomplete information, the strategic space is constructed from the type and action space. Player i 's set of possible strategies is the set of all possible functions with range A_i and domain T_i .

It is seemingly unnecessary for the player i to specify actions for each of player i 's possible type. Once nature has revealed a specific type to the player, that player should not be concerned about the other possible types. However, in choosing a strategy, player i has to consider what other players will do. What other players will do largely depend on what they think player i will do...

[Ref. 10:pp. 12-13]

3. Normal-form Representation of Bidding for Contract Game

The first price sealed bidding is a non-cooperative game with incomplete information. These games are often called Bayesian games. In these games, each bidder knows their cost, but does not know any other bidders' cost. Bids are submitted in sealed

envelopes and the envelopes are opened publicly, so we can assume that the bidders act simultaneously.

The normal form representation of the bidding for contract game with two bidders competing for the contract is represented as:

$$u_i(b_i, b_j, c_i, c_j) = \begin{cases} b_i - c_i & \text{if } b_i < b_j \\ (1/2) * (b_i - c_i) & \text{if } b_i = b_j \\ 0 & \text{if } b_i > b_j \end{cases}$$

When more than two bidders are competing for a contract, the normal-form representation of the bidding for contract game is represented as:

$$u_i(b_1, \dots, b_n; c_i, \dots, c_n) = \begin{cases} b_i - c_i & \text{if } b_i = \min(b_1, \dots, b_n) \\ (1/z) * (b_i - c_i) & \text{if } b_i = b_j = \dots = b_n \\ 0 & \text{if } b_i > \min(b_1, \dots, b_n) \end{cases}$$

$$i=1, 2, \dots, n$$

Assuming no more than z bidders submit the same bid.

C. ORDER STATISTICS: AN OVERVIEW

This section of the chapter will outline the basics of order statistics theory using the guidelines provided in R.Hogg and A. Craig book of mathematical statistics. [Ref. 11]

Theory of order statistics deals with the ranked values of a sample of random variables having drawn from a probability distribution. The ranking of the order statistics goes from the smallest to largest. Order statistics have some remarkable characteristics. For example: properties of the order statistics do not depend upon the distribution from which the random sample has been drawn.

This thesis uses order statistics to analyze the outcome of the bidding simulations. Bids and the bidders' production costs are considered to be random variables drawn from a probability distribution. Ranking the submitted bids and the expected costs in ascending order we generate order statistics.

Let X_1, X_2, \dots, X_n denote a random sample from a continuous distribution, having probability density function of $f(x)$ that is positive over the interval of $a < x < b$. Let Y_1 be the smallest of these X_i , Y_2 be the second smallest, ..., and Y_n be the largest. That is, $Y_1 < Y_2 < \dots < Y_n$ represents X_1, \dots, X_n when they are arranged in ascending order. It can be proven that the joint probability distribution of $Y_1 < Y_2 < \dots < Y_n$ is given by:

$$g(y_1, y_2, \dots, y_n) = n! * [f(x_1) * f(x_2) * \dots * f(x_n)] \quad \text{if } a < y_1 < y_2 < \dots < y_n < b$$

$$= 0 \quad \text{otherwise}$$

The proof of this theorem is found in R. Hogg and A. Craig. [Ref. 11] Marginal probability density function represents the probability density function of the order statistics. This is given by:

$$g_k(y_k) = [n! / (n-k)!] * [F(y_k)]^{n-1} * [1-F(y_k)]^{n-k} * f(y_k) \quad \text{for } a < y_k < b$$

$$= 0 \quad \text{otherwise}$$

D. SUMMARY

This chapter has explored elements of sealed bidding process and theory of FPSB. At this point, the chapter analyzed and defined the non-cooperative games with incomplete information as well as providing information about the Bayesian Nash Equilibrium. The third section summarized the basics of order statistics.

III. SIMULATION AND EXPERIMENTATION

This chapter of the thesis will explain the model used to develop the simulation of the FPSB and then provide the results obtained from the experimentation. While developing the simulation, two different cost distributions were used, uniform and triangular distributions. First section of this chapter explains the model used to simulate the FPSB. The next two sections summarize the results of the simulations.

A. SIMULATION OF FPSB

This section of the thesis will develop a FPSB model for simulating the bidding for contract game. The mathematical model for simulation will be transformed into computer code using Microsoft Excel's macro editor.

1. Simulation of FPSB

A model is defined as representation of a system for the purposes of studying the system. For most studies, it is not necessary to consider all the details of a system; thus a model is not only a substitute for the system, it is also a simplification of the system. On the other hand, the model should be sufficiently detailed to permit valid conclusions to be drawn about the real system. [Ref. 12]

By developing a simulation model we can study the processes and behavior of the system, and its changes over time.

Models can be classified as being mathematical or physical. A mathematical model uses symbolic notation and mathematical equations to simulate a system. Mathematical models can be classified as deterministic or stochastic simulation models.

A stochastic model uses one or more random variables as inputs. These random inputs generate random outputs. Since the outputs of the model are random, stochastic models can imitate the real system. However, the simulation results must be treated as a statistical estimate of the real-world system's characteristics.

FPSB model represents the bidders. It includes the rules and regulations of the bids and the bidders' behavior. The actual market conditions, the availability of resources and other factors also affect the FPSB system. Using more than one cost distribution incorporated some of these variables. However, to keep the model manageable, a number of simplifications and assumptions have been made.

2. Information Space of the Game and Strategic Behavior of the Bidder

This subsection will analyze one of the most important aspects of the model, the bidders' and the buyer's characteristics. The bidding environment will also be analyzed as well. Because of the wide variety of bidding regulations, this thesis assumes both bidders and buyers meet regulations. Both parties comply with the applicable laws and regulations at the time of bidding.

a. Information Space of the FPSB

We assume that the FPSB is a non-cooperative game in which the players have limited information. However, the players know the following:

1. The buyer is fully committed not to deviate from the FPSB rules

during the bidding process, even if the deviation is in the buyer's best interest. All participants know the rules of bidding.

2. The bidders' utility function is defined by a Von Neuman Morgenstein utility function, $U(\cdot)$, and it is common for all bidders.

3. The bidders know their production cost c_i , with certainty when they bid and the cost is private information known only to the bidder. However, the bidders have subjective assumptions about the range and distribution of production costs for other bidders. This thesis assumes that the probability distribution for each bidder is the same; it follows either a uniform or a triangular probability distribution over the production cost range.

4. The bidders know the number of the bidders with certainty.

5. While preparing the bid proposals, the bidders send signals, sometimes misleading, about their cost to other bidders. However, cooperation among bidders is restricted. [Ref. 10]

6. Bidders' proposal preparation costs are not incorporated into the model. Therefore, unsuccessful bidders' losses because of bid preparation activities were neglected.

b. The Players' Strategic Behavior

One of the assumptions about the bidders is that the buyer and the bidders are expected to act rationally. The rationality of the bidders means that:

1. The bidders pursue their own interest; they attempt to maximize their

profit by bidding for the contract. Bidders maximize profits by submitting the highest possible bid. However, the bidders recognize that they are constrained by the other bidders' bids. The higher the bid the lower the probability of winning the award. This self-regulating mechanism provides an efficient solution for the game.

2. Bidders consider their production costs and the production costs of the other bidders. The resulting bids form a Bayesian Nash equilibrium. Bidders using the equilibrium strategy simultaneously maximize the expected profit regarding both their and other participants' expected bids. [Ref. 10]

3. Model Description

Consider a competitive bidding model in which the buyer solicits an IFB to procure a specified commodity or service. This thesis assumes that there are n bidders for a particular procurement and they are responsive and responsible. It is assumed that bids are submitted and the contract awarded to the lowest responsive and responsible bidder. The contract specifies the winning firm's total receipts from the buyer. The winner's expected profit depends both on the bid submitted and the cost incurred. In turn, the bids are influenced by the firm's expectation about the competing bids.

The constructed model attempts to capture the major and decisive characteristics of the real FPSB process. However, this model is only a first approximation of many procurement procedures. The model provides an opportunity to experiment with the bidders' possible actions and decisions during FPSB. The proposed FPSB model assumes symmetry of information and preferences, which makes it possible to concentrate exclusively on a symmetric Bayesian Nash equilibrium.

4. Computer Simulation Methodology

This thesis uses a personal computer and Excel 7.0 spreadsheet software to simulate the FPSB process. Two different computer programs were written to conduct the necessary operations to simulate the process. The simulation results were collected in separate worksheets. The composed computer programs allow some of the data to be input interactively. Other necessary experiment data may be input using the Microsoft Excel 7.0 macro editor.

The bidders' initial production costs, which were random variables during the entire simulation process, were generated by Excel's built-in random number generator. The built-in random number generator provides a random number distributed uniformly over the interval $[0, 1]$.

B. EXPERIMENTATION UNDER UNIFORM COST DISTRIBUTION

This section of the chapter describes the FPSB simulation when the production costs are distributed according to the uniform distribution.

1. Mathematical Model for Simulation

For products and services for which fairly mature, simple and common technologies are used, the costs of production are almost identical. Assume that the bidders' potential production costs are distributed uniformly over an interval $[h, k]$.

For the uniform distribution:

The probability density function of cost: $f(c)=1/(h-k)$

The cumulative distribution function of cost: $F(c)=(c-a)/(h-k)$

Where h is the upper limit and k is the lower limit of the cost range.

A bidding strategy defines the relationship between the bidder's proposal, b_i , and cost, c_i . The following section is the derivation of the bidding function under uniform cost distribution.

2. General-Form Bidding Function for the Uniform Cost Distribution

Assume that n players are competing for a contract and the bidders' production/service cost c_i has a uniform cost distribution over the range of $[k, h]$.

Assume players 1, 2, 3,..., n-1, n adopt the strategy $b(\cdot)$, and $b(\cdot)$ is strictly increasing and differentiable. For a given value of the player i's cost, player i's optimal bidding strategy solves:

$$\text{Max}\{(b_i - c_i) * \text{Prob}[b_i < b(c_1), \dots, b_i < b(c_n)]\} \text{ where } n: \text{number of bidders.}$$

Let $b^{-1}(b_i), b^{-1}(b_{i+1}), \dots, b^{-1}(b_n)$ denote the costs that the bidders must have in order to bid b_i, b_{i+1}, \dots, b_n . That is, $b^{-1}(b_i) = c_i, \dots, b^{-1}(b_n) = c_n$ if $b_i = b(c_i), \dots, b_n = b(c_n)$.

We can define the probability of a given player i's bid to be the lowest of all as:

$$\text{Prob}(b_i < b_{i+1}, \dots, b_i < b_n) = [1 - b^{-1}(b_i)]^{n-1}$$

The first order condition for player i's optimization problem is then:

$$d\{(b_i - c_i) * [1 - b^{-1}(b_i)]^{n-1}\} / db_i = 0$$

$$[1 - b^{-1}(b_i)]^{n-1} + (b_i - c_i) * (n-1) * [1 - b^{-1}(b_i)]^{n-2} * d[1 - b^{-1}(b_i)] / db_i = 0$$

The first order condition is an implicit equation for i^{th} bidder's best response to the strategy $b(\cdot)$ played by other bidders, given that the i^{th} bidder's cost is c_i . If the strategy $b(\cdot)$ is to be a symmetric Nash equilibrium, we require that the solution of the first order condition be $b(c_i)$. That is, for each of i^{th} bidder's costs, the bidder does not want to deviate from the strategy $b(\cdot)$, given that the other players play the same strategy. To impose this, we substitute b_i with $b(c_i)$ into the first order condition:

$$\{1-b^{-1}[b(c_i)]\}^{n-1} + [b(c_i)-c_i] * (n-1) * \{1-b^{-1}[b(c_i)]\}^{n-2} * d\{1-b^{-1}[b(c_i)]\}/db_i=0$$

$$\text{where } b^{-1}[b(c_i)]=c_i \text{ and } d\{1-b^{-1}[b(c_i)]\}/db_i=-1/b'(c_i)$$

By substituting these two into the first order differential equation we get:

$$(1-c_i)^{n-1} - \{[b(c_i)-c_i] * (n-1) * (1-c_i)^{n-2}\} / b'(c_i)=0$$

$$(1-c_i)^{n-1} = \{[b(c_i)-c_i] * (n-1) * (1-c_i)^{n-2}\} / b'(c_i)$$

$$b'(c_i) * (1-c_i)^{n-1} - (n-1) * b(c_i) = - (n-1) * c_i$$

The left-hand side of the equation can be written as:

$$b'(c_i) * (1-c_i)^{n-1} - (n-1) * b(c_i) = [1/(1-c_i)^{n-2}] * \{d[b(c_i) * (1-c_i)^{n-1}]/dc_i$$

Then, the original equation can be written as:

$$[1/(1-c_i)^{n-2}] * \{d[b(c_i) * (1-c_i)^{n-1}]/dc_i\} = - (n-1) * c_i$$

$$\{d[b(c_i) * (1-c_i)^{n-1}]/dc_i\} = - c_i * (n-1) * (1-c_i)^{n-2}$$

Integrating both sides of this equation using by parts method results in:

$$\int \{d[b(c_i) * (1-c_i)^{n-1}]/dc_i\} = - \int c_i * (n-1) * (1-c_i)^{n-2}$$

$$b(c_i) * (1-c_i)^{n-1} = c_i * (1-c_i)^{n-1} - \int (1-c_i)^{n-1} dc_i$$

$$b(c_i) * (1-c_i)^{n-1} = c_i * (1-c_i)^{n-1} + [(1-c_i)^n]/n + k$$

Finally, by using the boundary conditions we determine the value of k . That is, $b(c_i) \geq c_i$. If $c_i=1$ then $b(1)$ is finite, which is true; thus $k=0$.

Therefore, the bidding function, $b(c_i)$, takes the form:

$$b(c_i) = c_i + [(1-c_i)]/n$$

$$b(c_i) = [1 + (n-1) * c_i]/n$$

Under the assumption that the players' strategies are strictly increasing and differentiable, we have a linear and symmetric Nash equilibrium in the n -person bidding game.

We can determine the bidding function for the cost interval, $[k, h]$, then, modify the function as:

$$b(c_i) = [(h-k) + (n-1) * c_i]/n$$

Where $k < h$ and,

k : the lower limit of the uniform cost distribution.

h : the upper limit of the uniform cost distribution.

3. Experimentation

During the experimentation, five different scenarios were used. In these scenarios, the number of bidders was changed to analyze the FPSB process and also verify the accuracy of the simulation. The number of bidders used are two, three, five, 10 and 15. Therefore, the bidding functions for these scenarios over the cost interval $[0, 1]$ are:

$$b_i = (1+c_i)/2, \quad b_i = (1+2*c_i)/3, \quad b_i = (1+4*c_i)/5, \quad b_i = (1+9*c_i)/10, \quad b_i = (1+14*c_i)/15$$

To allow different cost intervals to be used in the simulation, the required random number was generated as:

$$R=(h-k)*\text{Rand}()+k$$

Where R is the required random number and Rand() is the Excel generated random number.

The simulation program for FPSB contract game is in Appendix A.

a. Experimentation With Two Bidders

During the simulation, three different scenarios were experimented with and analyzed. The first scenario was conducted as if both bidders used equilibrium strategies. In the second scenario, one of the bidders underbid while the other used the equilibrium strategy. Finally, in the third scenario, both of the bidders did not use the equilibrium strategy and underbid.

The different scenarios were designed to show that using the equilibrium bidding strategy maximizes the bidders' profit and any deviation from the equilibrium strategy would reduce the expected profit for the deviating bidder.

Figure 1 shows the graphical results of the simulation when both bidders used equilibrium strategies. The results show that there exists an equilibrium in the contract game and the winners' expected profits approach 0.333 as the number of simulated games approaches 2000. Another result was that the bidders won almost equally; bidder1 won 982 of the games while bidder2 won the remaining 1018 games.

Average profit per game considers all games in which the bidders bid. As shown in Table 1 in Appendix C, bidder1 won 44 of the first 100 games and its total profit is 15.2916. Therefore its average profit is $15.2916/100$, which is 0.1529. The average profit is another indicator of bidders' expectations, motivations and behavior.

The average profit graph for this particular simulation is shown in Figure 2. This graph also reinforces the result of existing equilibrium in the bidding game. Both bidders' average profit approaches to 0.167 by bidding in 2000 games.

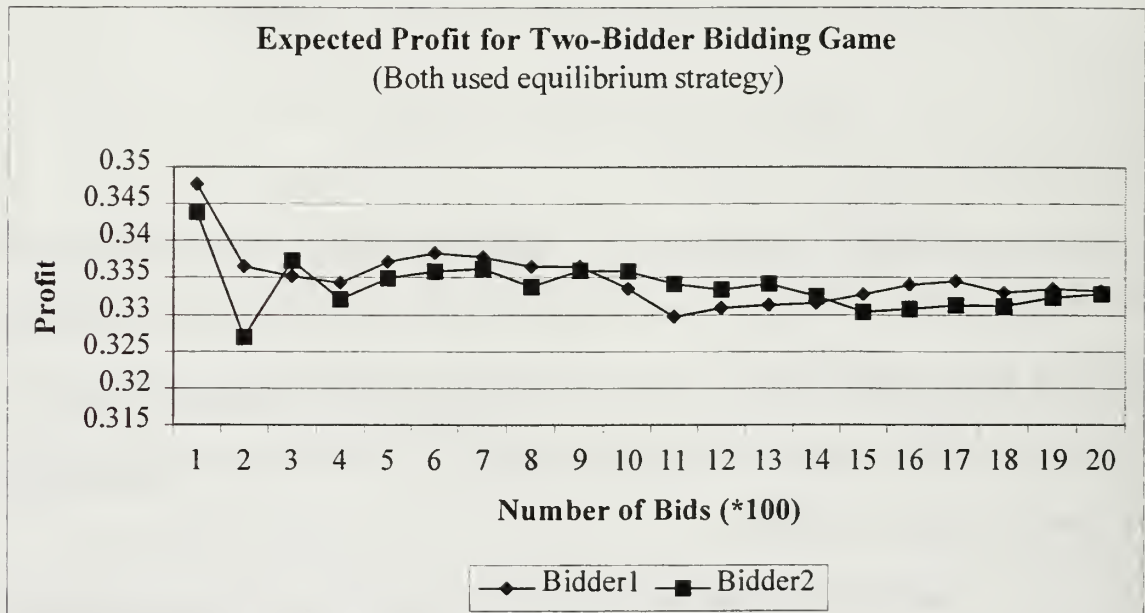


Figure 1

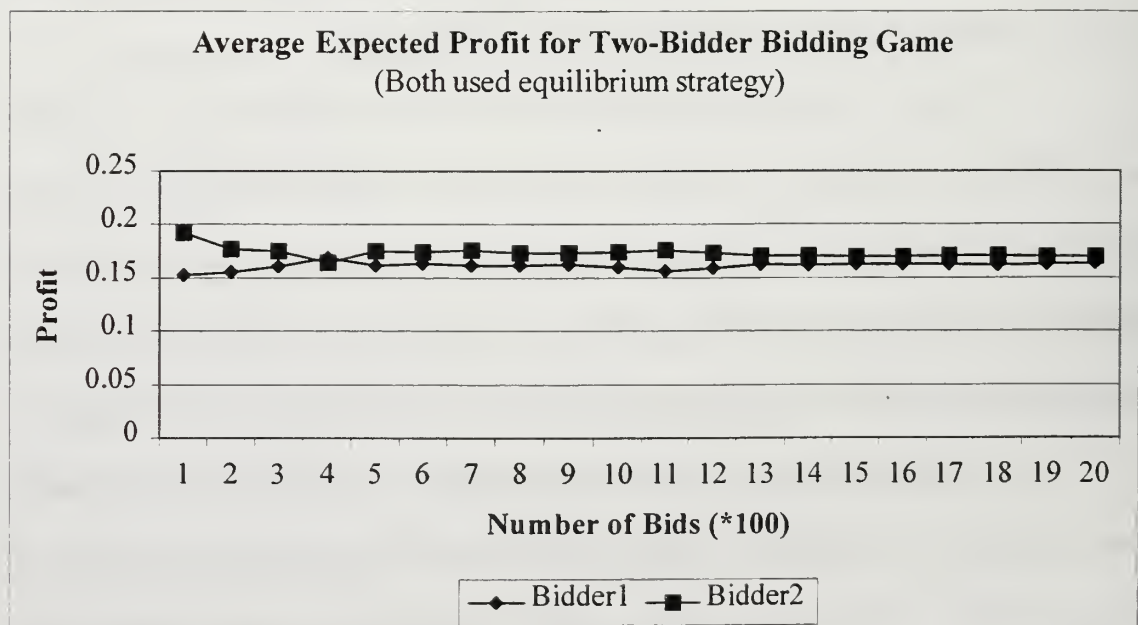


Figure 2

Total profits for both bidders resulted almost the same; 327.263 for bidder1 and 338.872 for bidder2 in 2000 games and are shown graphically in Figure 3.

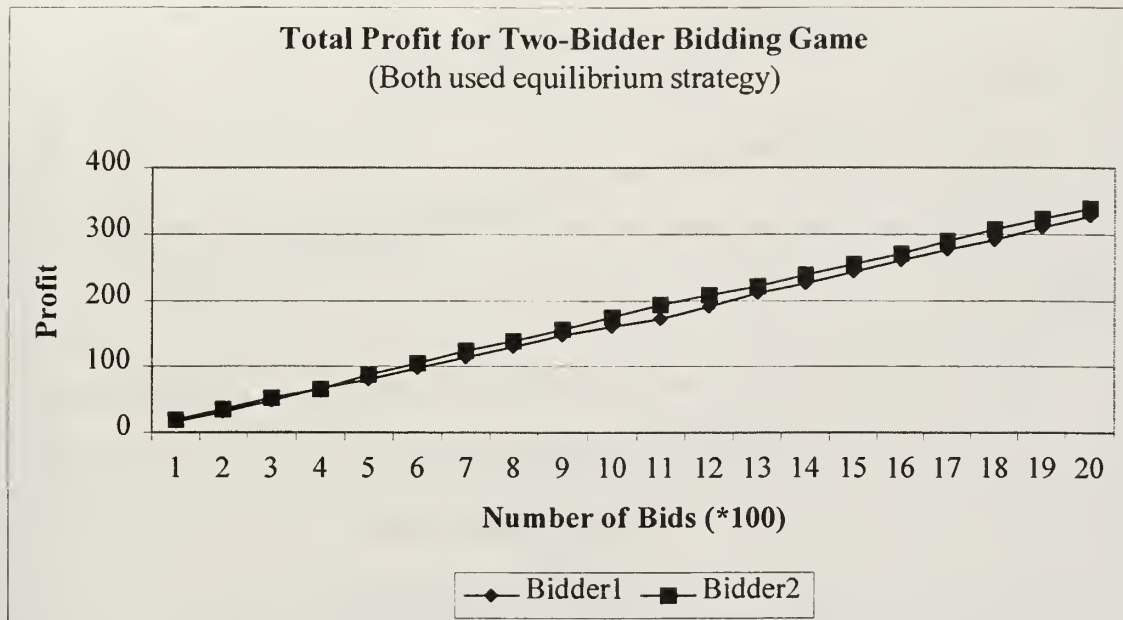


Figure 3

The next experimentation was conducted as bidder1 underbid by 0.2 while bidder2 used equilibrium strategy. Bidder1's bidding function therefore, was:

$$b_1 = [(1+c_1)/2] - 0.2$$

The results of the experimentation is shown in Table 2 in Appendix C. The experimentation resulted in lower profit for bidder1, 0.085, while bidder2 made 0.40 which is more than the equilibrium amount of 0.333. The expected profit graph is shown in Figure 4.

The average expected profit for bidder1 decreased to 0.069 and 0.074 for bidder2. The graph for average profit is shown in Figure 5.

Total profit for bidder1 also decreased to 104.514 and 111.956 for bidder2

as shown in Figure 6. Bidder1 won 1221 of 1500 games while bidder2 won remaining 279.

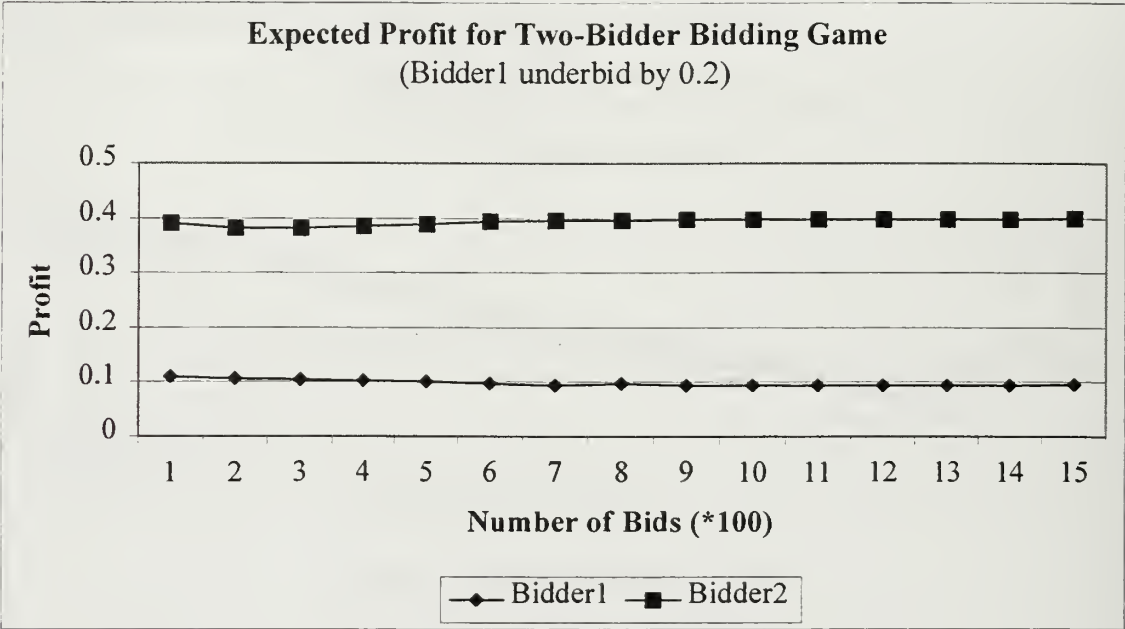


Figure 4

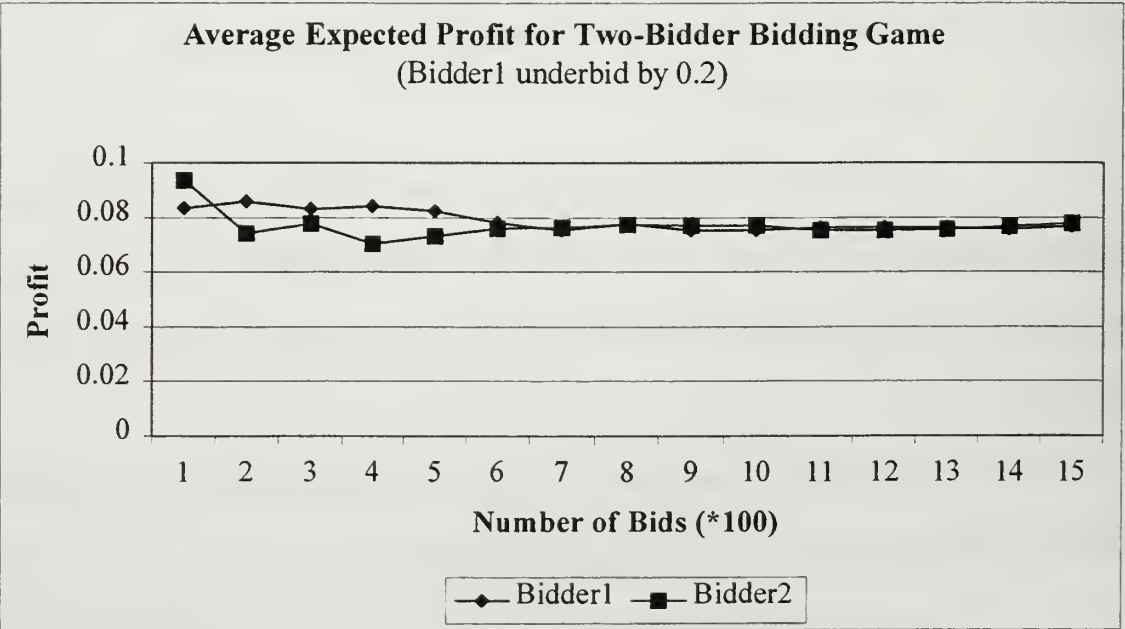


Figure 5

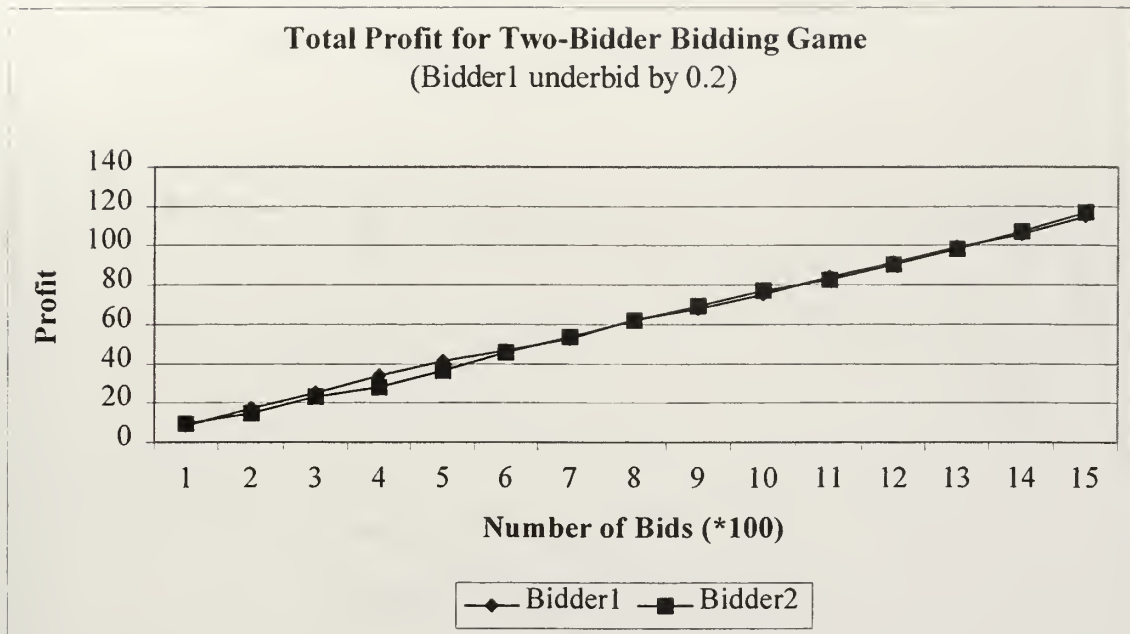


Figure 6

The last experiment with two bidders was conducted as they both did not use equilibrium strategies. Both bidders underbid by 0.1. Therefore, bidding function for both bidders was:

$$b = [(1+c)/2] - 0.1$$

In this scenario, both bidders' expected profits approached 0.23 in 1500 games, which is lower than the equilibrium amount of 0.33. The graphical representation of expected profit is shown in Figure 7.

The average profit for both bidders also decreased to 0.11 and is shown in Figure 8 graphically. Total profits in 1500 games as shown in Table 3, 171.08 for bidder1 and 175.54 for the other, were also lower than the equilibrium total profit of 243.236 and 253.983 respectively. The graphical representation of total profit curves is in Figure 9. The bidders won almost equal number of games. Bidder1 won 742 games while bidder2 won 758.

Expected Profit for Two-Bidder Bidding Game
(Both underbid by 0.1)

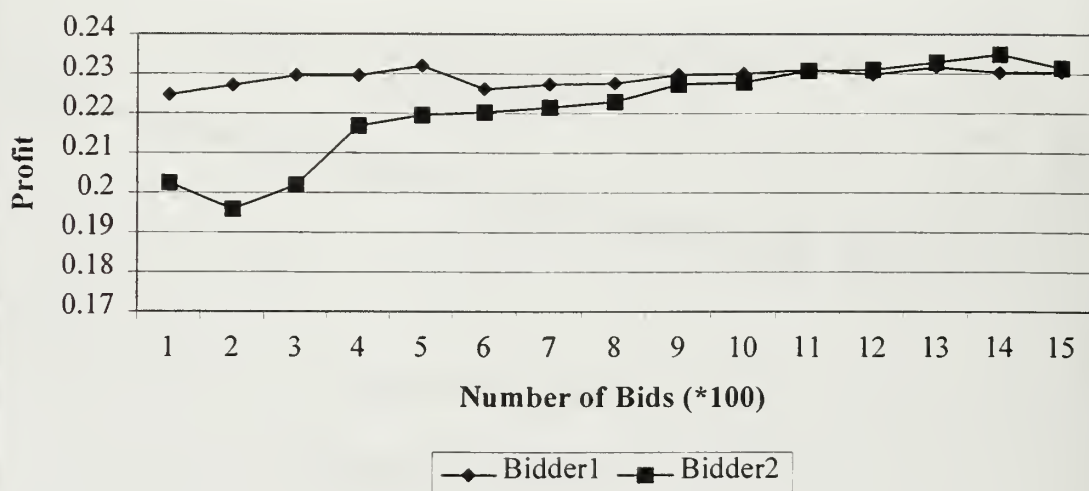


Figure 7

Average Expected Profit for Two-Bidder Bidding Game
(Both underbid by 0.1)

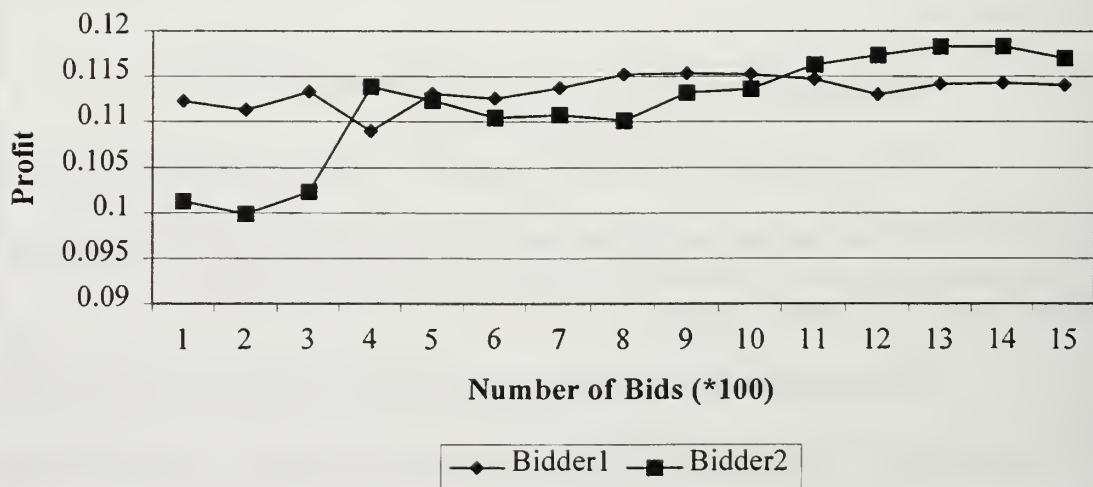


Figure 8

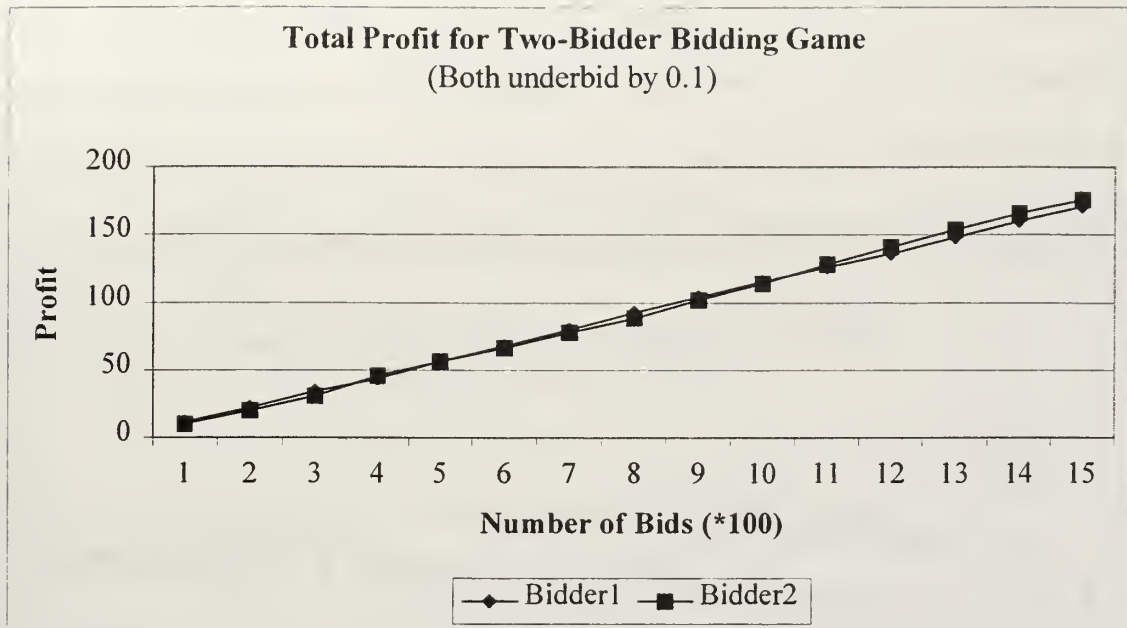


Figure 9

b. Experimentation With Three Bidders

This simulation along with the other multiple bidder simulations were conducted to find out the effect of number of bidders on the game as well as analyzing the behavior of bidders and the accuracy of the simulation. The simulation was conducted with three bidders and the cost was within the interval $[0, 1]$ under uniform distribution.

During the simulation, three different scenarios were analyzed. In the first scenario, all off the bidders used equilibrium strategy. In the second scenario, bidder1 underbid while others used equilibrium strategy. Finally in the third scenario, bidders “1” and “2” underbid by different amounts while bidder3 used equilibrium strategy.

Figure 10 shows the results of the first experimentation of which all bidders used equilibrium strategy. As seen in the graph, there is a well-defined equilibrium in the three-bidder bidding game as well as in the two-bidder bidding game.

According to the simulation results, the winners' expected profit from bidding approaches 0.25 as the number of games approaches 2000. Introducing another bidder to the game reduced the expected profit from bidding. This is true, because as the number of bidders increase, the probability of winning the award decreases, thus bidders bid lower.

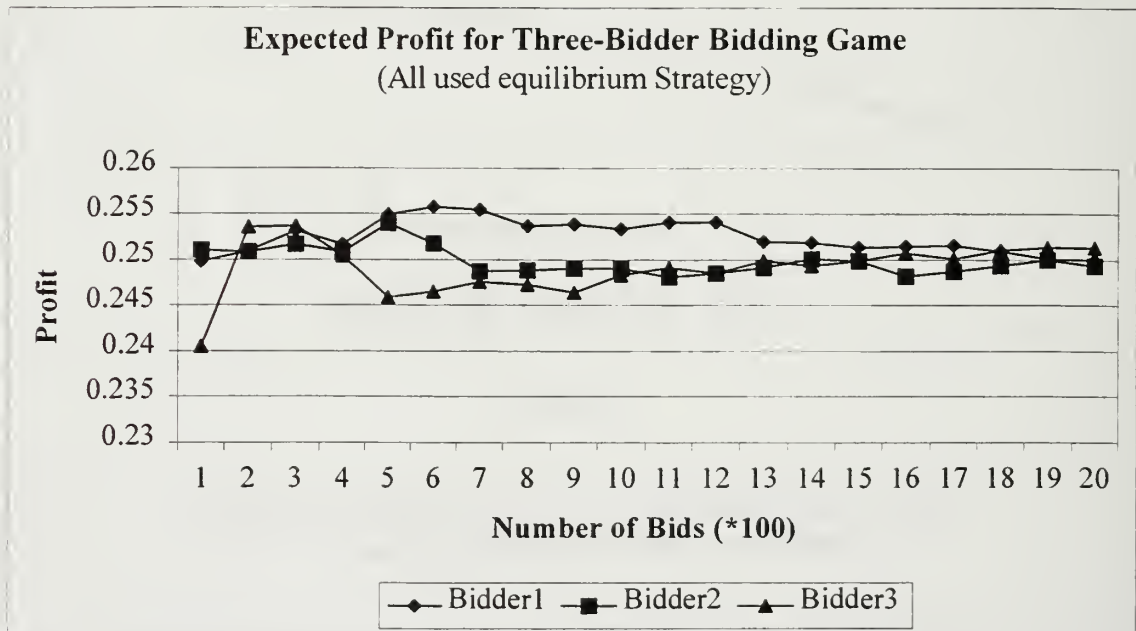


Figure 10

The average expected profit graph is shown in Figure 11 and the overall results of the experiment are shown in Table 4 in Appendix C. The average expected profit graph also reinforces the equilibrium of the game. In this simulation, bidders' average expected profit approached 0.83 while the number of games approached 2000.

As seen in Table 4, each bidder won the games almost equally. Overall, bidders won 683, 652 and 665 games respectively. Total profit for bidders as seen in Figure 12, were almost the same; 170.72, 162.6 and 167.13 respectively.

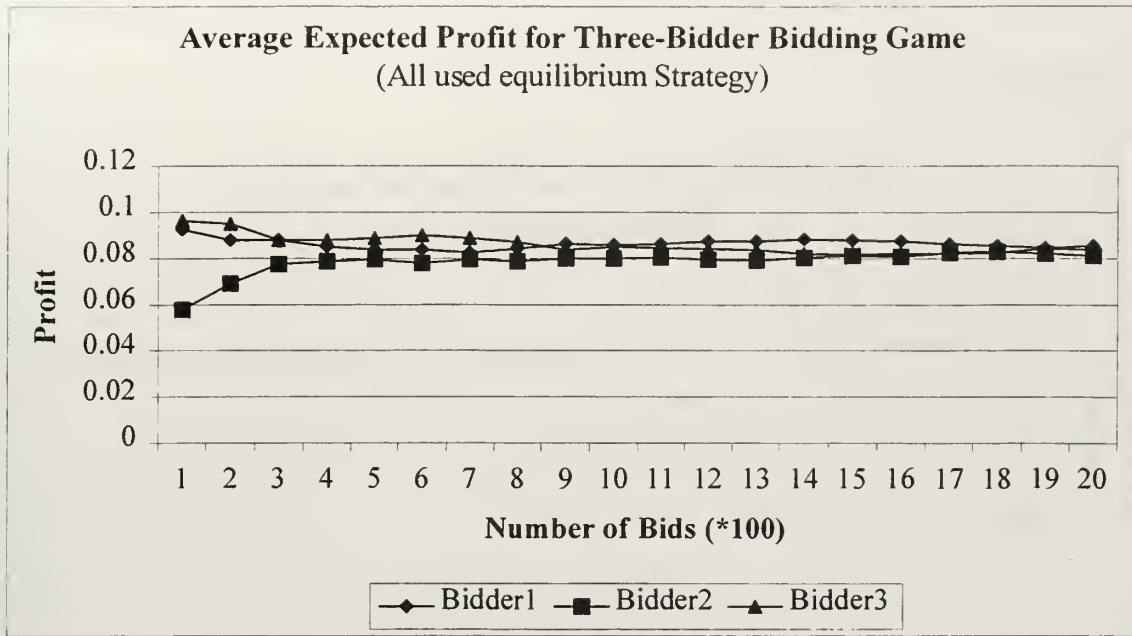


Figure 11

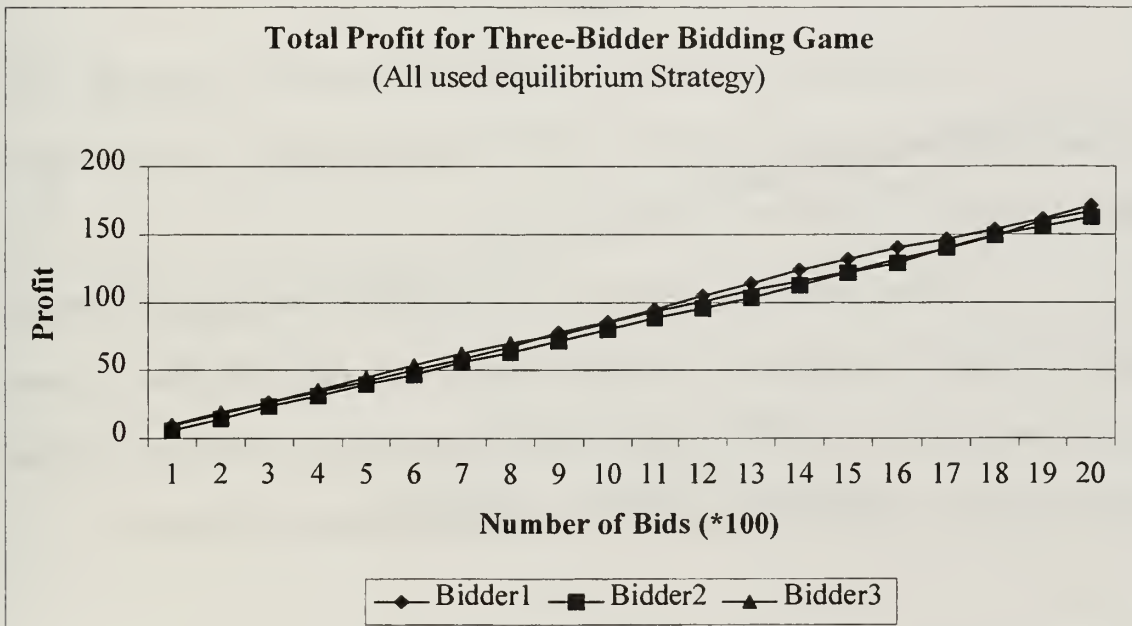


Figure 12

In the next experiment, bidder1 underbid by 0.1 while other two used equilibrium strategy. As a result of 1500 games, bidder1's expected profit decreased to

0.13 while the other two bidders' slightly increased to 0.258 and .259 respectively. The expected profit graph is shown in Figure 13.

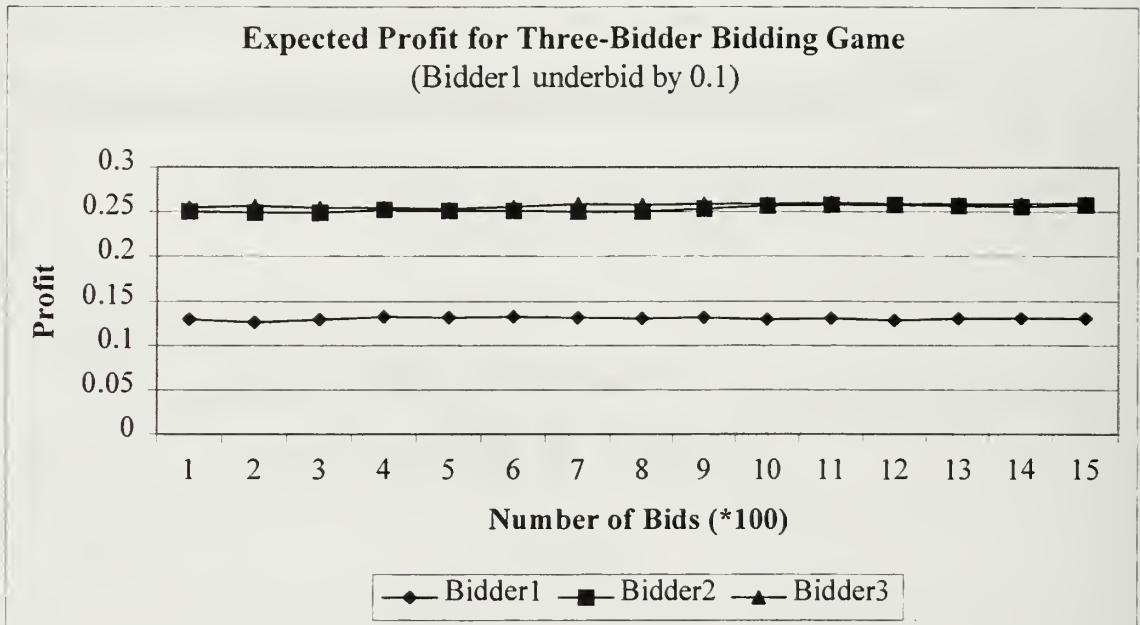


Figure 13

Average profit scheme was different than the equilibrium game experiment. Bidder1's average profit was 0.059 while others higher, 0.708 and 0.706 as graphically shown in Figure 14.

Bidder1 won 680 of 1500 games while others won 412 and 408 games as shown in Table 5 in Appendix C. Although bidder1 won more games than the others won, its total profit was 88.5022 which is lower than the others'; which were 106.2 and 105.87 respectively. Total profit graph for this experiment is shown in Figure 15.

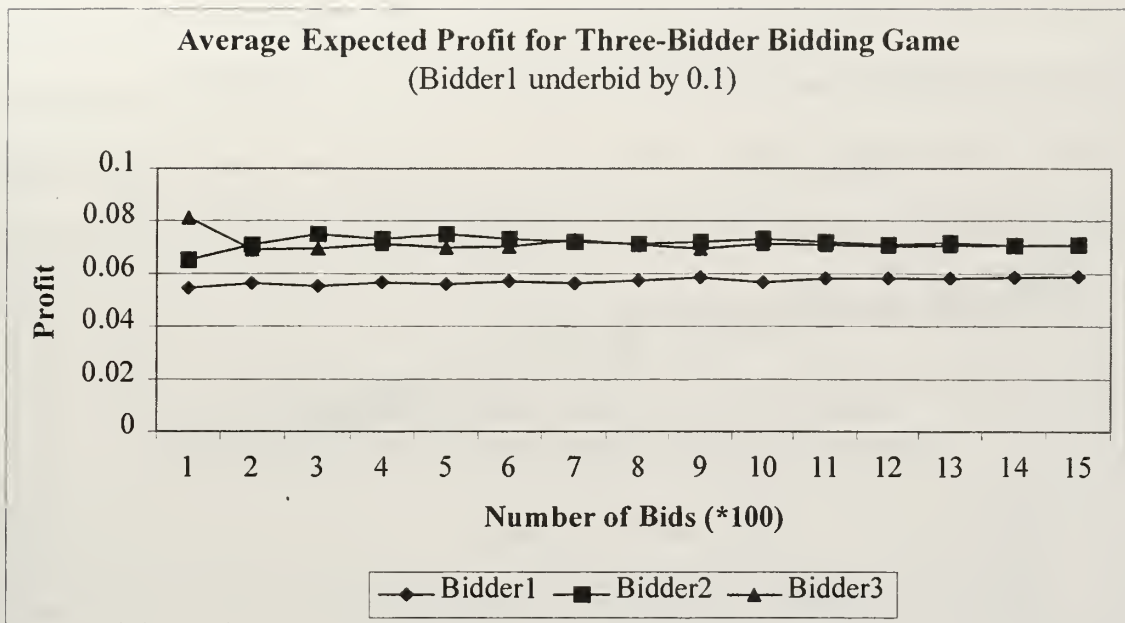


Figure 14

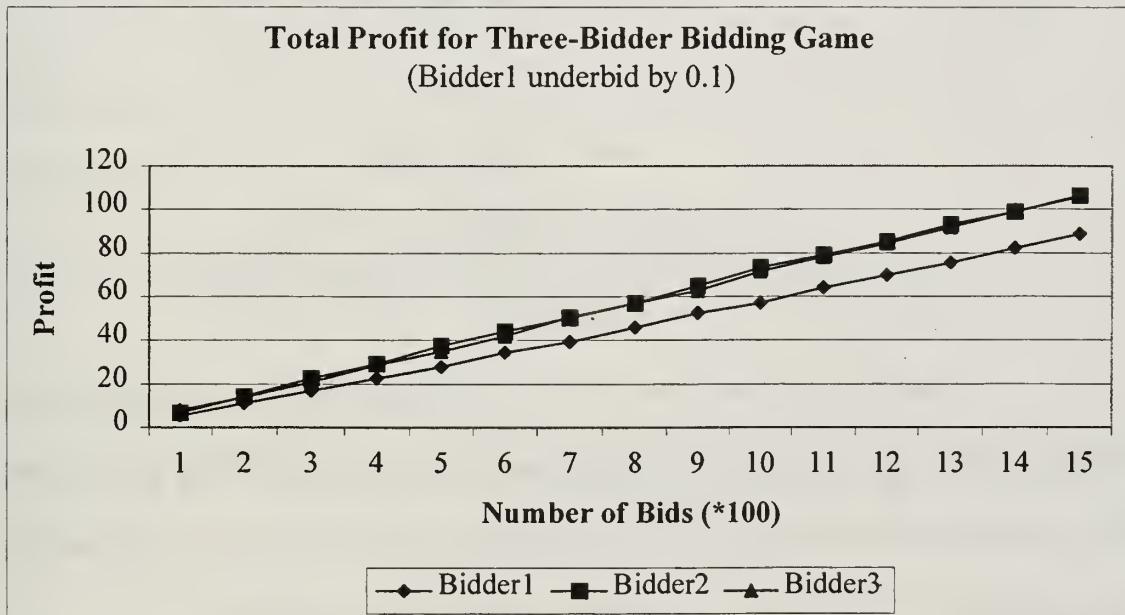


Figure 15

The last scenario for three-bidder simulation was bidder3 used equilibrium strategy while bidders “1” and “2” did not. Bidders “1” and “2” underbid by 0.1 and 0.15 respectively. The experiment resulted in lower expected profits for the first two bidders

while it was even higher for the third. Bidder1's profit approached 0.1441, which is slightly higher than in the previous game, which was 0.13 and bidder2's was even lower, 0.0857 in 1500 games. Bidder3, which had used equilibrium strategy, made slightly higher profit than it did in the first two experiments, 0.2645 as opposed to 0.25 and 0.259. The expected profit scheme is shown in Figure 16:

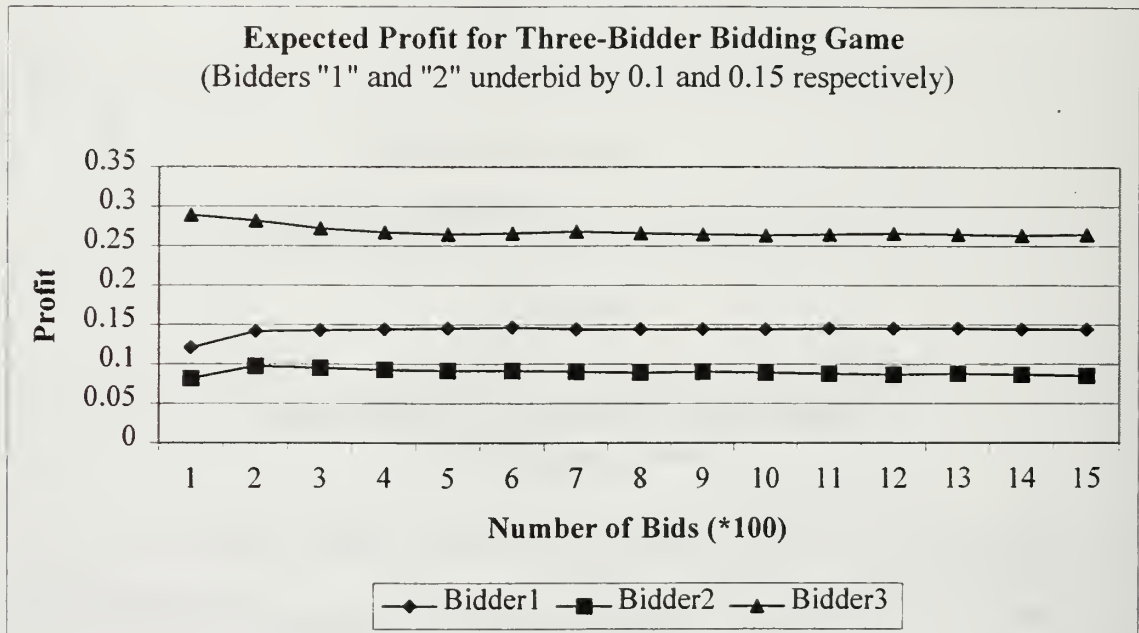


Figure 16

Total profit scheme was also very different than previous experiments. Bidder1's total profit turned out to be higher than bidders "2" and "3". It made 77.961 while others made 60.2550 and 67.722 in 1500 games. Bidders won 541, 703 and 256 games respectively. Due to the high number of games won by bidder1 made him gain more profit. In spite of winning more games, bidder2 made the lowest profit as seen in Table 6 in Appendix C and shown in Figure 17.

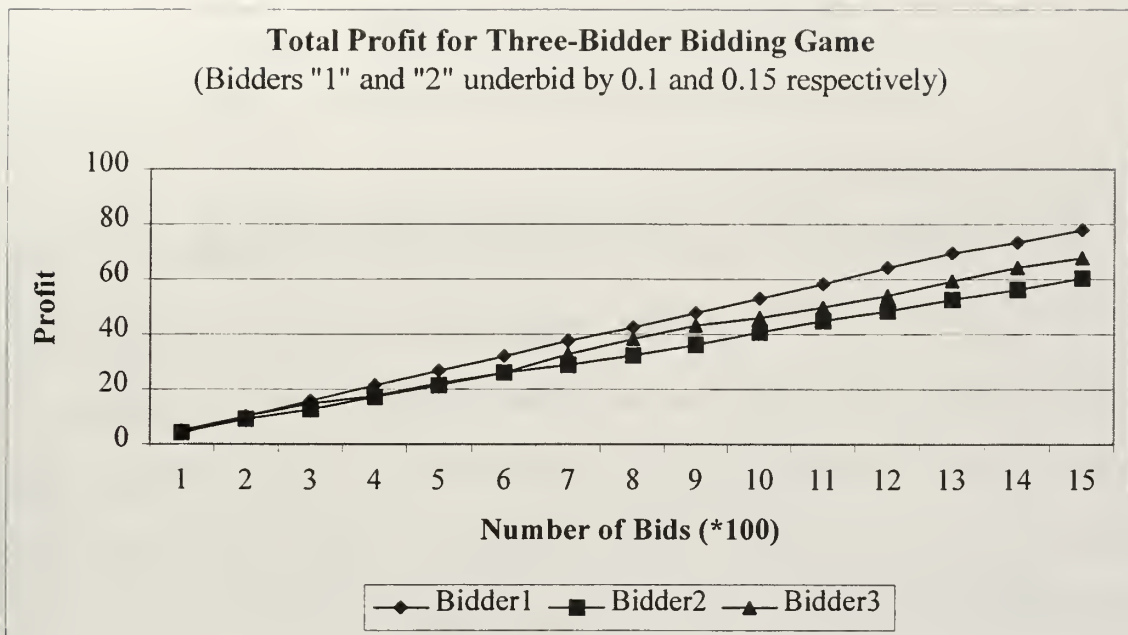


Figure 17

Though, bidder1 had higher average profit than the others did. It made 0.052 as opposed to 0.0402 and 0.0451 made by bidders "2" and "3" respectively. The average profit graph is shown in Figure 18:

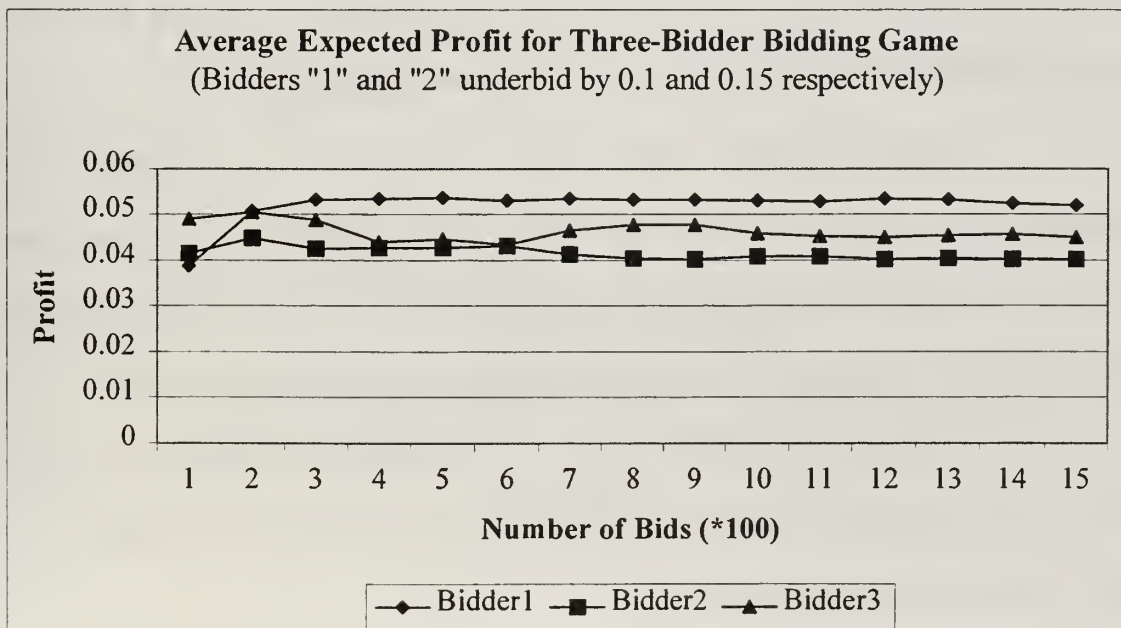


Figure 18

c. Experimentation With Five Bidders

The simulation was conducted with five bidders according to uniform cost distribution within interval $[0, 1]$.

During the simulation four different scenarios were analyzed. In the first scenario, all bidders used the equilibrium strategy. The second scenario was conducted as bidder1 underbid while others used equilibrium strategy. In the third scenario, both bidders “1” and “2” underbid by the same amount while others used equilibrium strategy and in the last scenario, bidders “1” and “2” underbid by different amounts while others used equilibrium strategy.

Figure 19 shows the results obtained from the simulation of which all bidders used equilibrium strategy. As seen in the graph, there is a well-defined equilibrium in the game as in the two and three-bidder bidding games. According to the simulation, each winner’s expected profit approached approximately 0.166 as the number of games approached 1500. As the number of bidders increased, the expected profit from bidding decreased as it was the case in the three-bidder bidding game.

Average expected profit graph is shown in Figure 20 and overall simulation results obtained are shown in Tables 7 and 8 in Appendix C. One more time, the average expected profits reinforced the game equilibrium. The average expected profits for the bidders approached 0.033 as the number of games approached 1500.

The experiment resulted in very close total profits, ranging from 45.383 to 51.889, as seen in Table 8. The total profit graph is shown in Figure 21. The bidders won 277 to 313 games during the experiment.

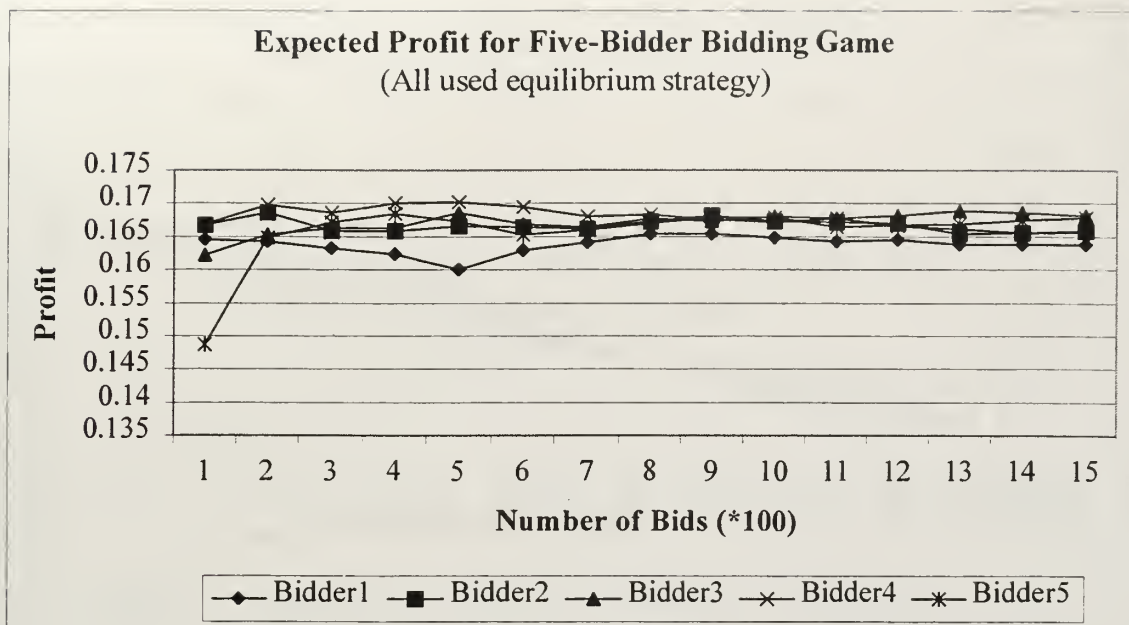


Figure 19

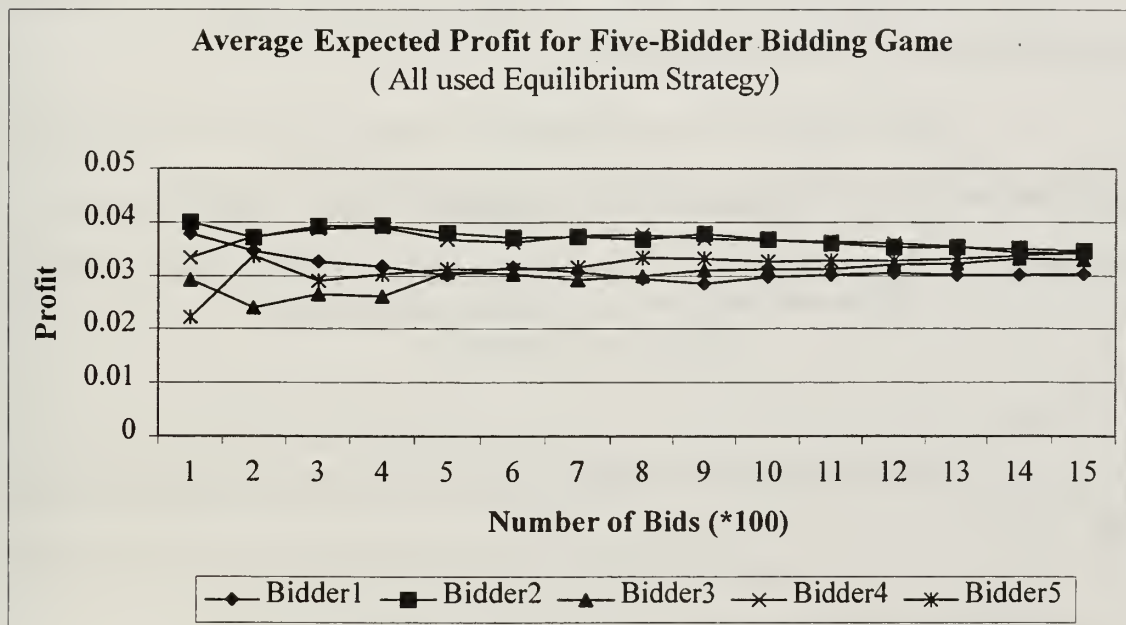


Figure 20

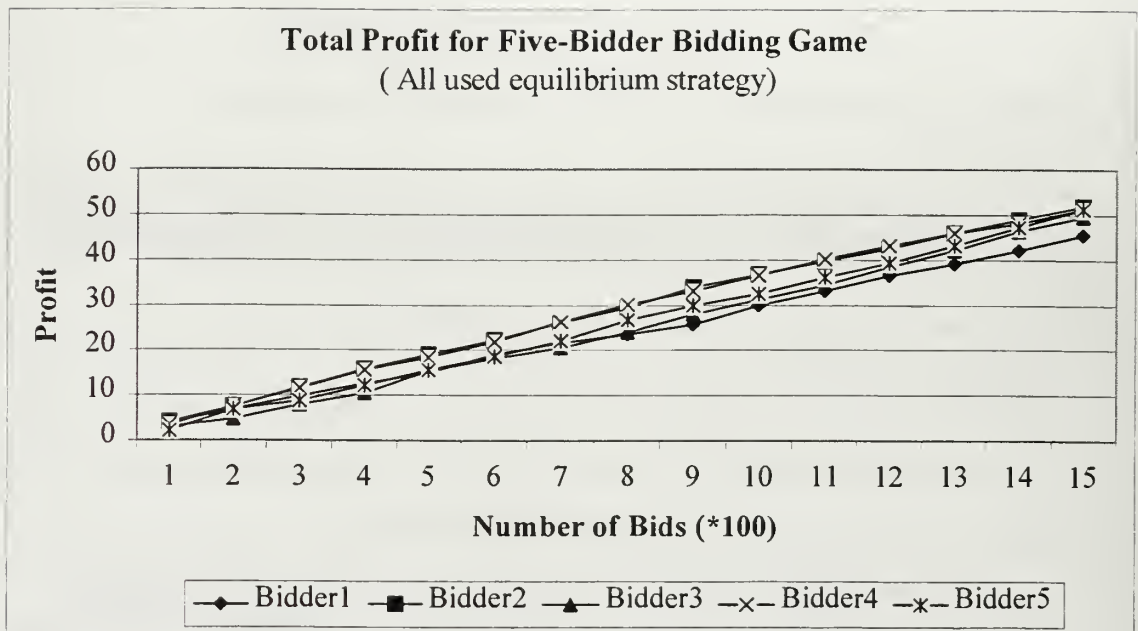


Figure 21

The second experiment, where bidder1 underbid by 0.09 and others used the equilibrium strategy, resulted in following:

Bidder1 made a lower expected profit, 0.07, than the others did, which is approximately 0.168, as the number of games approached 1500 as shown in Figure 22:

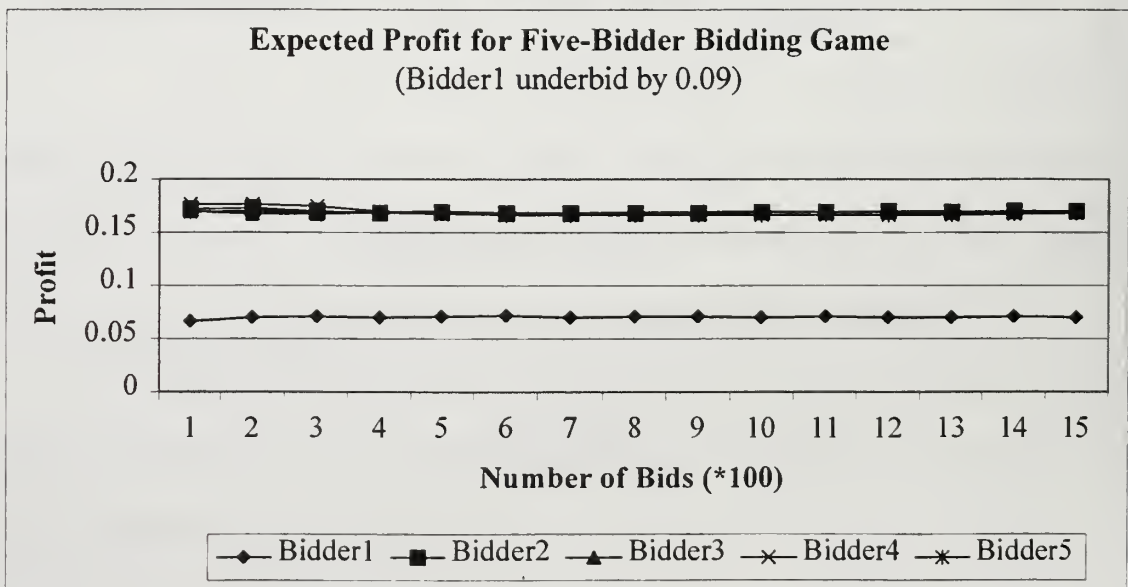


Figure 22

The average expected profit for bidder1 approached 0.0209 while others approached to a range from 0.0269 to 0.0328. The average profit curves are shown in Figure 23:

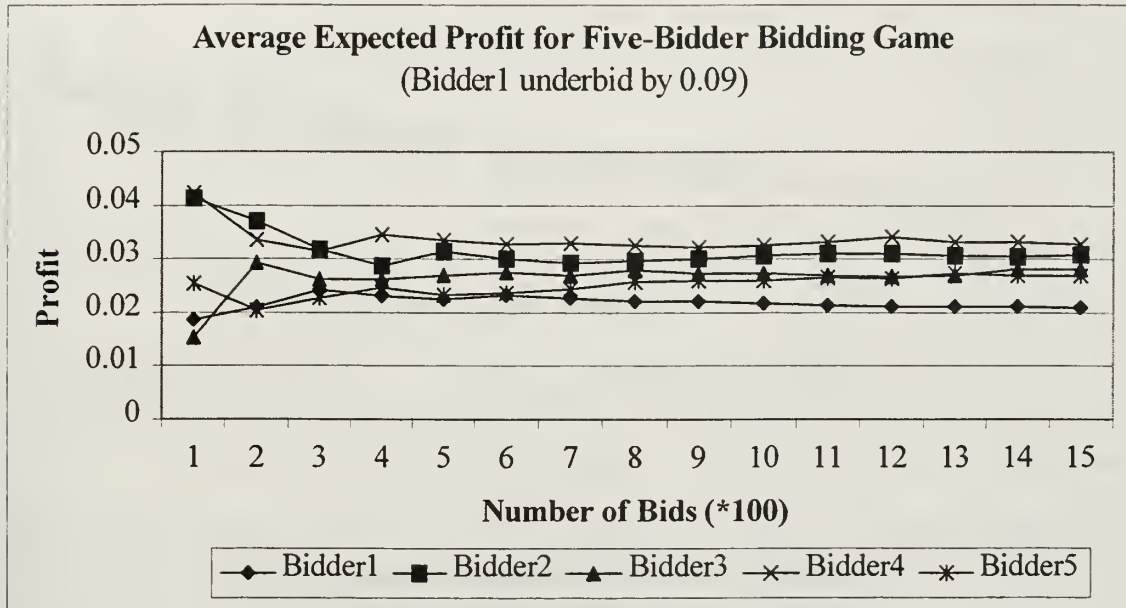


Figure 23

Bidder1 won 446 games while others won the games in a range of 241 to 292 times. Overall results of the experiment is outlined in Tables 9 and 10 in Appendix C.

Total profit for bidder1 was also lower than the others', 31.3722 as opposed to 40.4257-49.2515, even though bidder1 won more games than the others did. The total profit curves are graphically shown in Figure 24.

In the third scenario, both bidders "1" and "2" underbid by 0.12 while the rest of the bidders used equilibrium strategy. Bidders "1" and "2" made lower profits, 0.0378 and 0.0407 respectively, than the rest did, approximately 0.169, as shown in Figure 25.

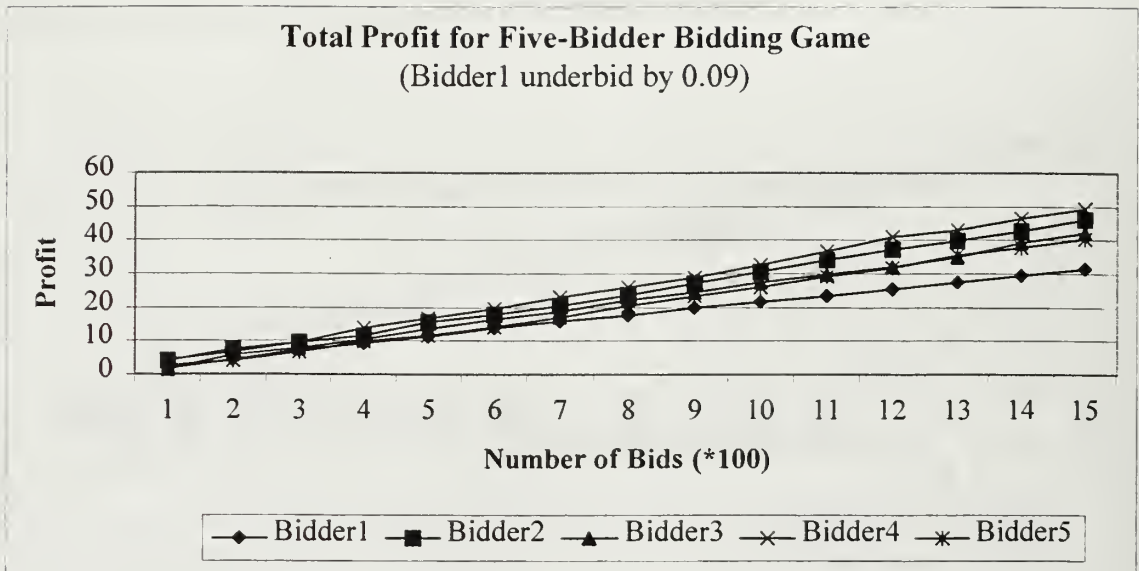


Figure 24

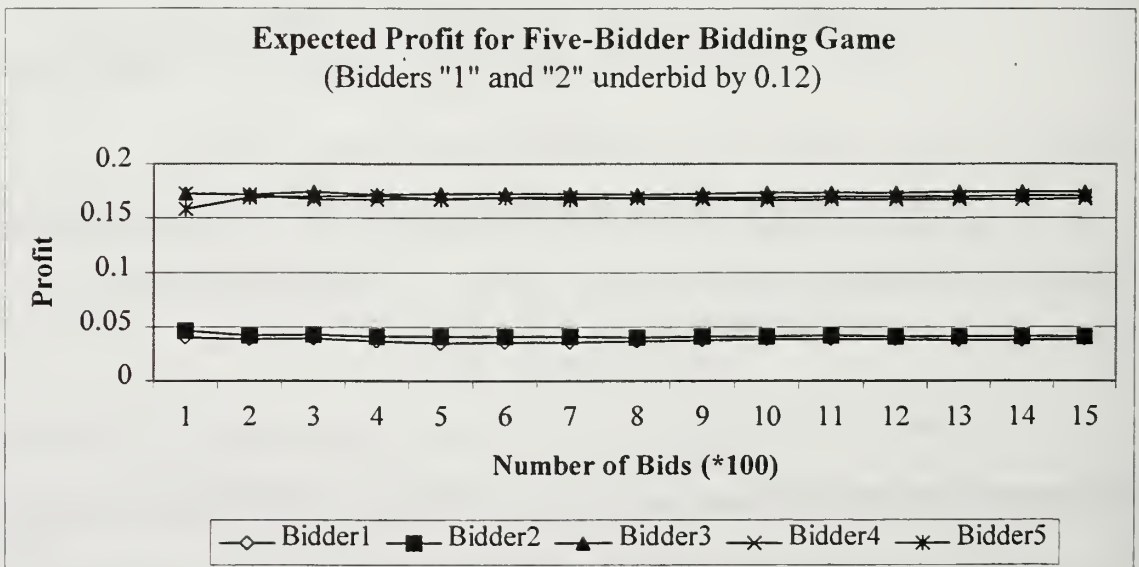


Figure 25

The average profit scheme resulted the same; bidders "1" and "2" made 0.011 while others did 0.024, 0.0245 and 0.0215 respectively, as seen in Figure 26.

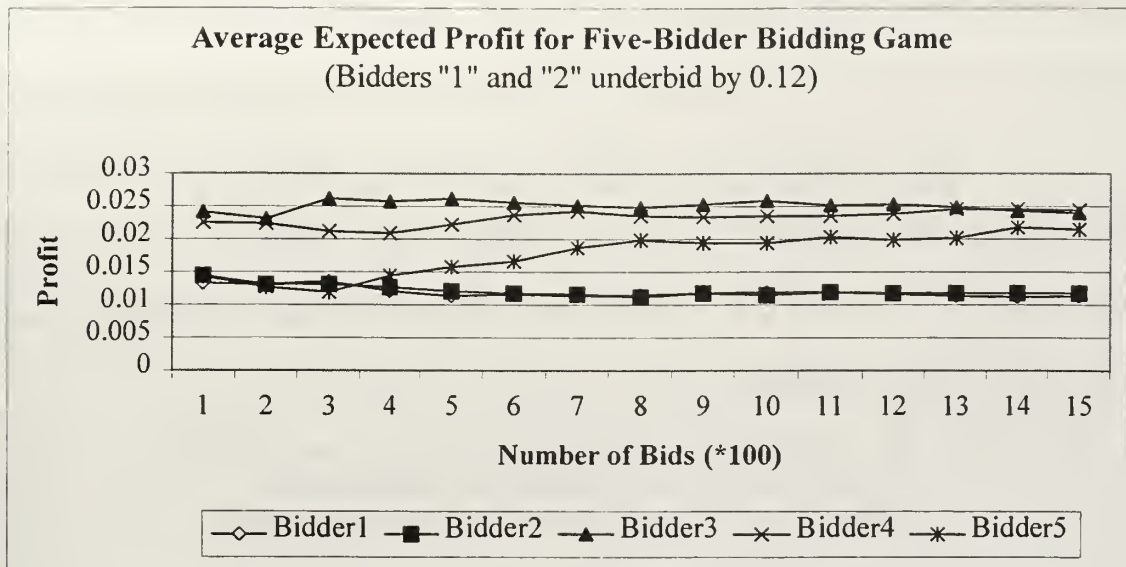


Figure 26

Total profit for bidders "1" and "2" also were lower than the others' profits. They made 16.992 and 17.737 respectively, while others profit ranged from 32.21 to 36.711. Total profit curves for bidders are shown in Figure 27.

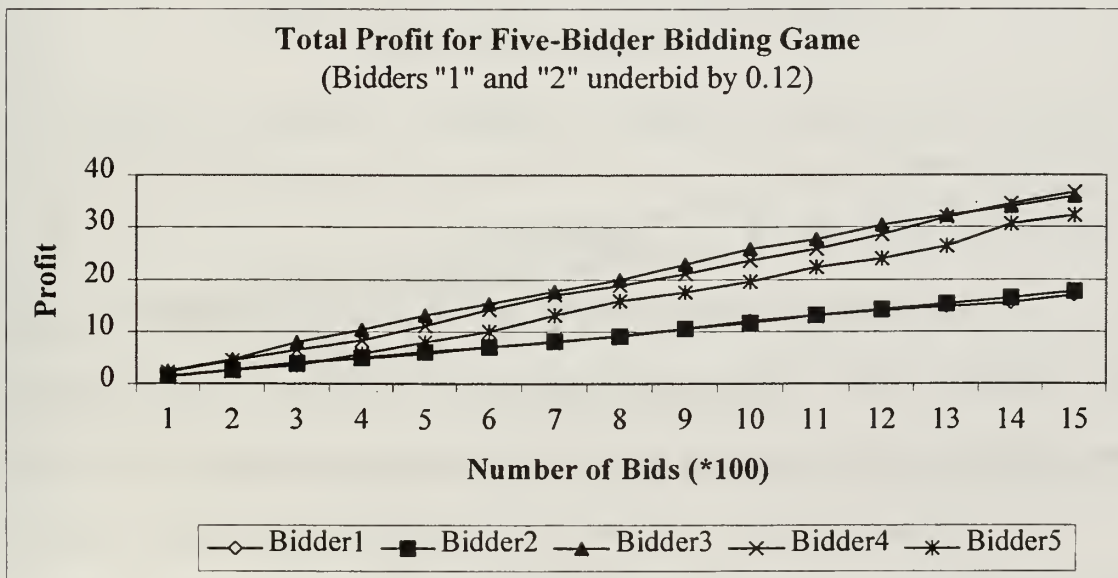


Figure 27

Bidders "1" and "2" won 449 and 436 games out of 1500 games. The rest won the games 189-219 times. As it was the case in previous experiments, underbidding

bidders made lower profits, although they won more games than the equilibrium strategy bidders. Overall results of the experiment are shown in Tables 11 and 12 in Appendix C.

In the last scenario, bidders “1” and “2” underbid by 0.12 and 0.1 respectively while others used equilibrium strategy. The underbidding bidders made lower profits of 0.03781 and 0.06149 respectively while others made 0.169. The expected profit graph is shown in Figure 28.

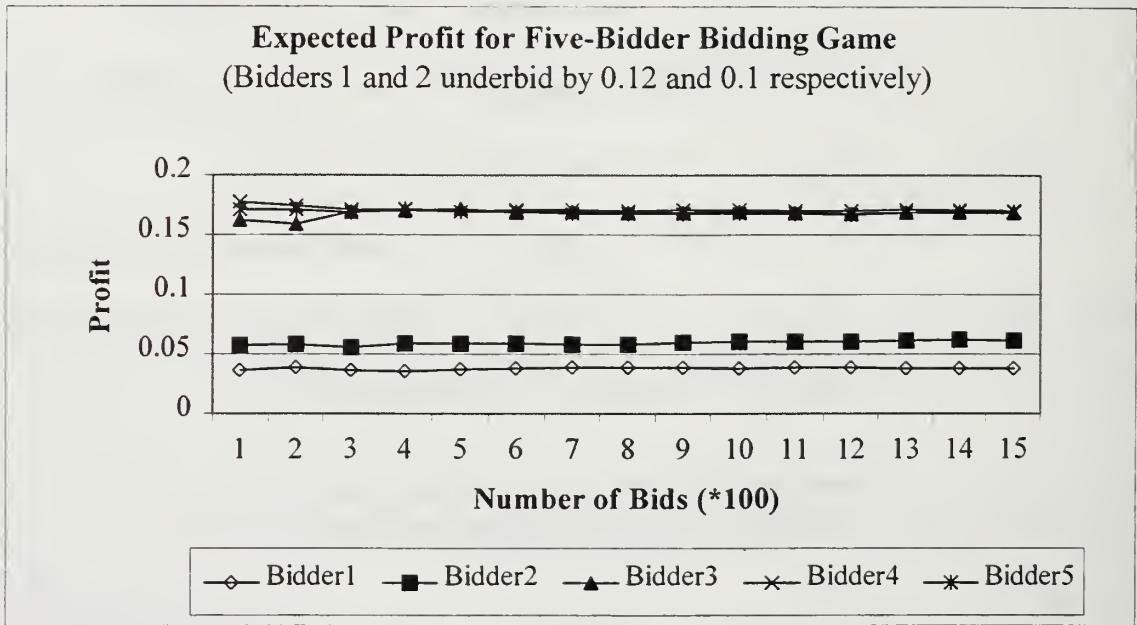


Figure 28

The average profit for bidders “1” and “2” were 0.01129 and 0.0169 while others’ approached almost 0.024 in 1500 games as seen in Figure 29. The equilibrium strategy bidders made higher expected and average profits than underbidding bidders did.

Bidders “1” and “2” won 448 and 414 games while the others won very close number of games ranging from 209 to 218 as seen in Table 13 in Appendix C. Despite bidders “1” and “2” won much more games than others, they made lower total

profits in 1500 games. The lowest bidder, bidder1, made the lowest total profit, 16.9386. The second lowest total profit was bidder2's, 25.4564.

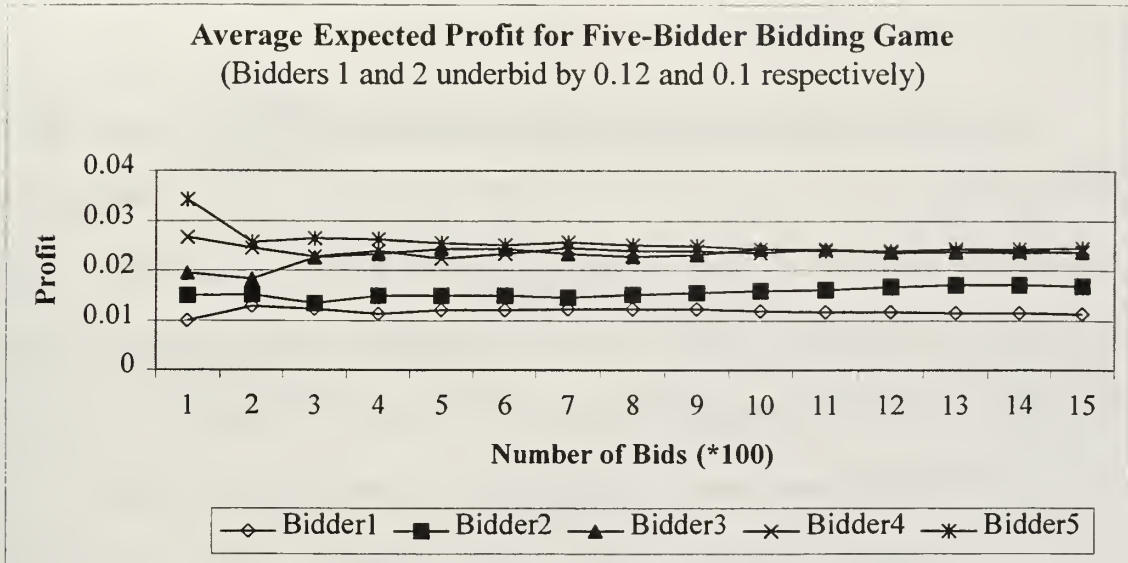


Figure 29

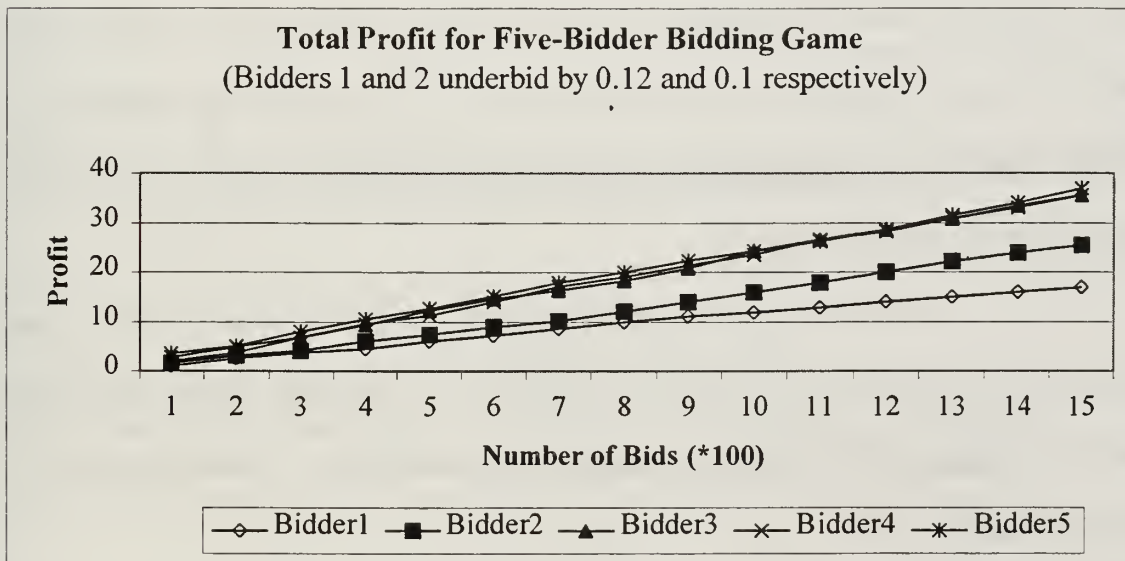


Figure 30

Finally, equilibrium strategy bidders made very close total profits ranging from 35.57 to 36.84. Total profit curves are shown in Figure 30. Results of the experiment are shown in Tables 13 and 14 in Appendix C.

d. Experimentation With Ten Bidders

The simulation was conducted with ten bidders according to uniform cost distribution within interval $[0, 1]$.

In these series of experiments, four different scenarios used to simulate the bidding process. In the first scenario, all bidders used the equilibrium strategy. The second one was conducted as bidder1 underbid while others used the equilibrium strategy again. In the third one, both bidders “1” and “2” underbid by the same amount while all others used the equilibrium strategy. Finally, in the last scenario, bidders “1”, “2”, “3” and “4” underbid by different amounts while the rest used the equilibrium strategy.

The first experiment, which all used equilibrium strategy, resulted in a well-defined equilibrium. All bidders’ expected profit approached approximately 0.091 as the number of games approached 1500. The graphical representation of the expected profit is shown in Figure 31. As can be seen in the overall results of the experiment in Tables 15 through 18 in Appendix C, introducing more bidders to the game reduced the expected profit from bidding more.

The average expected profit graph is shown in Figure 32. The average expected profit graph also reinforces the equilibrium of the game. In this simulation, bidders’ average expected profit approaches approximately 0.009 while the number of games approaches 1500.

As seen in Table 16 in Appendix C, each bidder won the games approximately equal number of times. Overall, bidders won in a range of 139 to 162 games. Total profits as seen in Figure 33, were almost the same, ranging between 12.6196 and 14.7461.

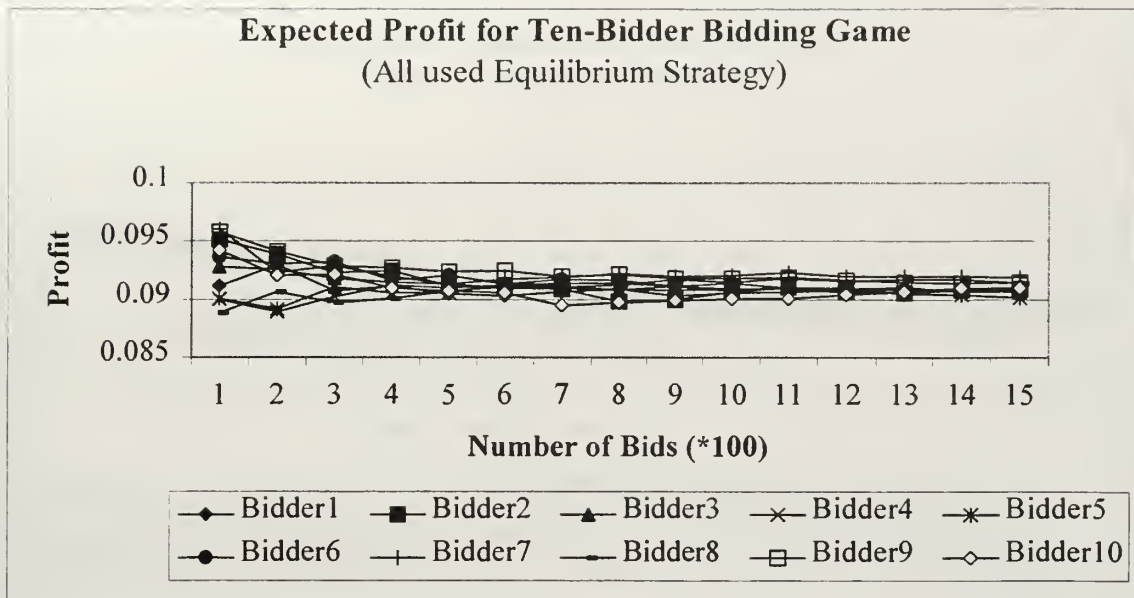


Figure 31

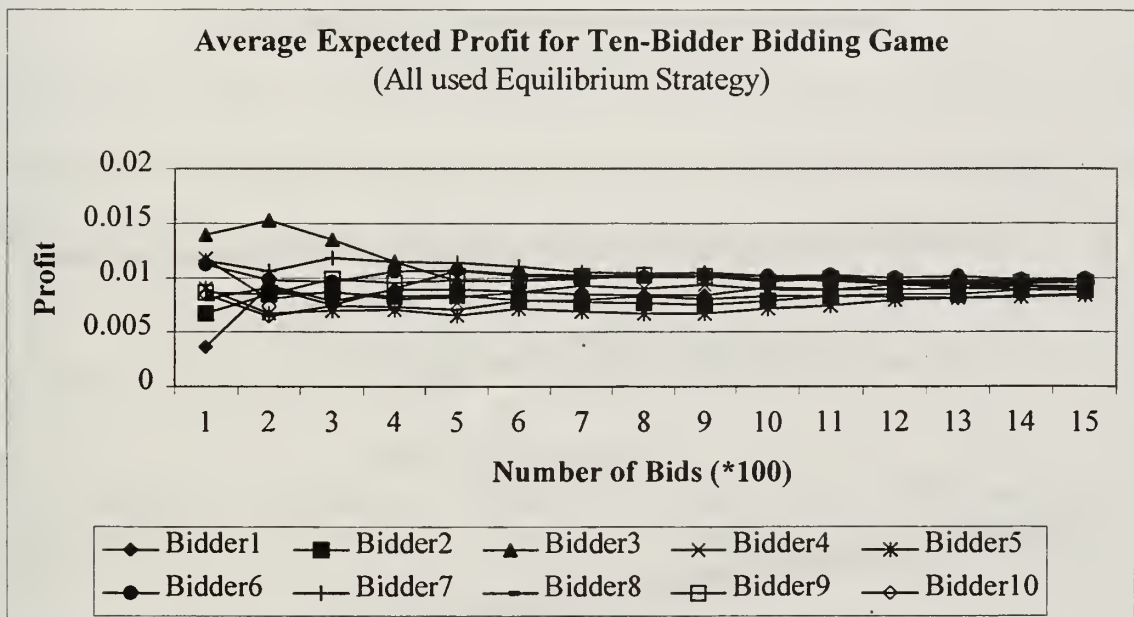


Figure 32

In the next experiment, bidder1 underbid by 0.0825, which was 15 percent of the average bid price, while others bid according to the equilibrium strategy. As a result of 1500 games, bidder1's expected profit decreased to 0.057 while other bidders' remained at 0.091. The expected profit graph is shown in Figure 34.

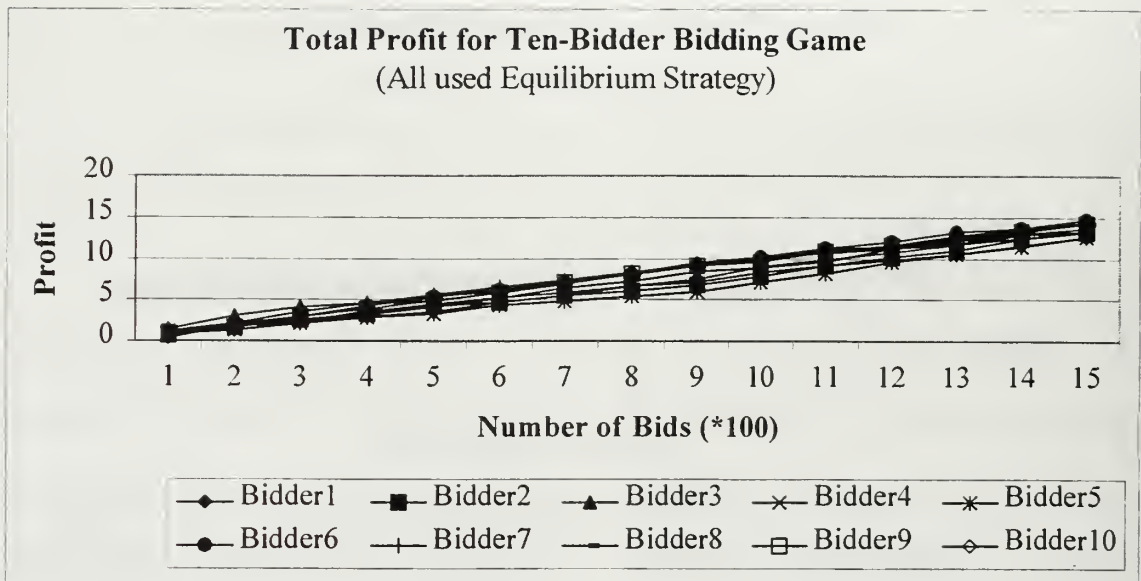


Figure 33

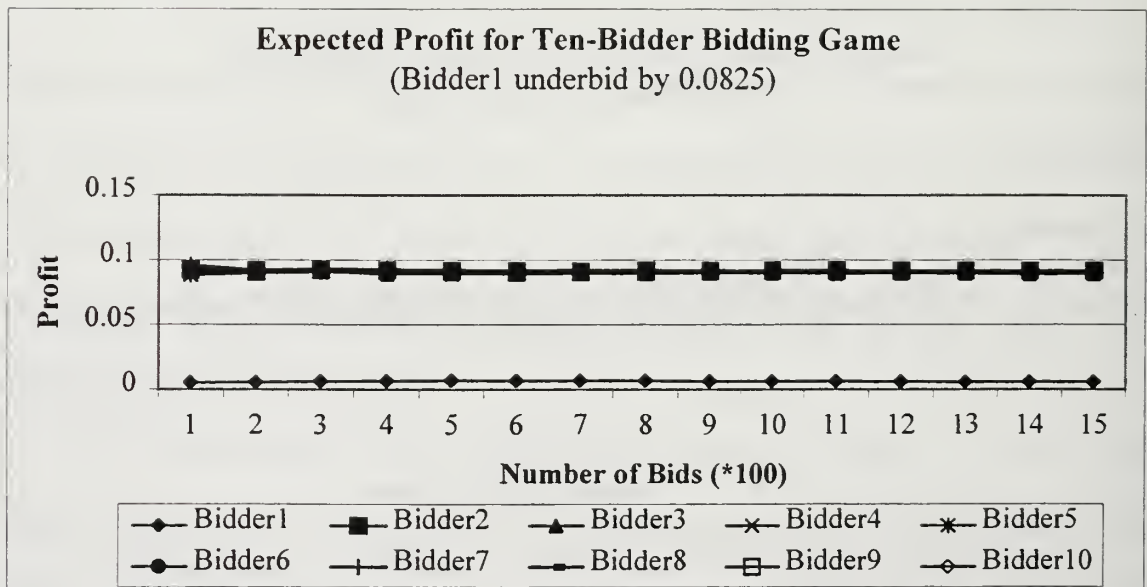


Figure 34

Average profit for the bidders also reinforced the same scheme as the equilibrium game had shown. Bidder1's average profit was 0.0011 while others were very close, approximately 0.008, as graphically shown in Figure 35. Overall results of the experiment are outlined in Tables 19 through 21 in Appendix C.

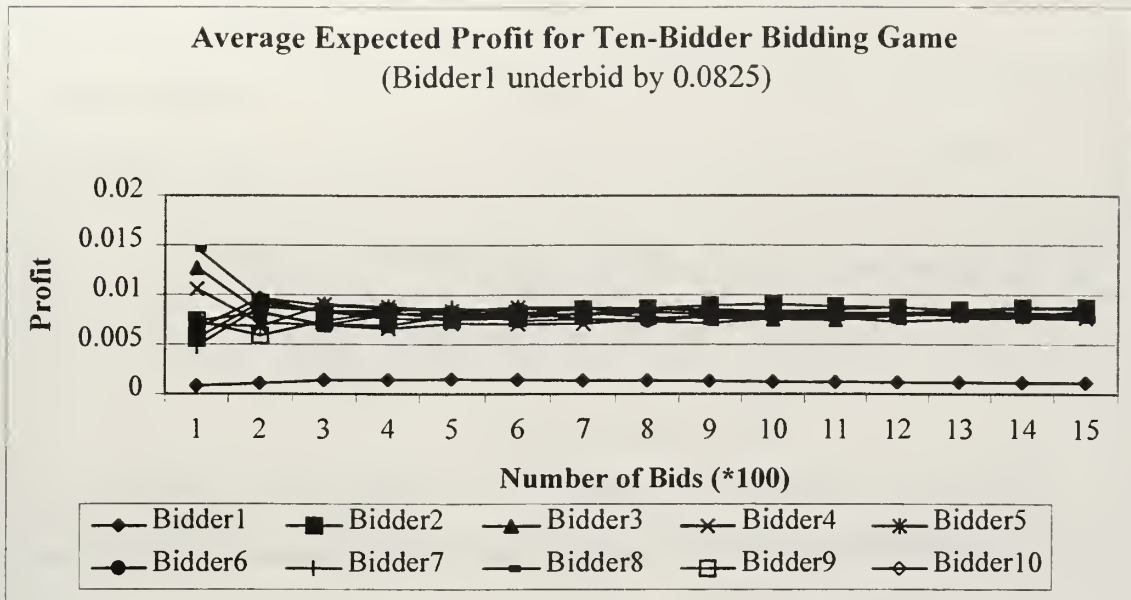


Figure 35

Bidder1 won 296 of 1500 games while others won in a range of 124 to 143 games as shown in Table 20. Although bidder1 won more games than the others won, its total profit was 1.6872 which is much lower than the others' profits ranging from 11.3013 to 13.1131. Total profit graph for this experiment is shown in Figure 36:

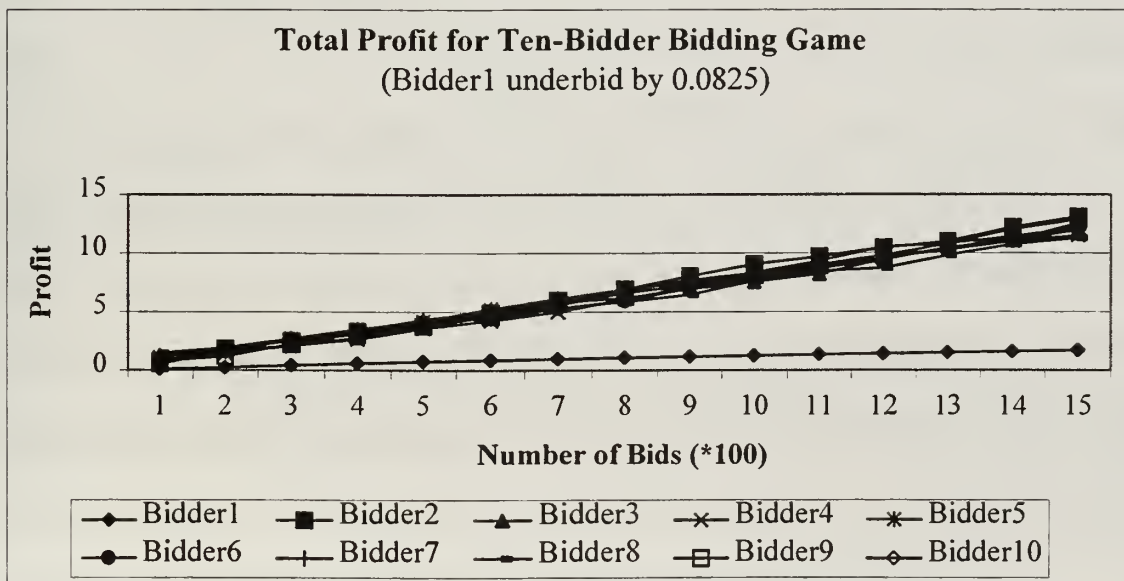


Figure 36

In the third experiment, bidders “1” and “2” underbid by 0.0825 while others bid according to the equilibrium strategy. As a result of 1500 games, bidders “1” and “2” made expected profits, 0.055 and 0.059, while the other bidders’ profits remained at 0.091. The expected profit graph is shown in Figure 37.

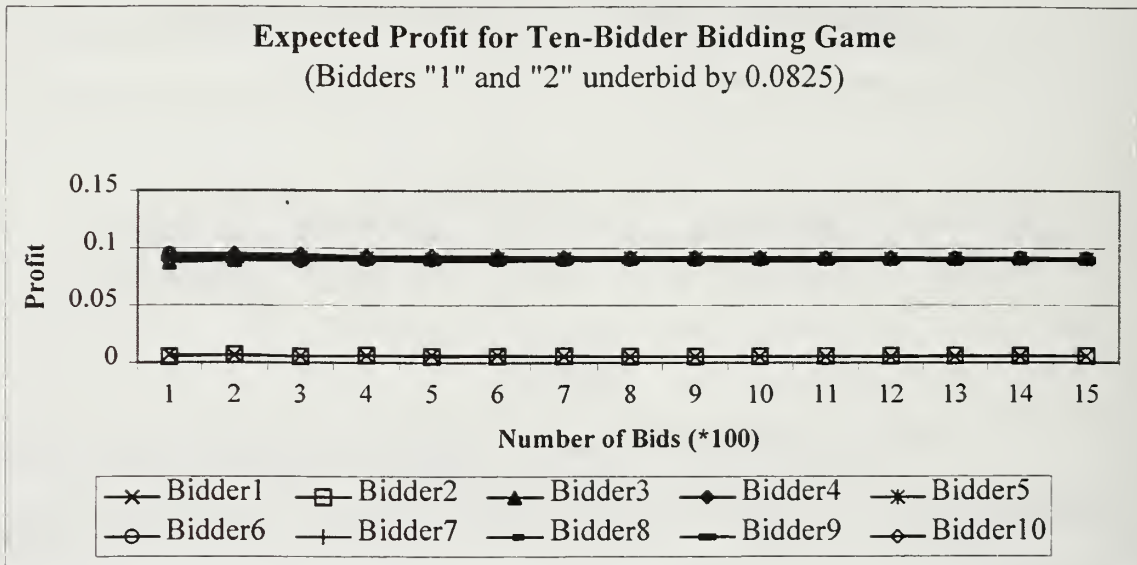


Figure 37

Average profit for the bidders also reinforced the same scheme as the expected profit results had shown. Underbidding bidders’ average profits were 0.001 while others were ranging from 0.0064 to 0.0082, as graphically shown in Figure 38.

Bidders “1” and “2” won 264 and 259 games respectively while others won the games in a range of 109 to 135 times as shown in Table 24 in Appendix C. Total profits for underbidding bidders were 1.4591 and 1.5343 which are much lower than the others’ profits ranging from 9.7221 to 12.3395. Total profit graph for this experiment is shown in Figure 39.

The overall results of the experiment are shown in Tables 23 through 26 in Appendix C.

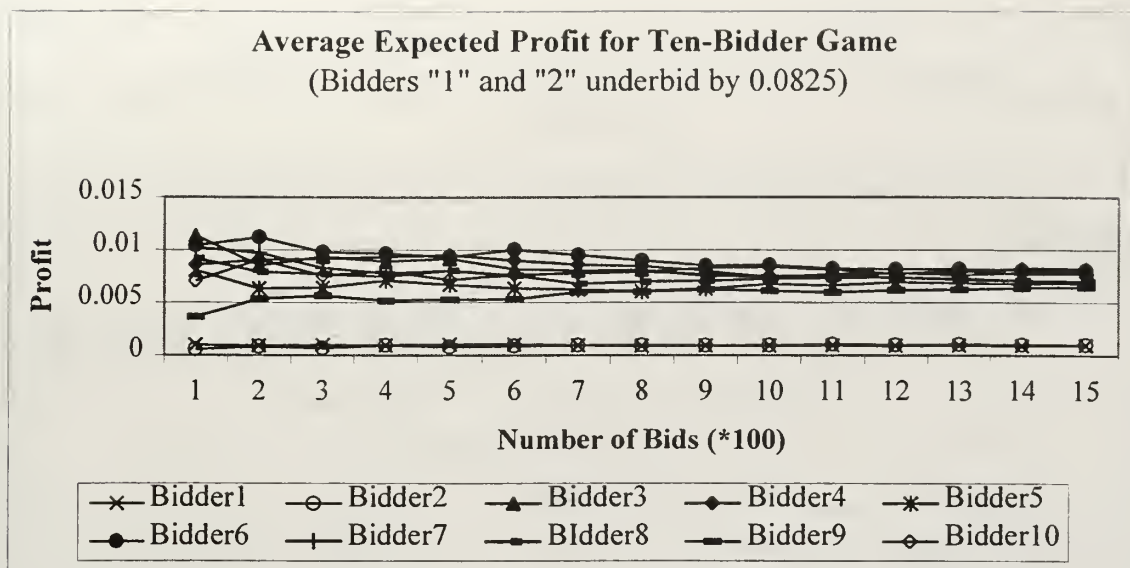


Figure 38

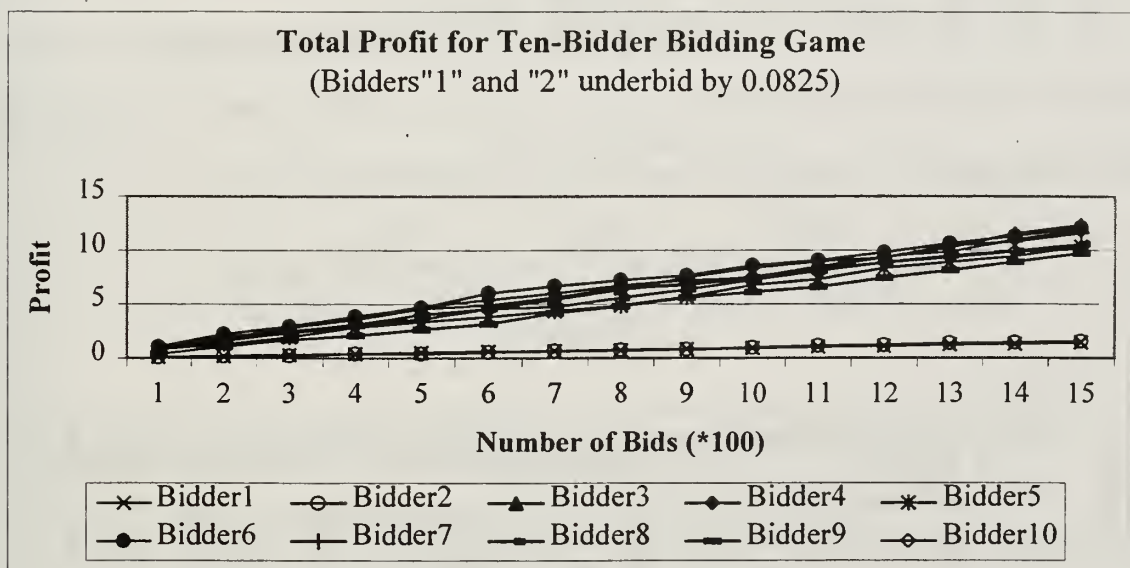


Figure 39

In the last experiment, bidders "1", "2", "3" and "4" underbid by 0.0825, 0.075, 0.0675 and 0.06 respectively in 1500 games. The remaining six bidders used equilibrium strategy. The underbidding bidders made lower profits, 0.0063, 0.0148, 0.0229 and 0.03, than the equilibrium strategy bidders did, which was 0.091. The expected profit curves are shown in Figure 40.

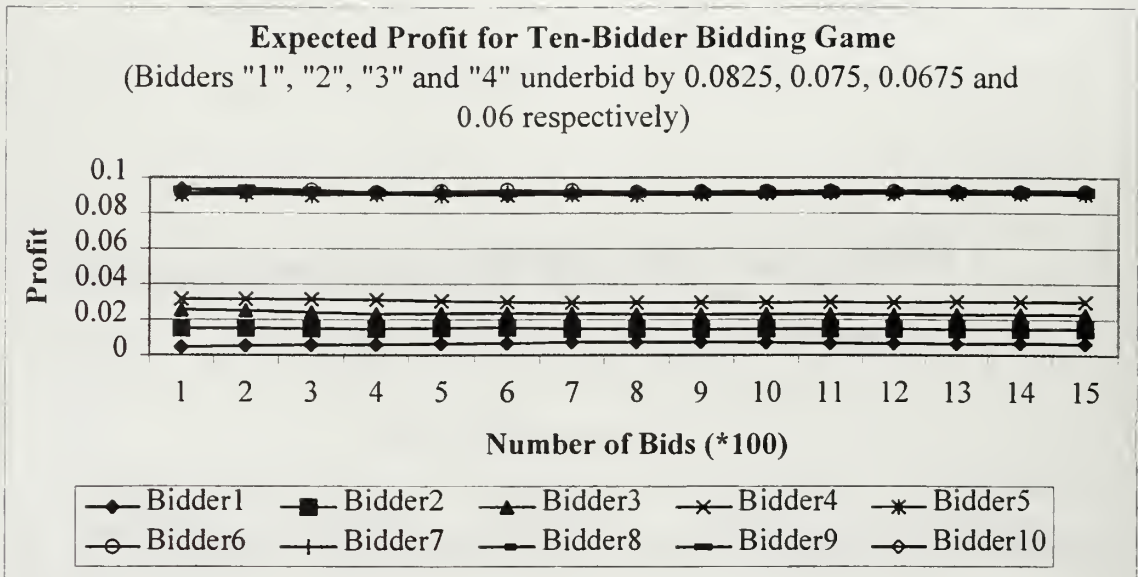


Figure 40

The equilibrium strategy bidders made higher average profits as seen in Figure 41. The underbidding bidders average profits were 0.00098, 0.00234, 0.00328 and 0.004 while equilibrium strategy bidders' ranged from 0.0053 to 0.0073.

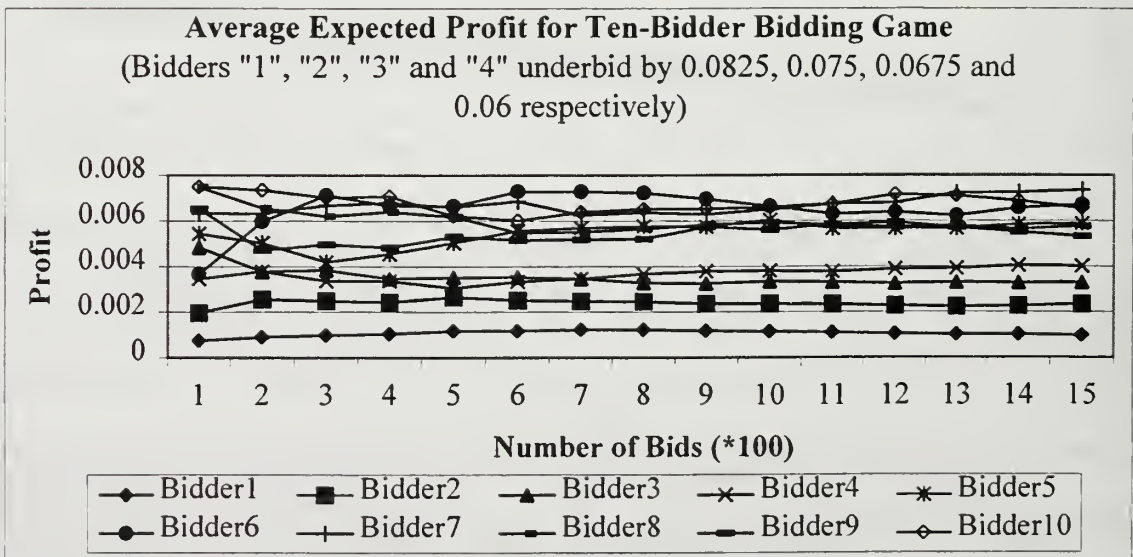


Figure 41

The underbidding bidders won more games than the others did. They won the games in a range of 200 to 233 times while equilibrium strategy bidders won in a

range of 88 to 121 times. Despite this result, the equilibrium bidders made higher total profits than underbidding bidders. The underbidding bidders' total profits ranged from 1.4674 to 6.0083 while others' ranged from 7.9767 to 11.0308. Total profit curves are shown in Figure 42. Overall results of the experiment are written in Tables 27 through 30 in Appendix C.

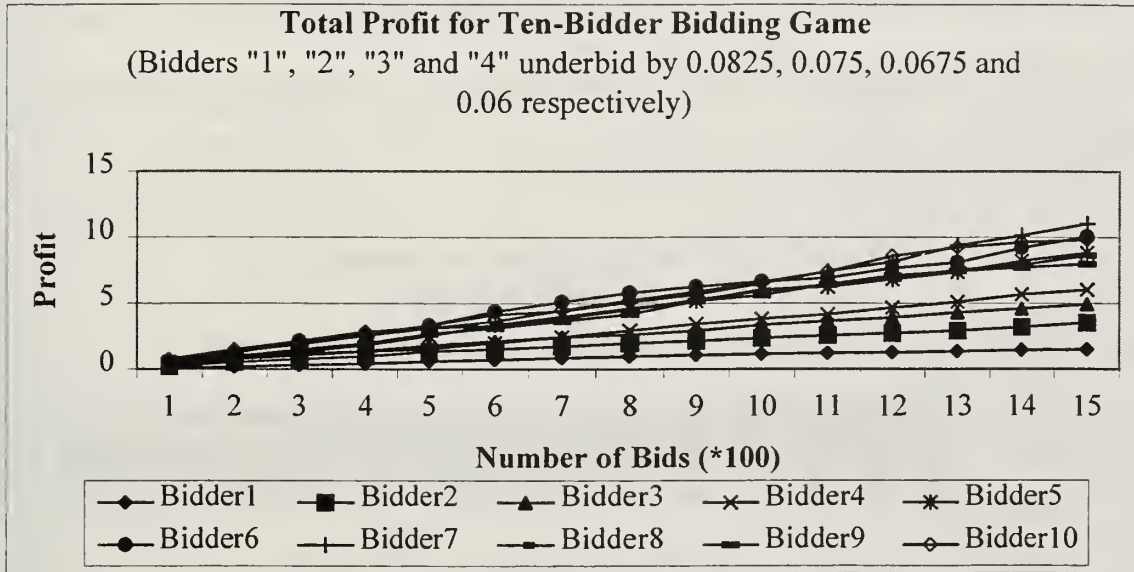


Figure 42

e. Experimentation With Fifteen Bidders

The simulation was conducted with fifteen bidders according to uniform cost distribution within interval $[0, 1]$.

In these last series of experiments under uniform cost distribution, four different scenarios used to simulate the bidding process. In the first scenario, all bidders used the equilibrium strategy. In the second one, bidder1 underbid while others used the equilibrium strategy again. In the third one, bidders "1", "2" and "3" underbid by the same amount while all others used the equilibrium strategy. Finally, in the last scenario,

bidders “1”, “2”, “3” and “4” underbid by different amounts while the rest used the equilibrium strategy.

The first experiment of which all used equilibrium strategy resulted in a well-defined equilibrium. All bidders’ expected profit approached approximately 0.062 as the number of games approached 1500. The graphical representation of the expected profit is shown in Figure 43. As it was the result of the previous experiments, introducing more bidders to the game reduced the expected profit from bidding more. The results are outlined in Tables 31 through 34 in Appendix C.

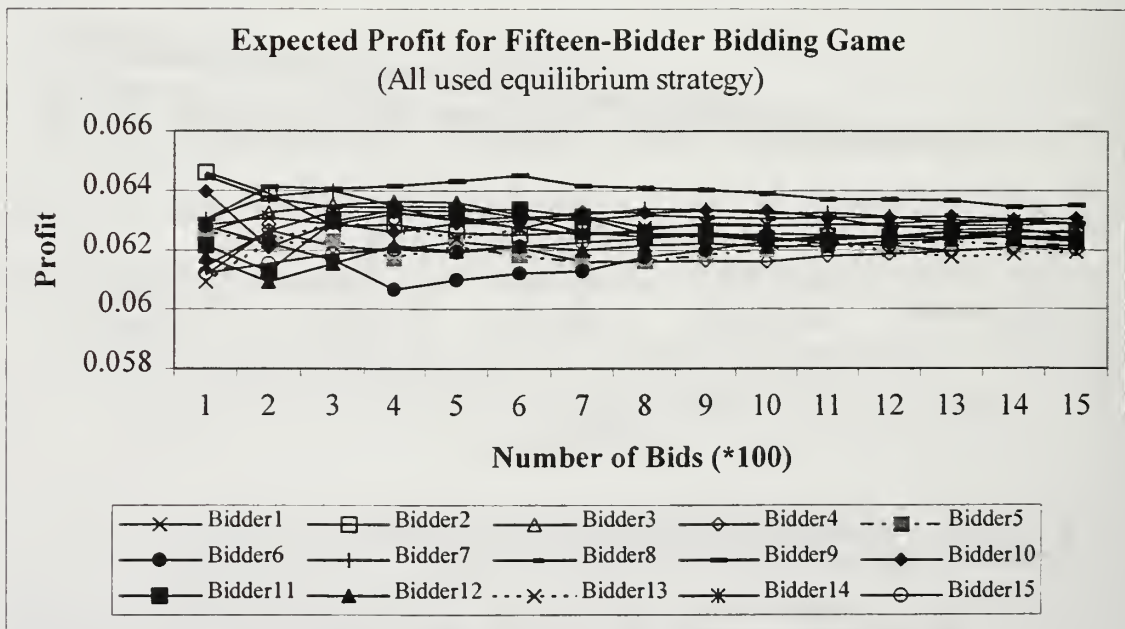


Figure 43

The average expected profit graph is shown in Figure 44. The average expected profit results also reinforce the equilibrium of the game. In this simulation, bidders’ average expected profit approaches 0.004 while the number of games approaches 1500.

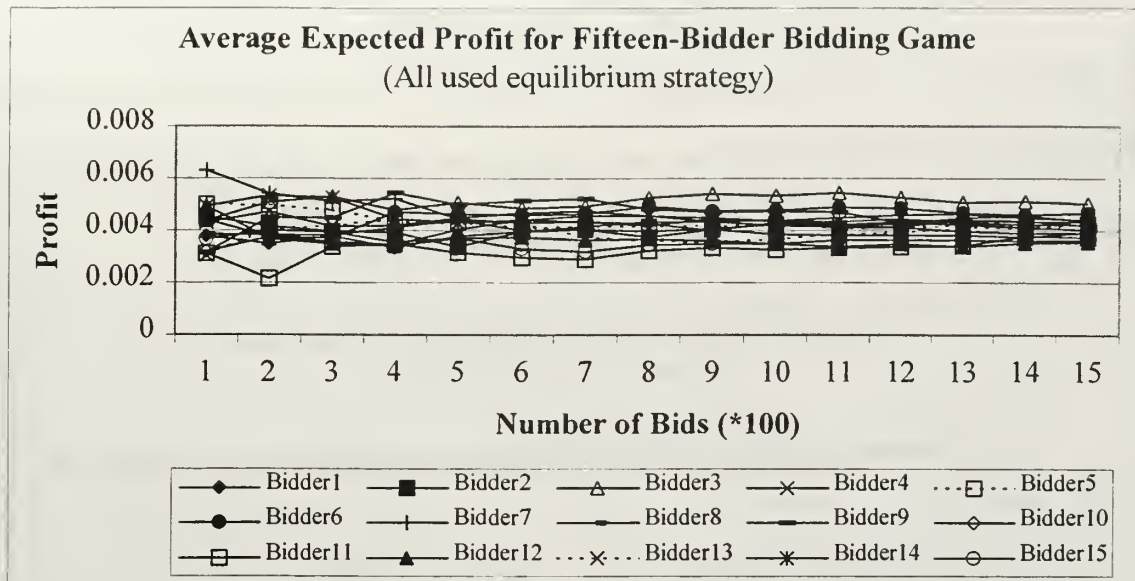


Figure 44

As seen in Table 32, each bidder won the games approximately equal number of times. Overall, bidders won in a range of 86 to 121 games each. Total profit for bidders as seen in Figure 45, ranged between 5.361 to 7.578.

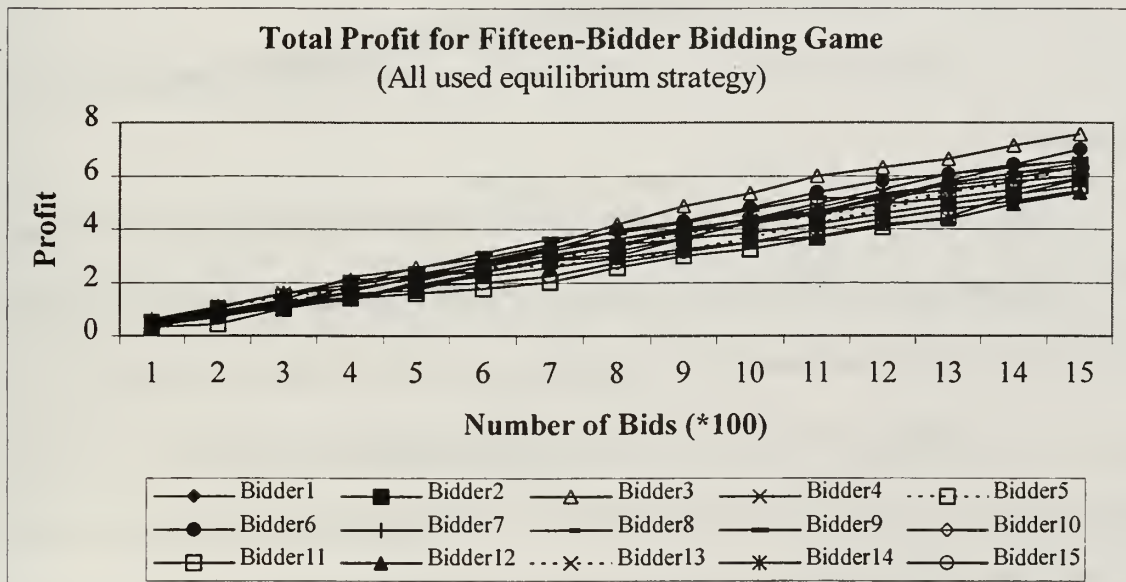


Figure 45

In the next experiment, bidder1 underbid by 0.075 while others bid according to the equilibrium strategy. As a result of 1500 games, bidder1's expected profit decreased to 0.0116 while the other bidders' remained at 0.062. The expected profit graph is shown in Figure 46.

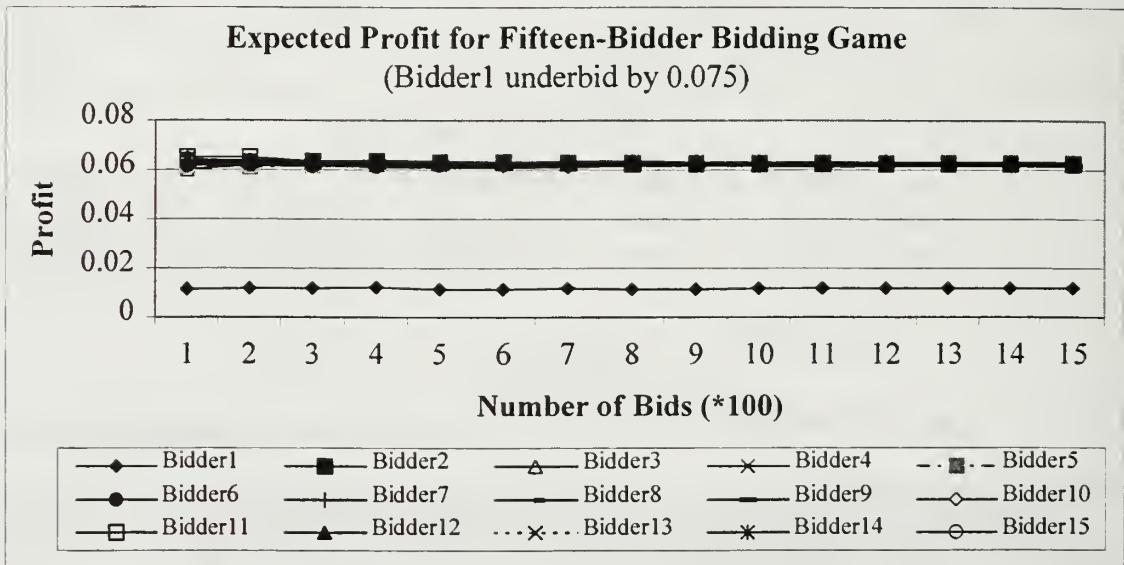


Figure 46

The overall results of the experiment are outlined in Tables 35 through 38 in Appendix C.

Average expected profit for the bidders also reinforced the same scheme as the equilibrium game had shown, except for bidder1. Bidder1's average profit was 0.00139 while others were almost the same, 0.004, as graphically shown in Figure 47.

Bidder1 won 179 of 1500 games while others won in a range of 69 to 109 games as shown in Table 36. Although bidder1 won more games than the others won, its total profit was 2.07798 which is quite lower than the others' profits ranging from 4.2818 to 6.8147. Total profit graph for this experiment is shown in Figure 48.

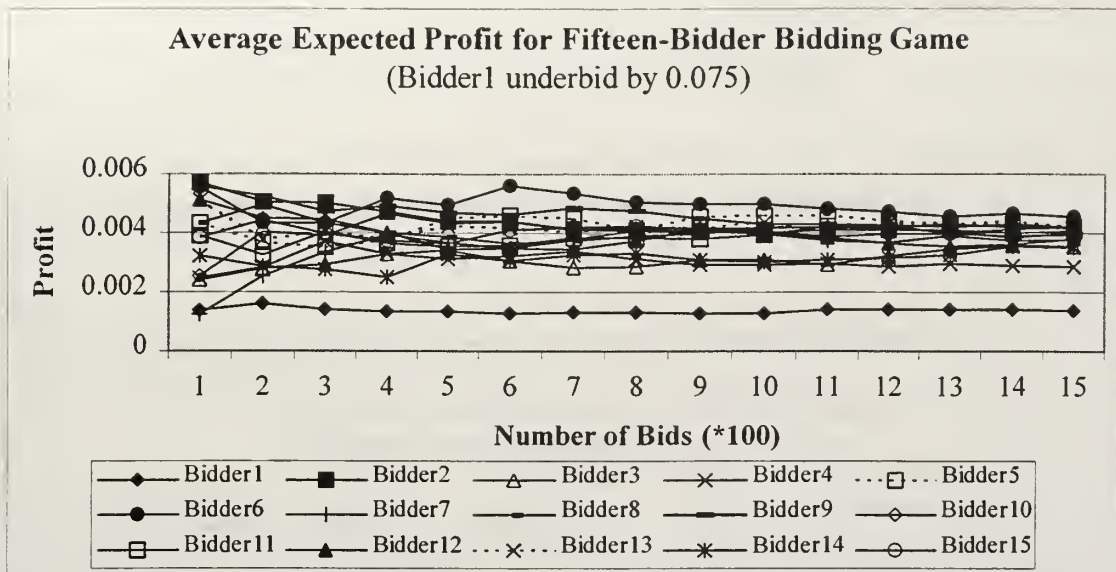


Figure 47

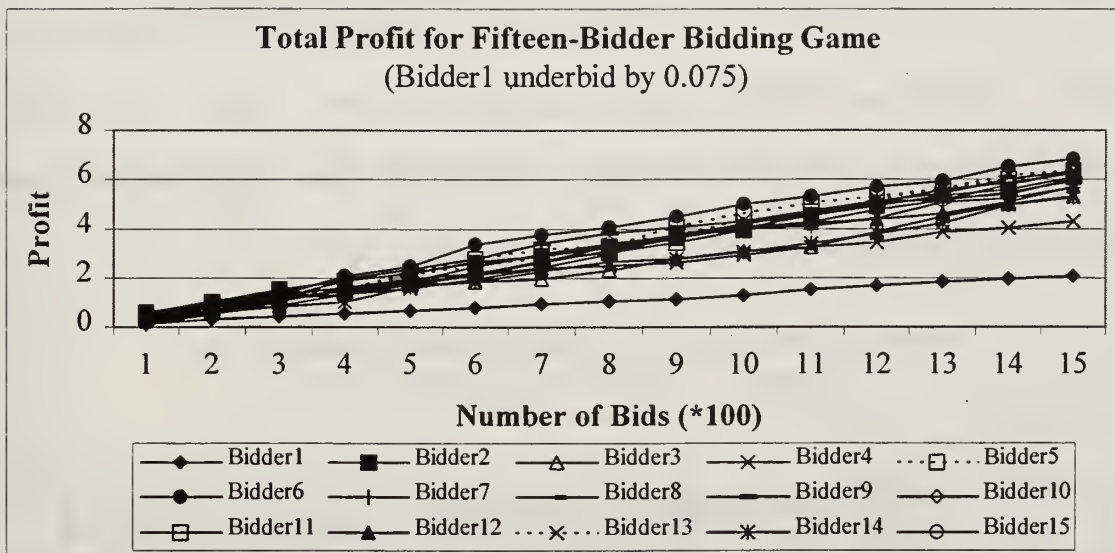


Figure 48

In the third experiment, bidders “1”, “2” and “3” underbid by 0.055 while others used equilibrium strategy in 1500 games. As a result of the experiment, underbidding bidders’ expected profit approached 0.0057 while others’ approached approximately 0.062. The expected profit graph is shown in Figure 49.

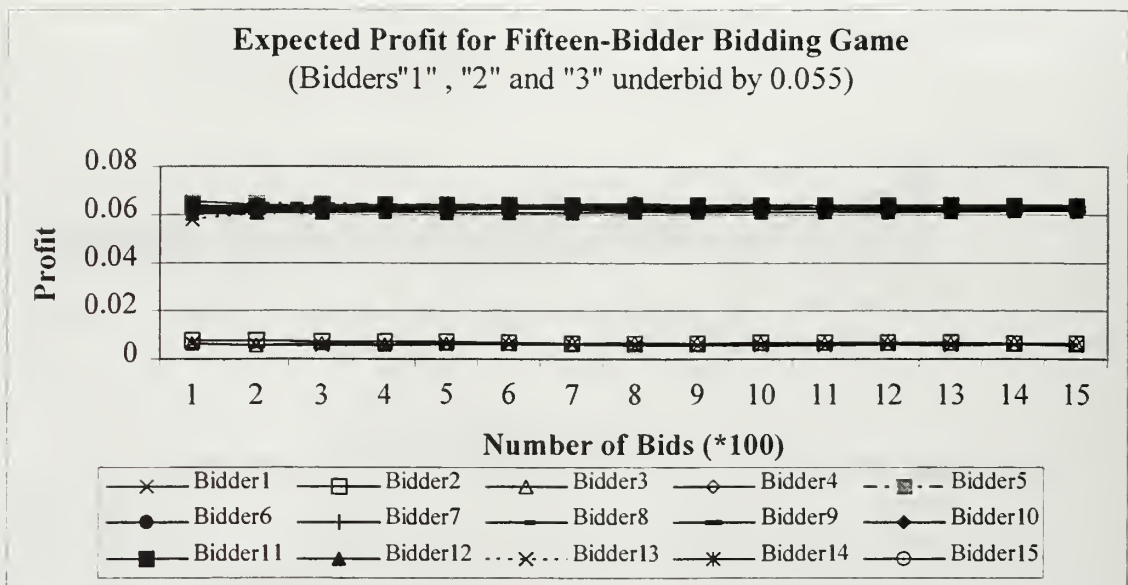


Figure 49

The average expected profit results were parallel to the expected profit results. Underbidding bidders' average profits approached 0.0007 while others' approached to a range of 0.0028 to 0.0041 in 1500 games. The average expected profit graph is shown in Figure 50.

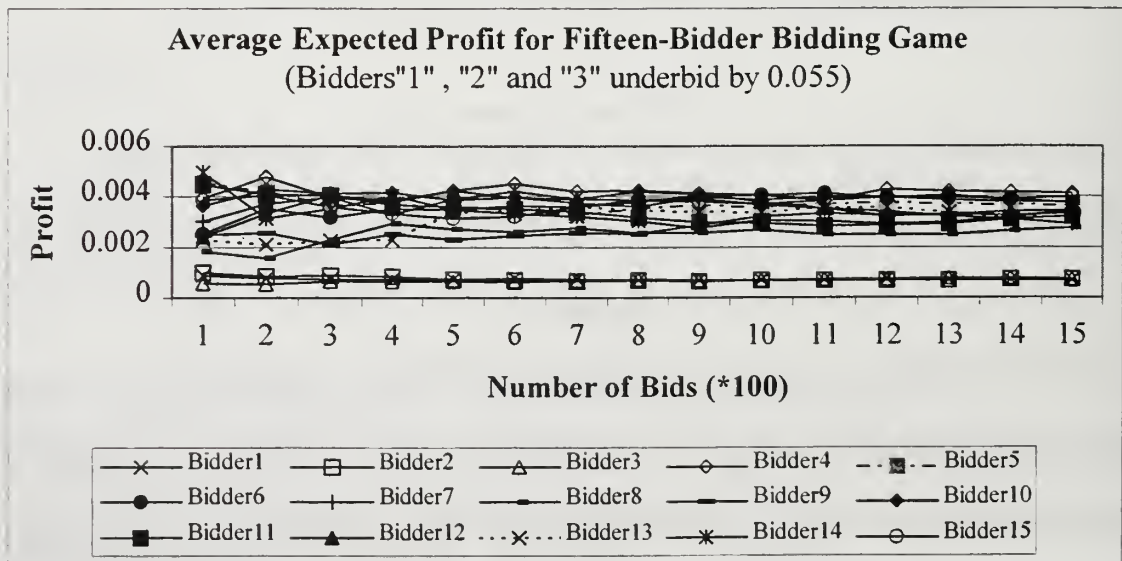


Figure 50

Total profits for underbidding bidders, ranging from 1.0074 to 1.0914, were also lower than the equilibrium strategy bidders' total profits, 4.1325 to 6.1872. The underbidding bidders won 163 to 176 games while others won 67 to 99 games as seen in Table 40 in Appendix C. The total profit curves are shown in Figure 51 and overall results of the experiment are shown in Tables 39 through 42 in Appendix C.

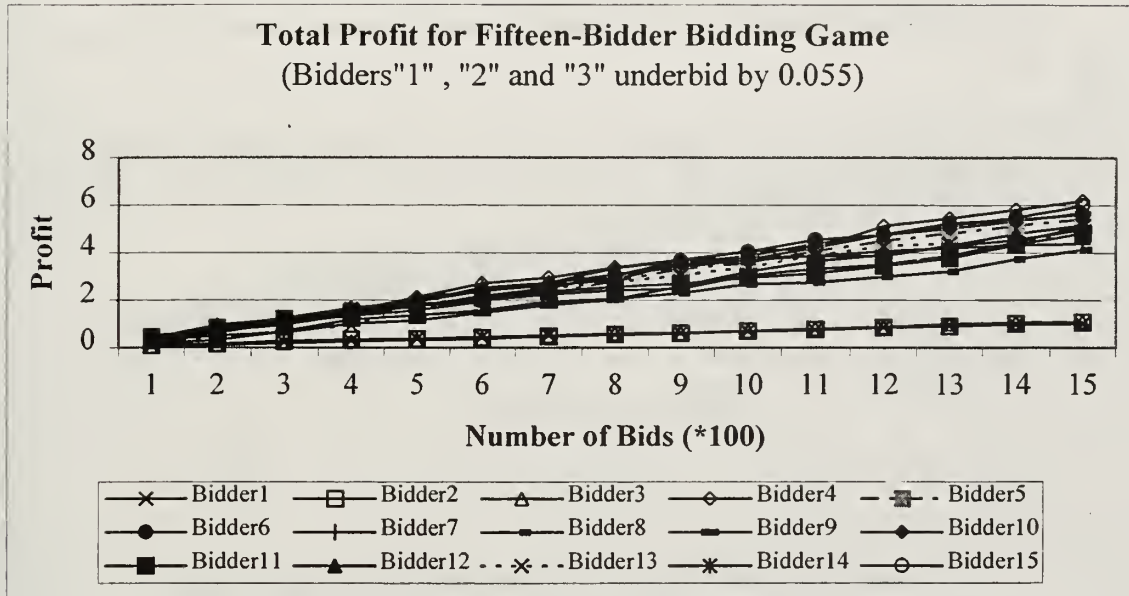


Figure 51

In the last experiment with fifteen bidders, bidders "1", "2", "3" and "4" underbid by 0.055, 0.05, 0.045 and 0.04 respectively, while others used equilibrium strategy in 1500 games. As a result of the experiment, underbidding bidders' expected profit approached 0.006, 0.0115, 0.0168 and 0.022 respectively, while others' approached approximately 0.063. The expected profit graph is shown in Figure 52.

The average expected profit results were parallel to the expected profit results. Underbidding bidders' average profits approached 0.0063, 0.00119, 0.00182 and 0.00229 while others' approached a range of 0.0028 to 0.004 in 1500 games. This result along with all previous experiments showed that there is indeed a strong equilibrium in

contract games. The deviant bidders lose profit even if they win more games than the equilibrium strategy bidders. The average profit graph for this experiment is shown in Figure 53.

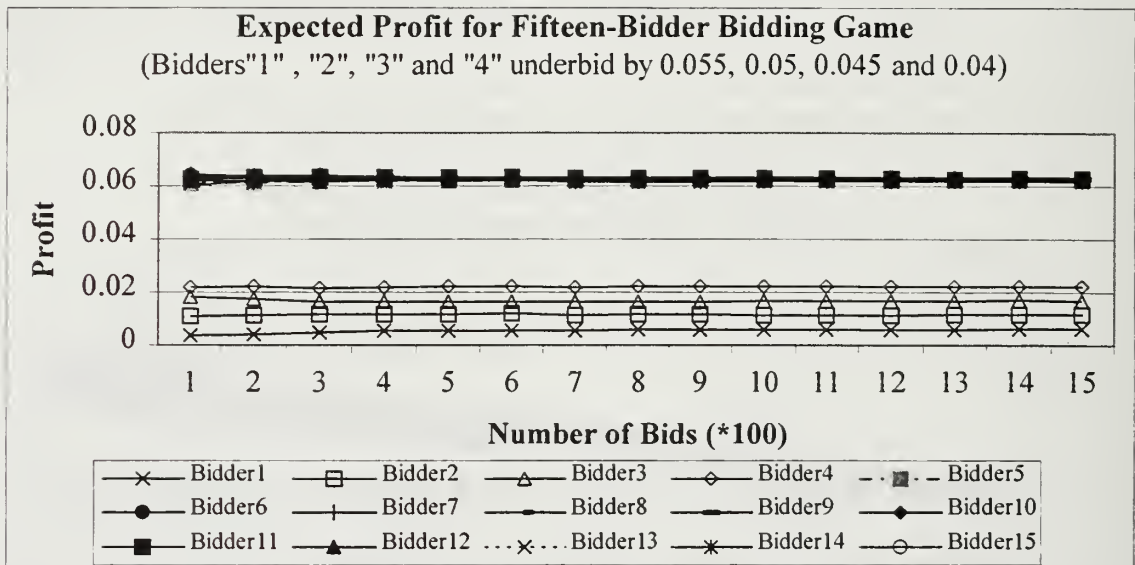


Figure 52

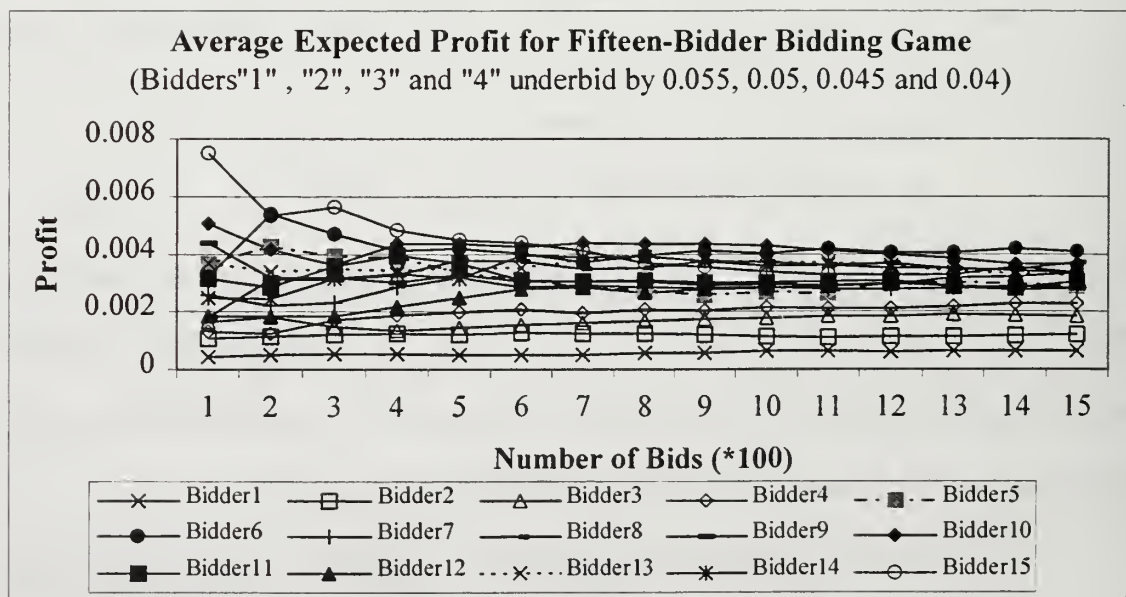


Figure 53

Total profits for underbidding bidders, ranging from 0.9441 to 3.4353, were also lower than the equilibrium strategy bidders' total profits, 4.2041 to 6.1235. The underbidding bidders won 155 to 162 games as opposed to others, which won 67 to 97 games as seen in Table 44. Total profit curves are shown in Figure 54.

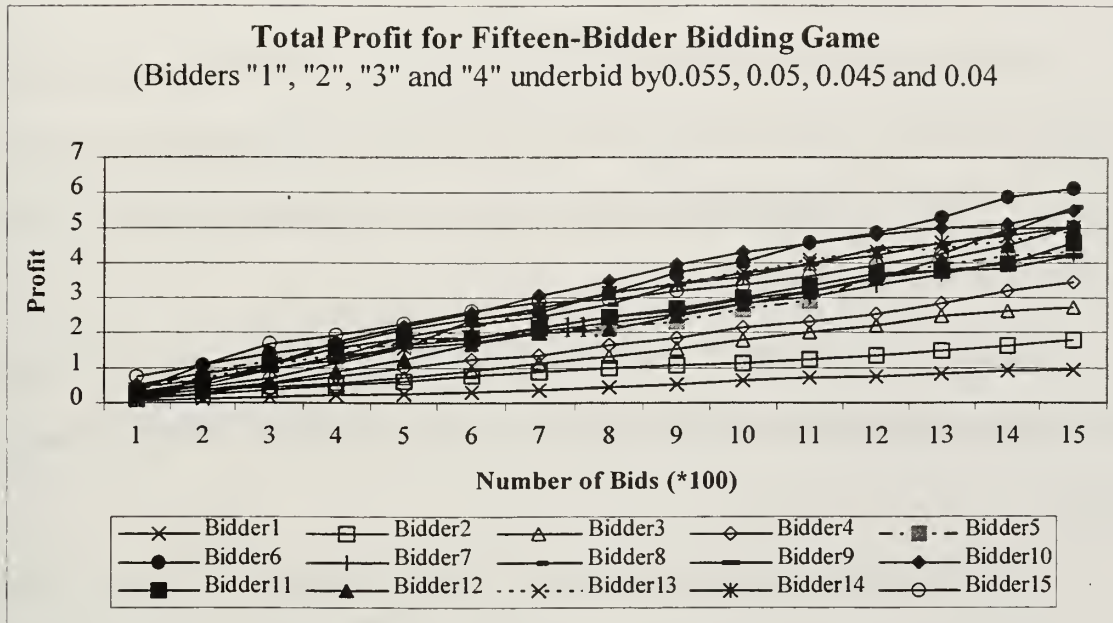


Figure 54

The overall results of the experiment are shown in Tables 43 through 46 in Appendix C.

C. EXPERIMENTATION UNDER TRIANGULAR COST DISTRIBUTION

This section of the chapter describes the FPSB simulation when the production costs are distributed according to triangular distribution. The FPSB simulation using a triangular production cost distribution defines the bidders' behavior under a different cost condition. The uniform cost distribution was primarily used for simplicity, though it was reasonable when bidders' production costs were identical due to the similar production

methods used. Triangular distribution may be a better reflection of reality when some bidders have competitive advantage over the others because of better production techniques, management controls or different technologies used for the product.

1. Mathematical Model for Simulation

For products for which relatively new, more complex and different manufacturing methods are used, the costs of production vary according to the manufacturers' business practices. Triangular distribution, then, becomes a closer representation of costs associated with that product.

Assume that the bidders' potential production costs are distributed according to the triangular distribution over an interval $[h, k]$ with a mode value, m , with probability $2/(h-k)$.

A bidding strategy defines the relationship between the bidder's proposal, b_i , and cost, c_i . The following section is the derivation of the bidding function under uniform cost distribution.

2. General-Form Bidding Function for the Triangular Cost Distribution

The probability density function, $f_X(x)$, of the triangular distribution is shown as:

$$f_X(x) = \begin{cases} [2*(x-k)]/[(h-k)*(m-k)] & \text{for } k < x < m \\ [2*(h-x)]/[(h-k)*(h-m)] & \text{for } m < x < h \\ 0 & \text{otherwise} \end{cases}$$

Where $k < m < h$ and,

k : the lower limit of the triangular distribution.

m: the mode of the distribution.

h: the upper limit of the triangular distribution.

The graphic representation of the triangular distribution is shown in Figure 55:

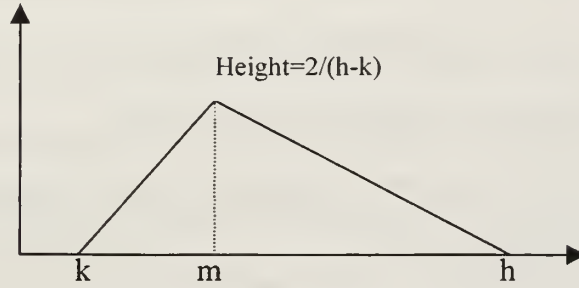


Figure 55 PDF of the triangular distribution

The cumulative distribution function, $F_X(x)$, of the triangular distribution is:

$$F_X(x) = \begin{cases} 0 & \text{for } x < k \\ [(x-k)^2]/[(h-k)*(m-k)] & \text{for } k < x < m \\ [1-(h-x)^2]/[(h-k)*(h-m)] & \text{for } m < x < h \\ 1 & \text{for } x > h \end{cases}$$

Assume players 1, 2, 3, ..., n-1, n adopt the strategy $b(\cdot)$, and $b(\cdot)$ is strictly increasing and differentiable. For a given value of the i^{th} player's cost, i^{th} player's optimal bidding strategy solves:

$$\text{Max}\{(b_i - c_i) * \text{Prob}[b_i < b(c_1), \dots, b_i < b(c_n)]\} \text{ where } n: \text{ number of bidders.}$$

The probability of a given player i 's bid to be the lowest of all is:

$$\text{Prob}(b_i < b_{i+1}, \dots, b_i < b_n) = [1 - b^{-1}(b_i)]^{n-1}$$

The first order condition for player i 's optimization problem is then:

$$d\{(b_i - c_i) * [1 - b^{-1}(b_i)]^{n-1}\} / db_i = 0$$

The triangular distribution has two different distributions over the interval $[k, h]$, that is, the first one is from the lower limit, k , through the mode, m , and the second one is from m through the upper limit, h , of the distribution. Because of this unique characteristic, the triangular distribution of the bidders' costs may be a better representative of the costs of items when the manufacturing methods are complex and the bidders use different techniques to manufacture those items.

This section of the thesis will define the bidding functions according to the interval costs fall into over $[k, h]$.

a. The Bidding Function for the cost interval $[k, m]$:

The bidders' expected profit, $E[\pi]$, has a maximum if the derivative of the expected profit with respect to b equals to zero. That is:

$$d(E[\pi])/db=0$$

The expected profit is:

$$E[\pi]=(b-c)*[1-F_X(x)]^{n-1}$$

By taking the derivative of expected profit with respect to b and then equal it to zero, we are going to find the bidding function that maximizes bidders' profits.

$$d(E[\pi])/db=0$$

$$d\{(b-c)*[1-F_X(x)]^{n-1}\}=0$$

$$d\{(b-c)*\{[1-(b-k)^2]/[(h-k)*(m-k)]\}^{n-1}\}/db=0$$

Let's use K instead of $[(h-k)*(m-k)]$ to make the calculations easier.

$$d\{(b-c)*\{[1-(b-k)^2]/K\}^{n-1}\}/db=0$$

$$\{[1-(b-k)^2]/K\}^{n-1} + \{(b-c)*(n-1)*\{[1-(b-k)^2]/K\}^{(n-2)}*[-2*(b-k)/K]\}=0$$

$$[K-(b-k)^2]^{n-1} + \{(b-c)*(n-1)*[K-(b-k)^2]^{(n-2)}*[-2*(b-k)]\}=0$$

by factoring out $[K-(b-k)^2]^{n-2}$:

$$K-(b-k)^2-2*(n-1)*(b-c)*(b-k)=0$$

$$K-(b^2-2*b*k-k^2)-2(n-1)*(b^2-c*b-b*k+c*k)=0$$

$$-[2*(n-1)+1]*b^2+2*[n*k+(n-1)*c]*b+[K-k^2-2*(n-1)*c*k]=0$$

$$-[(n-1)+0.5]*b^2+[n*k+(n-1)*c]*b+\{[K-k^2-2*(n-1)*c*k]/2\}=0$$

By solving the equation for b using the quadratic formula, we get the general formula for the bidding function under the triangular cost distribution when actual costs are in the interval $[k, m]$:

$$b = \frac{n*k+(n-1)*c + \{[n*k+(n-1)*c]^2 + 2*\{[(n-1)+0.5]*[K-k^2-2*(n-1)*c*k]\}^{0.5}}{2*[(n-1)+0.5]}$$

The bidding function is a function of number of bidders, cost and cost distribution functions. Using this general formula, we can derive all the necessary bidding functions where the cost is distributed triangularly and cost is in the interval $[k, m]$.

b. The Bidding Function For The Interval $[m, h]$

The bidders' expected profit, $E[\pi]$, has a maximum if the $dE[\pi]/db=0$.

$$E[\pi]=(b-c)*[1-F(x)]^{n-1}$$

$$dE[\pi]/db = d\{(b-c)*[1-\{1-(h-b)^2\}/[(h-k)*(h-m)]]^{n-1}\}/db=0$$

Let $T=(h-k)*(h-m)$ in order to simplify the derivation. The equation then becomes:

$$d\{(b-c)*[1-\{1-(h-b)^2\}/T]^{n-1}\}/db=0$$

$$\{1-[1-(h-b)^2]/T\}^{n-1}+(b-c)*(n-1)*\{1-[1-(h-b)^2]/T\}^{n-2}*[-2*(h-b)/T]=0$$

$$[-(h-b)^2/T]^{n-1} + (b-c)*(n-1)*\{-(h-b)^2/T\}^{n-2}[-2*(h-b)/T]=0$$

$$[-(h-b)^2]^{n-1} + (b-c)*(n-1)*[-(h-b)^2]^{n-2}[-2*(h-b)]=0$$

Factoring out $[-(h-b)^2]^{n-2}$ yields:

$$(h-b)^2 - [2*(n-1)*(h-b)*(b-c)]=0$$

Factoring out $(h-b)$ yields:

$$h-b-2*(n-1)*(b-c)=0$$

Solving for b gives the general formula for the bidding function when the cost distribution is triangular and the costs are within the interval $[m, h]$:

$$b = \frac{h + 2*(n-1)*c}{2*n-1}$$

3. Experimentation

During the experimentation, five different scenarios were used. In this scenarios, the number of bidders were changed to analyze the FPSB process and verify the accuracy of the simulation as well. The number of bidders used are two, three, five, ten and 15. Since the bidding function depends on the interval where the cost is distributed (i.e., either in $[k, m]$ or $[m, h]$), they take one of the either form of the below formulas using following parameters:

Lower limit of the triangular distribution, $k=0$.

Mode of the distribution, $m=1$.

Upper limit of the triangular distribution, $h=2$.

Two-bidder equilibrium strategy game:

$$bi = \frac{c_i + (c_i^2 + 6)^{0.5}}{3} \quad \text{if } k < c_i < m$$

$$bi = \frac{2 + 2 * c_i}{3} \quad \text{if } m < c_i < h$$

Three-bidder equilibrium strategy game:

$$bi = \frac{2 * c_i + (4 * c_i^2 + 10)^{0.5}}{5} \quad \text{if } k < c_i < m$$

$$bi = \frac{2 + 4 * c_i}{5} \quad \text{if } m < c_i < h$$

Five-bidder equilibrium strategy game:

$$bi = \frac{4 * c_i + (16 * c_i^2 + 18)^{0.5}}{9} \quad \text{if } k < c_i < m$$

$$bi = \frac{2 + 8 * c_i}{9} \quad \text{if } m < c_i < h$$

Ten-bidder equilibrium strategy game:

$$bi = \frac{9 * c_i + (81 * c_i^2 + 38)^{0.5}}{19} \quad \text{if } k < c_i < m$$

$$b_i = \frac{2+18*c_i}{19} \quad \text{if } m < c_i < h$$

Fifteen-bidder equilibrium strategy game:

$$b_i = \frac{14*c_i + (196*c_i^2 + 58)^{0.5}}{29} \quad \text{if } k < c_i < m$$

$$b_i = \frac{2+28*c_i}{29} \quad \text{if } m < c_i < h$$

a. Random Number Generation

For simulation, the Excel 7.0 built-in random number generator, which provides random numbers uniformly distributed over the interval [0, 1], was used. To convert these numbers to be distributed according to triangular distribution, inverse transformation technique was used. [Ref 12:p. 300]

According to this technique:

$$c = k + [\text{Rand()} * (m - k) * (h - k)]^{0.5} \quad \text{for } k < c < m \text{ and}$$

$$c = h - [\text{Rand()} * (h - m) * (h - k)]^{0.5} \quad \text{for } m < c < h$$

where c is triangularly distributed random number and $\text{Rand}()$ is uniformly distributed random number generated by Excel 7.0.

The simulation program for FPSB contract game is in Appendix B.

b. Experimentation With Two Bidders

During the simulation, three different scenarios were experimented and analyzed. In the first scenario, both bidders used equilibrium strategies. In the second scenario, one of the bidders underbid while the other used the equilibrium strategy. Finally in the third scenario, both of the bidders did not use the equilibrium strategy.

The different scenarios were designed to show that using the equilibrium bidding strategy maximizes the bidders' profit and any deviation from the equilibrium strategy would reduce the expected profit for the deviating bidder.

Figure 56 shows the graphical result of the simulation when both bidders used equilibrium strategies. The results show that there exists an equilibrium in the contract game and the winners' expected profit approaches 0.22 as the number of simulated games approaches 2500.

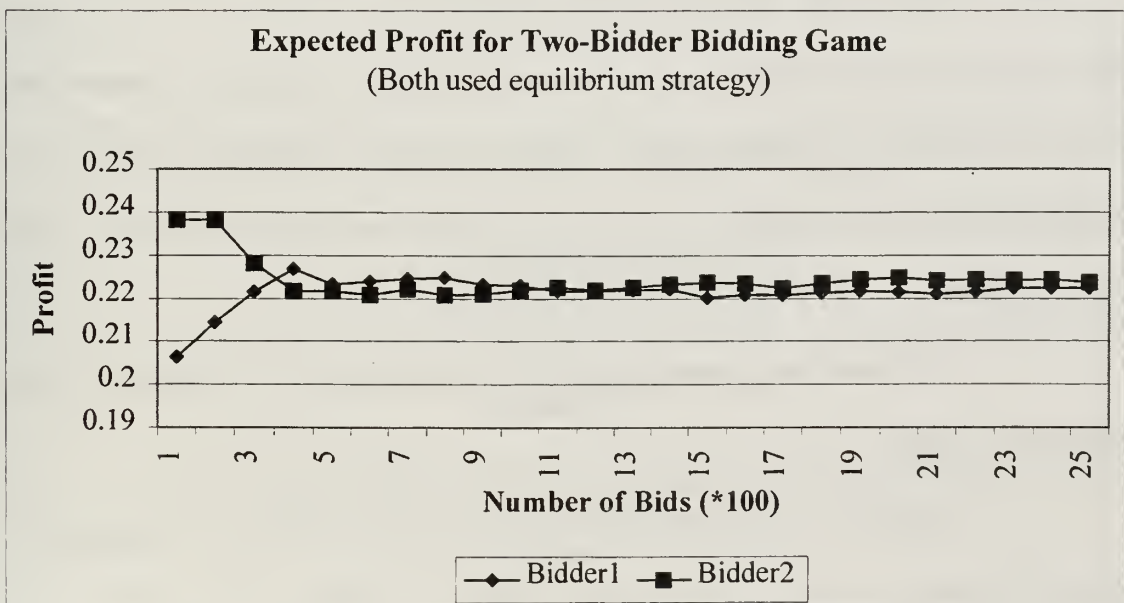


Figure 56

The average profit graph for this particular simulation is shown in Figure 57. This result also reinforces the result of existing equilibrium in the bidding game. Both bidders' average profit approaches approximately 0.11 by bidding in 2500 games.

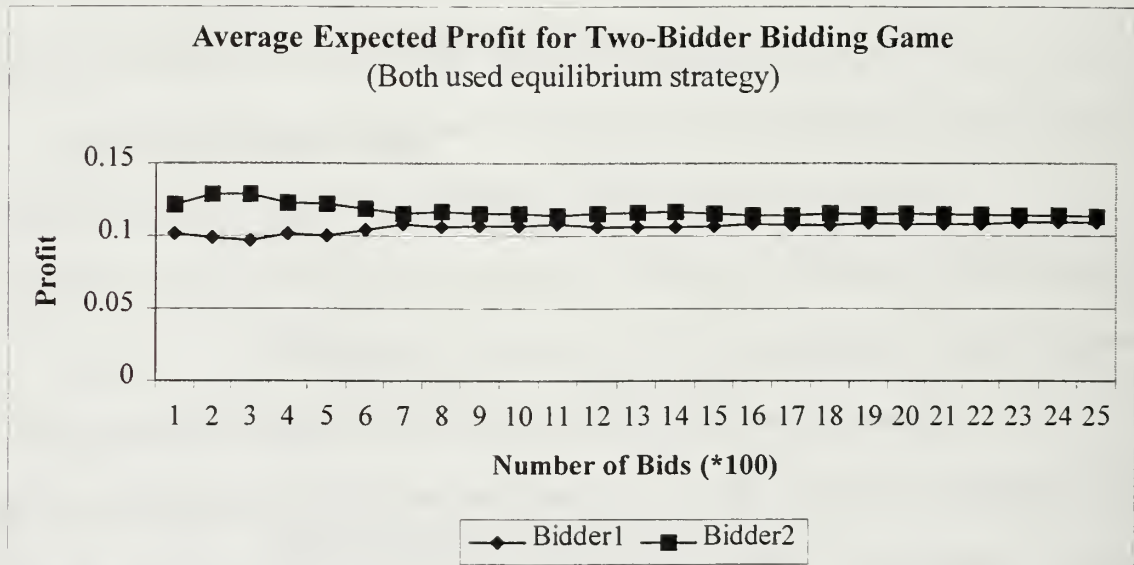


Figure 57

Total profits for both bidders resulted almost the same; 273.6609 for bidder1 and 283.7462 for bidder2 and are shown graphically in Figure 58. Another result was that the bidders won almost equally; bidder1 won 1231 of the games while bidder2 won the remaining 1269 games. Overall results of the experiment are outlined in Table 47 in Appendix C.

In the next experimentation, bidder1 underbid by 0.15 while bidder2 used equilibrium strategy.

The results of the experimentation are shown in Table 48 in Appendix C. The experimentation resulted in lower expected profit for bidder1, 0.048, while bidder2 made 0.243, which is more than the equilibrium amount of 0.22. The expected profit graph is shown in Figure 59.

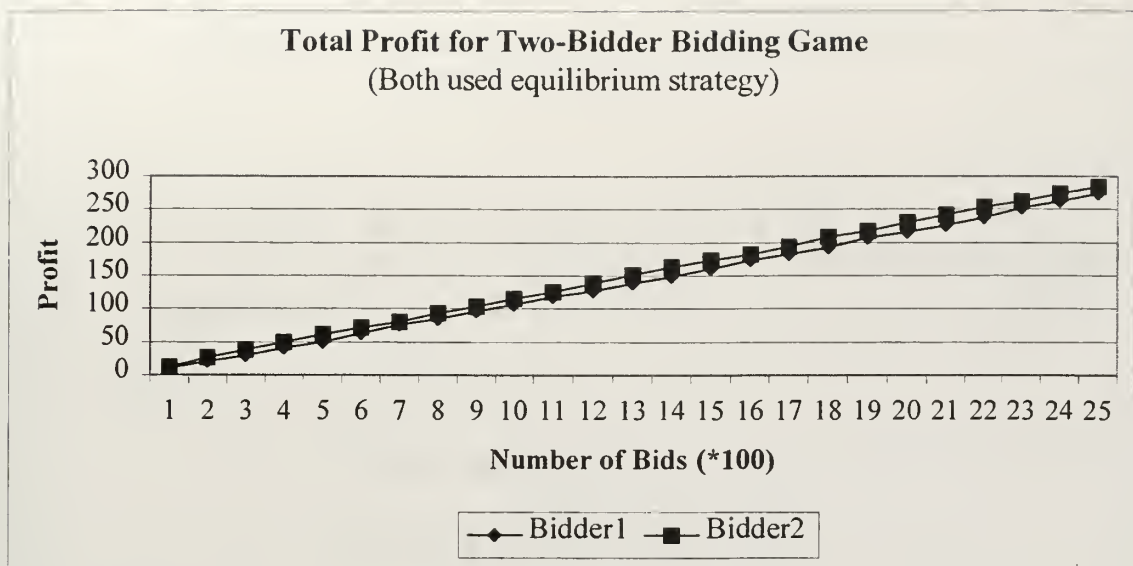


Figure 58

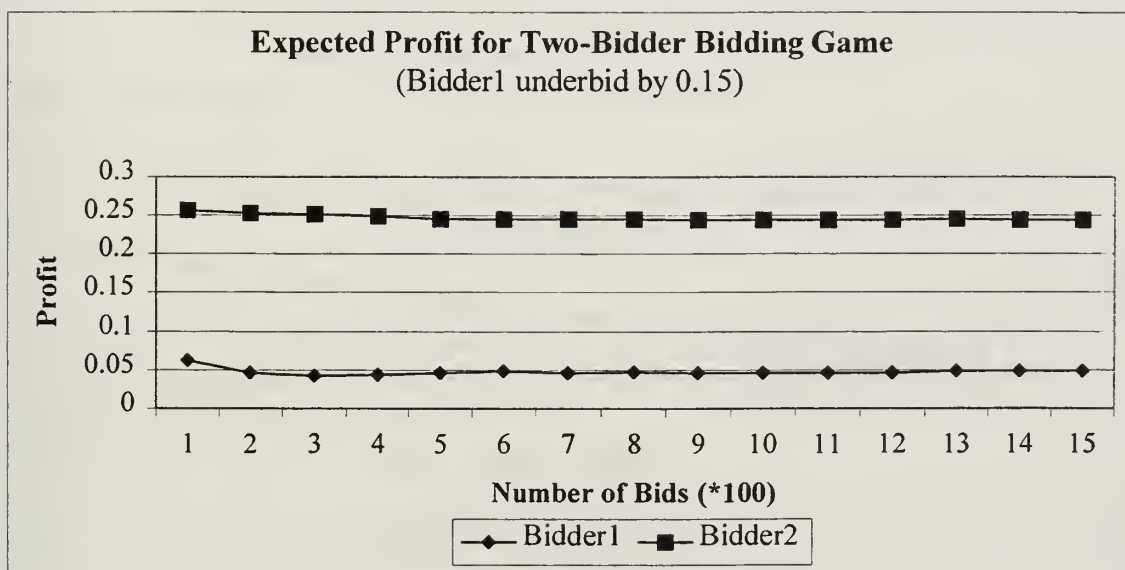


Figure 59

The average profit for bidder1 decreased to 0.034 and 0.07 for bidder2.

The graph for average profit is shown in Figure 60.

Total profit for bidder1 also dramatically decreased to 51.4114 and 105.1904 for bidder2 in 1500 games as shown in Figure 61. Bidder1 won 1068 of 1500 games while bidder2 won only 432.

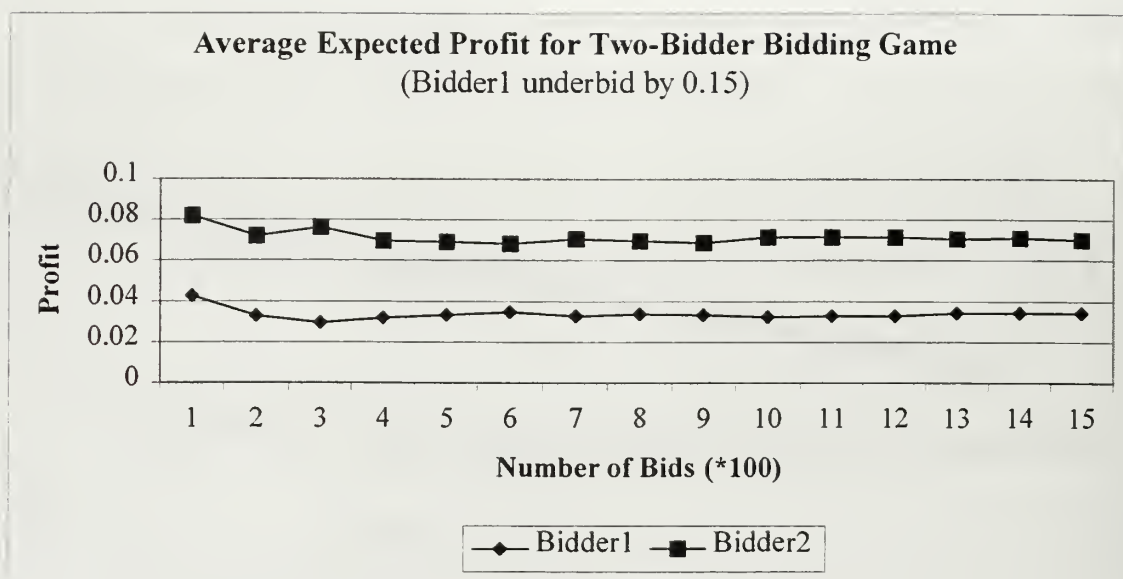


Figure 60

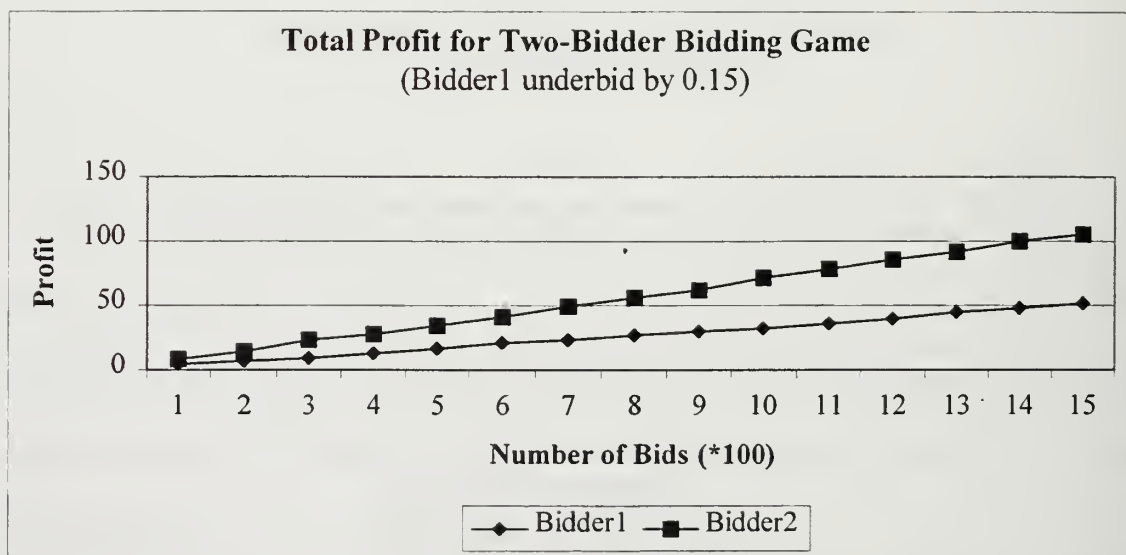


Figure 61

The last experiment with two bidders was conducted as they both did not use the equilibrium strategy. Both bidders underbid by 0.1, which was approximately five percent of their average bids. In this scenario, both bidders' expected profits approached 0.122, which is lower than the equilibrium amount of 0.22 as the number of games approached 2500. The graphical representation of expected profit is shown in Figure 62.

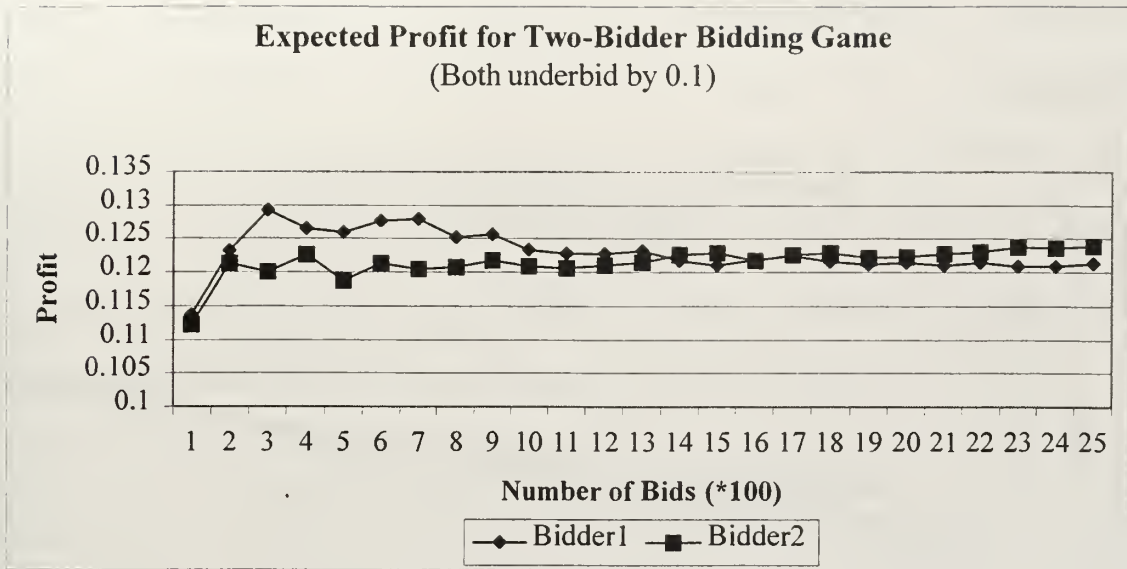


Figure 62

The average profit for both bidders also decreased to 0.06 and is shown in Figure 63 graphically.

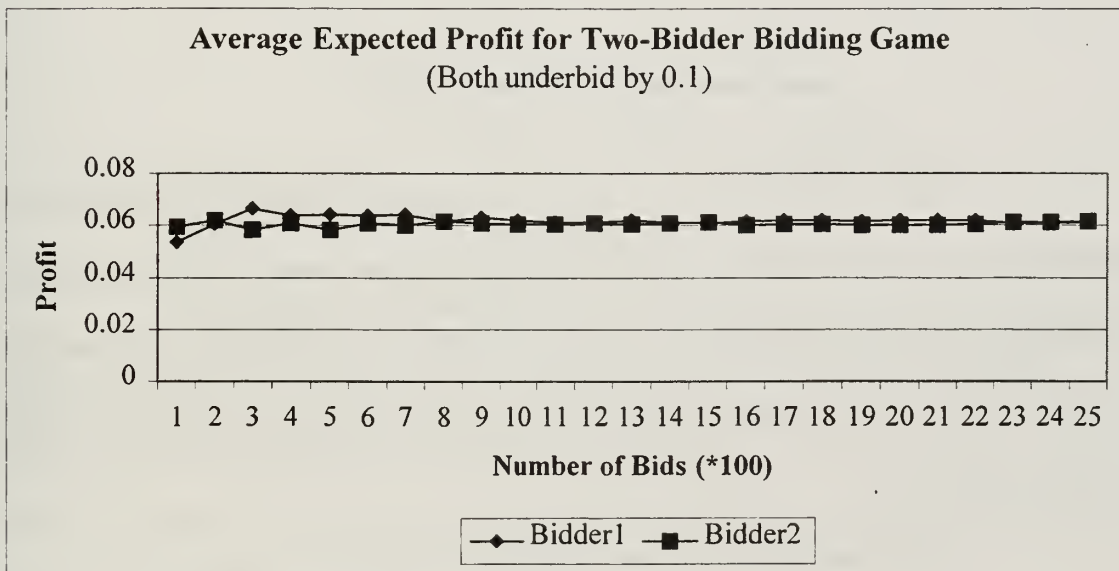


Figure 63

Total profits in 2500 games shown in Table 49 in Appendix C, 152.6738 for bidder1 and 153.8211 for the other, were also lower than the equilibrium total profit

of 273.6609 and 283.7462 respectively. The graphical representation of total profit curves is in Figure 64.

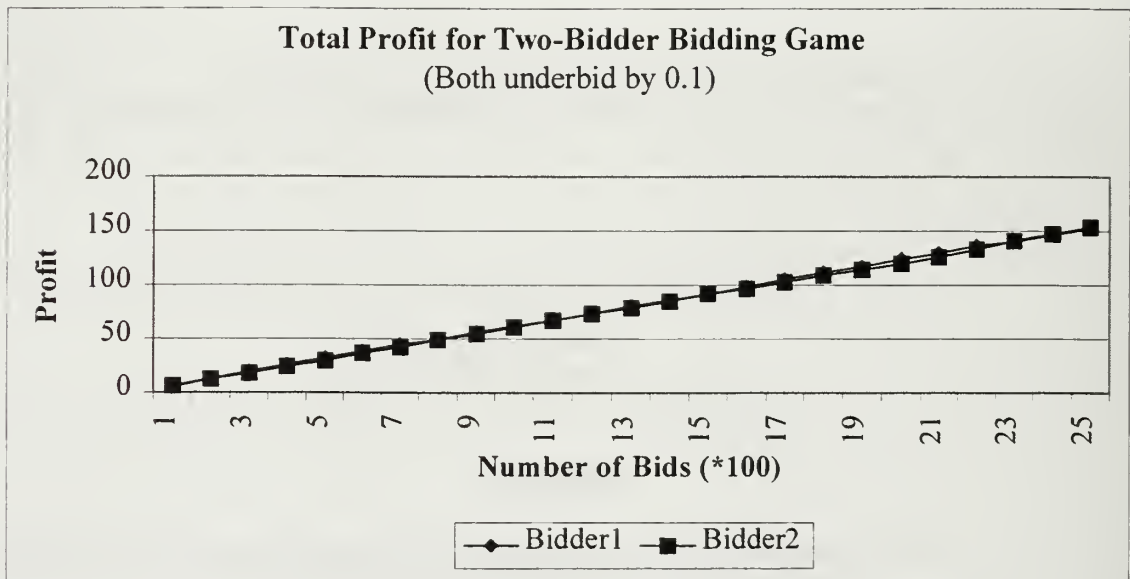


Figure 64

c. Experimentation With Three Bidders

This simulation along with the other multiple bidder simulations was conducted to find out the effect of number of bidders on the game as well as analyzing the behavior of bidders and the accuracy of the simulation. The simulation was conducted with three bidders and the cost is within the interval $[0, 2]$ with a mode value of 2 under triangular distribution.

During the simulation, three different scenarios were analyzed. In the first scenario, all off the bidders used equilibrium strategy. In the second scenario, bidder1 underbid while others used equilibrium strategy. Finally in the third scenario, bidders “1” and “2” underbid by different amounts while bidder3 used equilibrium strategy.

Figure 65 shows the results of the first experimentation of which all bidders used equilibrium strategy. As seen in the graph, there is a well-defined equilibrium in the three-bidder bidding game as well as in the two-bidder bidding game. According to the simulation results, the winners' expected profit from bidding approached approximately 0.149 as the number of games approached 2000. Introducing another bidder to the game reduced the expected profit from bidding.

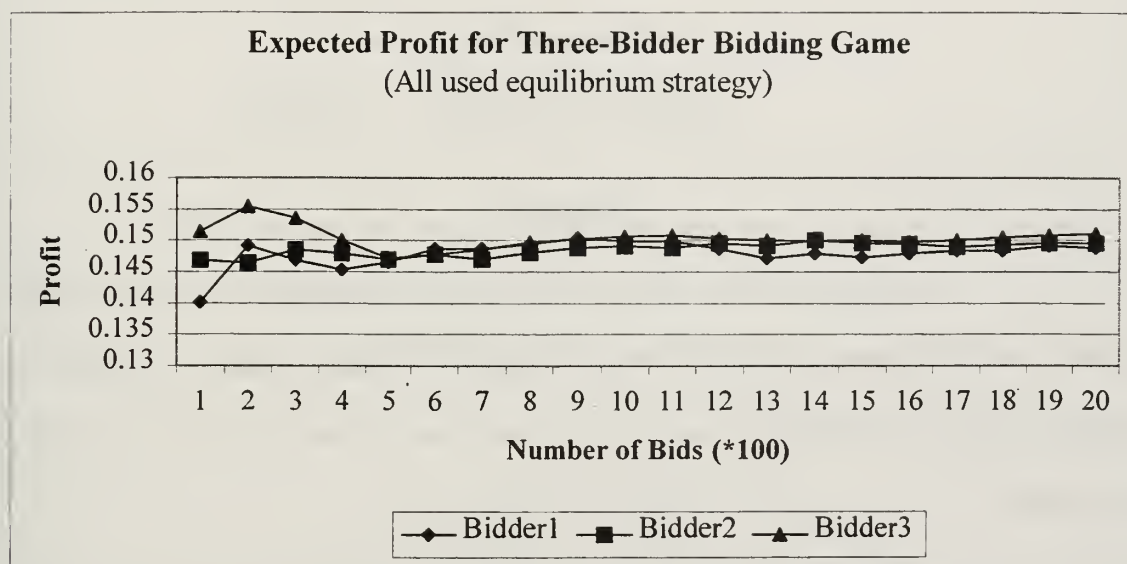


Figure 65

The average expected profit graph is shown in Figure 66 and the overall results of the experiment are shown in Table 50 in Appendix C. The average expected profit graph also reinforces the equilibrium of the game. In this simulation, bidders' average expected profit approached approximately 0.05 while the number of games approached 2000.

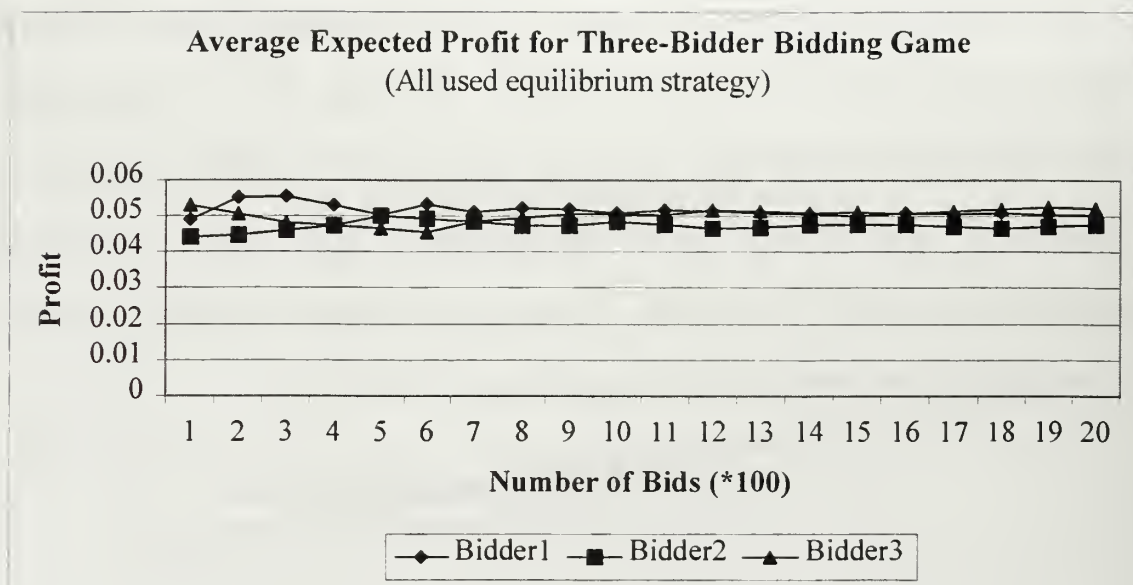


Figure 66

As seen in Table 50, each bidder won the games almost equally. Overall, bidder1 won 675 games, bidder2 635 and bidder3 690 games each. Total profit for bidders as seen in Figure 67, were almost the same; 100.41, 94.983 and 104.24 respectively.

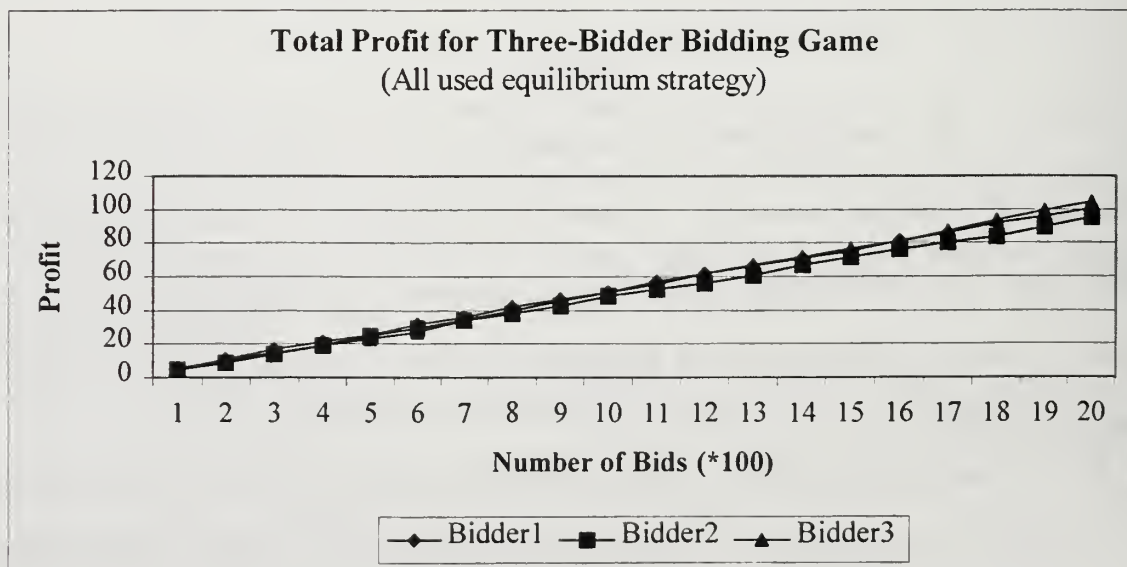


Figure 67

In the next experiment, bidder1 underbid by 0.1, which was approximately six percent of the average bid amount, while other two used equilibrium strategy. As a result of 1500 games, bidder1's expected profit decreased to 0.041 while the other two bidders' slightly increased to 0.1529 and .1541 respectively. The expected profit graph is shown in Figure 68

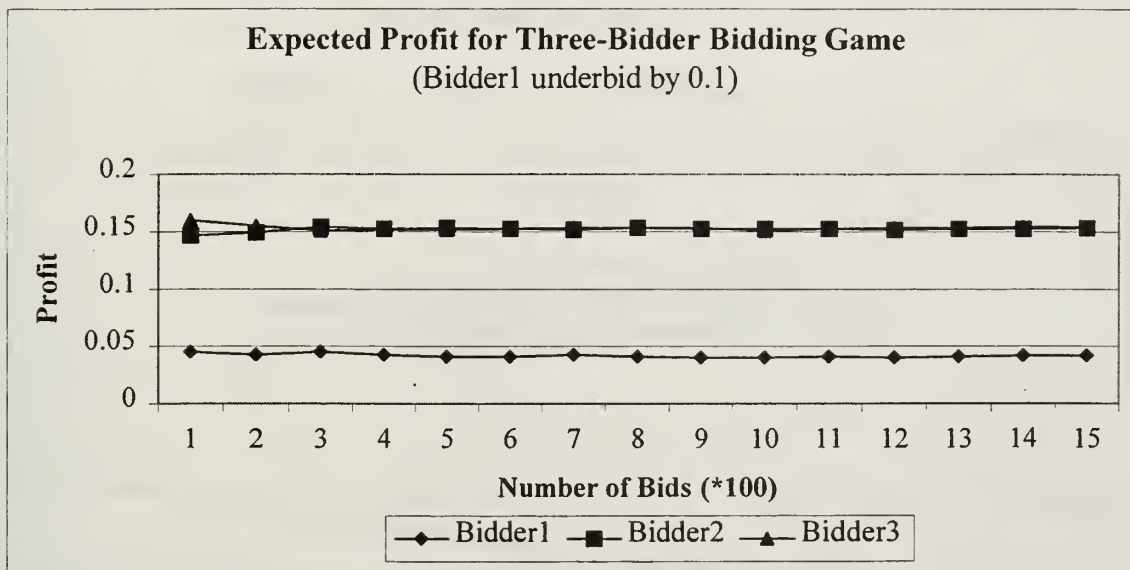


Figure 68

Average profit scheme was different than the equilibrium game experiment for bidder1. Bidder1's average profit was 0.019 while others higher, 0.041 as graphically shown in Figure 69.

Bidder1 won 687 of 1500 games while others won 405 and 408 games as shown in Table 51 in Appendix C. Although bidder1 won more games than the others won, its total profit was 28.554 which is lower than the others'; which were 61.925 and 62.879 respectively. Total profit graph for this experiment is shown in Figure 70.

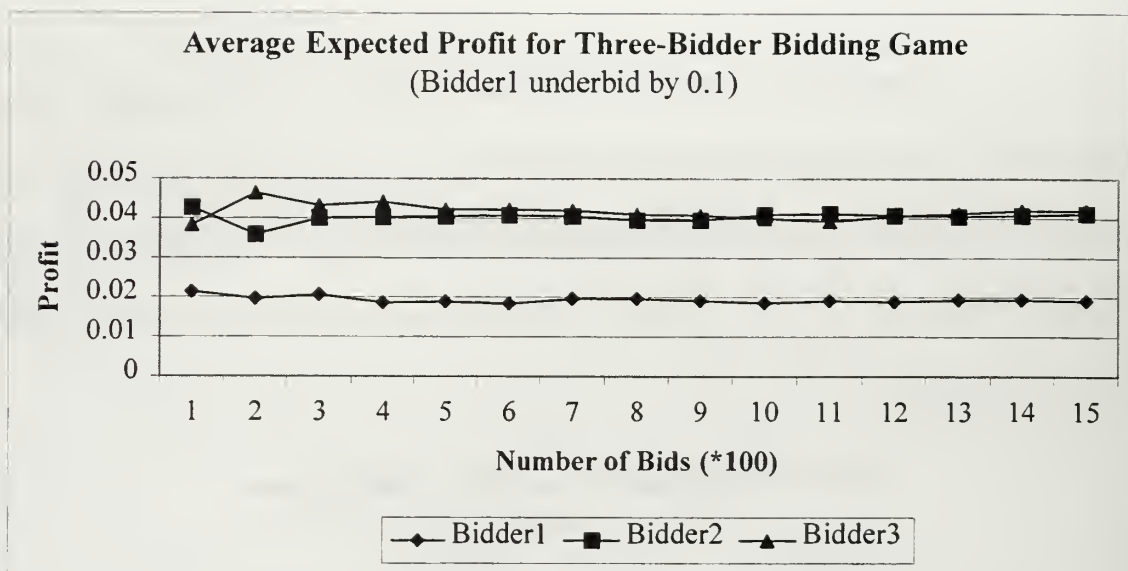


Figure 69

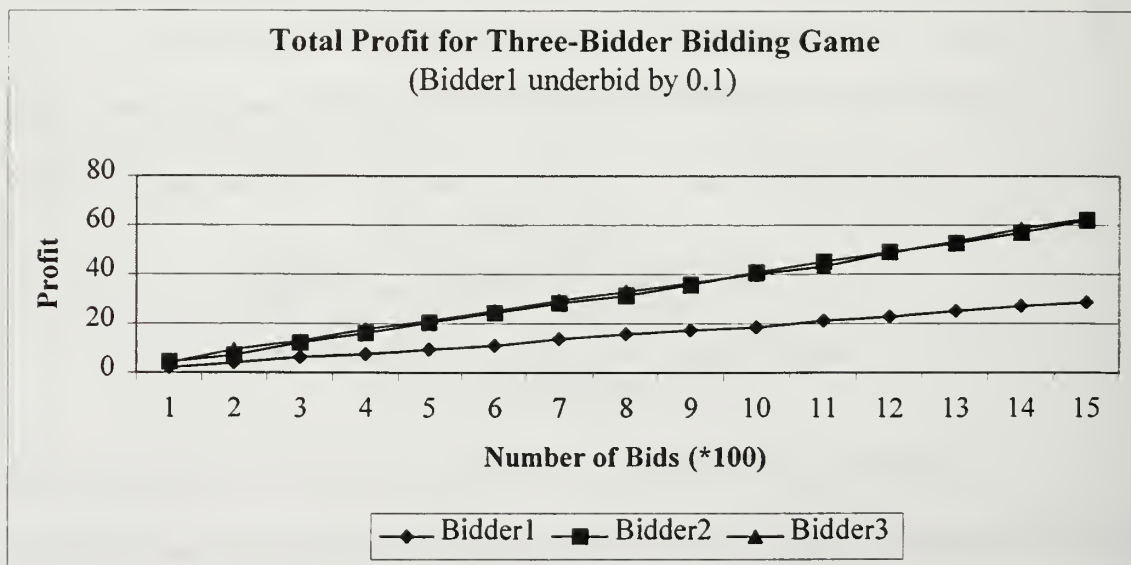


Figure 70

The last scenario for three-bidder simulation was while bidder3 used equilibrium strategy bidders “1” and “2” did not. Bidders “1” and “2” underbid by 0.1 and 0.12 respectively. The experiment resulted in lower expected profits for the first two bidders while it was even higher for the third. Bidder1’s profit approached 0.0469, which is slightly higher than in the previous game which was 0.041 and bidder2’s was even

lower, 0.0239 in 2000 games. Bidder3, which had used equilibrium strategy, made higher profit than it did in the first two experiments, 0.157 as opposed to 0.149 and 0.1541. The expected profit graph is shown in Figure 71:

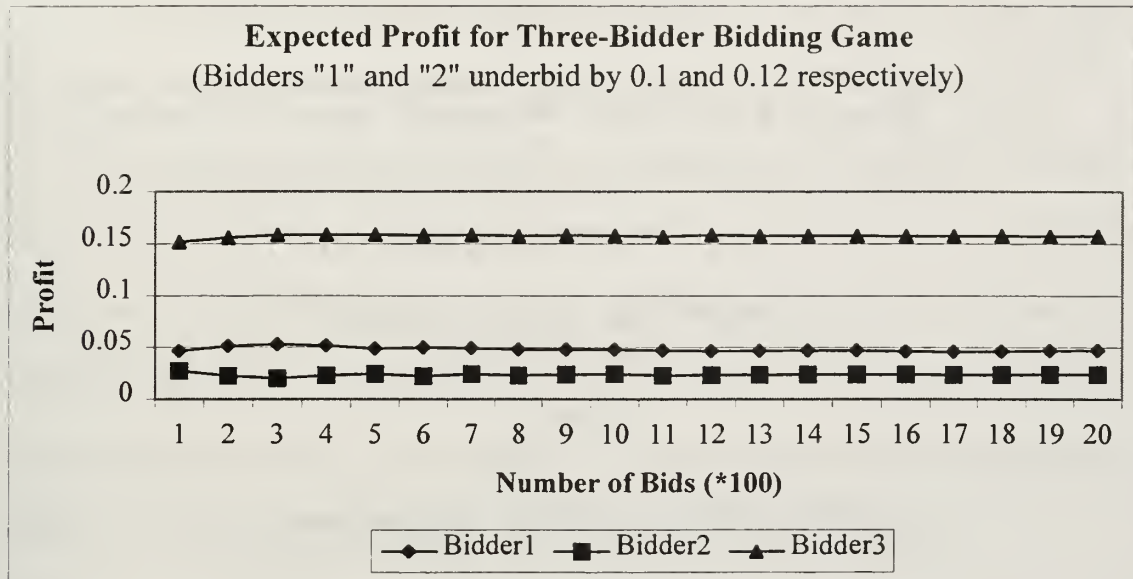


Figure 71

Bidder2 had lowest average profit than the others. It made 0.01 as opposed to 0.0168 and 0.033 made by bidders "1" and "3" respectively. The average profit graph is shown in Figure 72.

Total profit scheme was also different than previous experiments, but the general trend, which is the more a bidder underbids the less profit it makes, was the same. Bidder2 made the lowest profit, 20.3517 while bidder1 and bidder3 made 33.7713 and 67.483 respectively in 2000 games. Bidders won 720, 851 and 429 games respectively. Total profit curves are shown in Figure 73.

Overall results of the game are outlined in Table 52 in Appendix C.

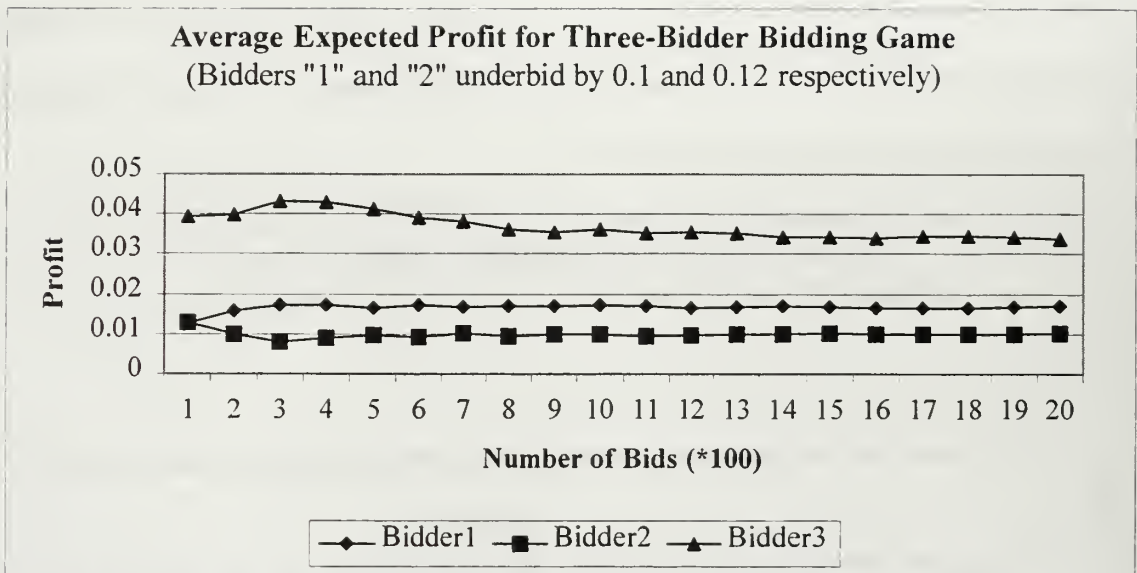


Figure 72

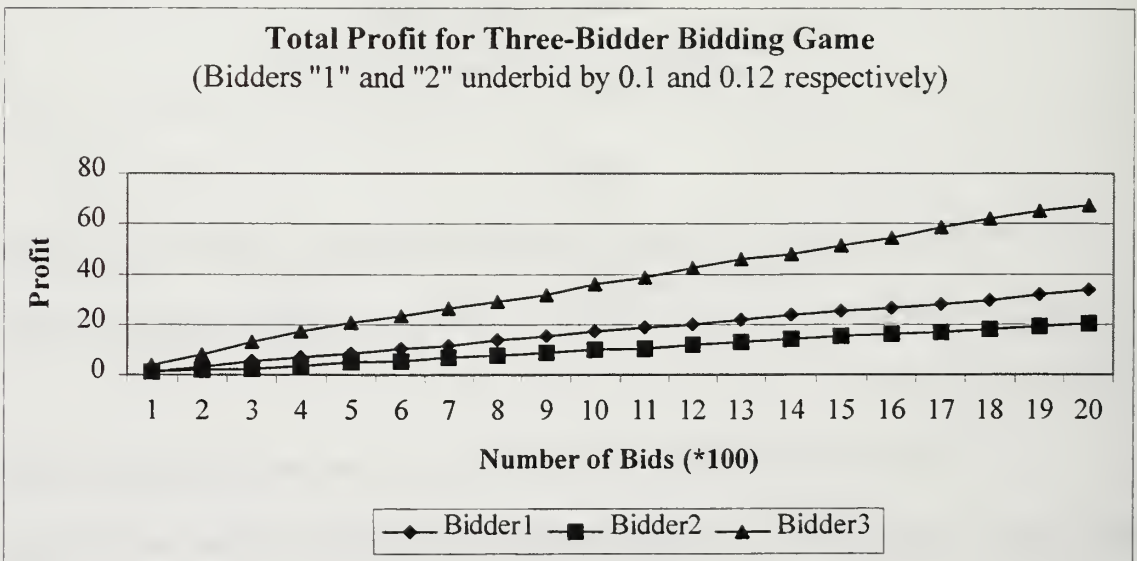


Figure 73

d. Experimentation With Five Bidders

The simulation was conducted with five bidders according to triangular cost distribution within interval $[0, 2]$ with a mode value of 2.

During the simulation four different scenarios were analyzed. In the first scenario, all bidders used the equilibrium strategy. In the second scenario, bidder1 underbid while others used equilibrium strategy. In the third scenario, both bidders “1” and “2” underbid while others used equilibrium strategy and in the last scenario, bidders “1” and “2” underbid by different amounts while others used equilibrium strategy.

Figure 74 shows the results obtained from the simulation of which all bidders used equilibrium strategy. As seen in the graph, there is a well-defined equilibrium in the game as in the two and three-bidder bidding games. According to the simulation, each winner’s expected profit approached approximately 0.092 as the number of games approached 1500. As the number of bidders increased, the expected profit from bidding decreased, as it was the case in the three-bidder bidding game.

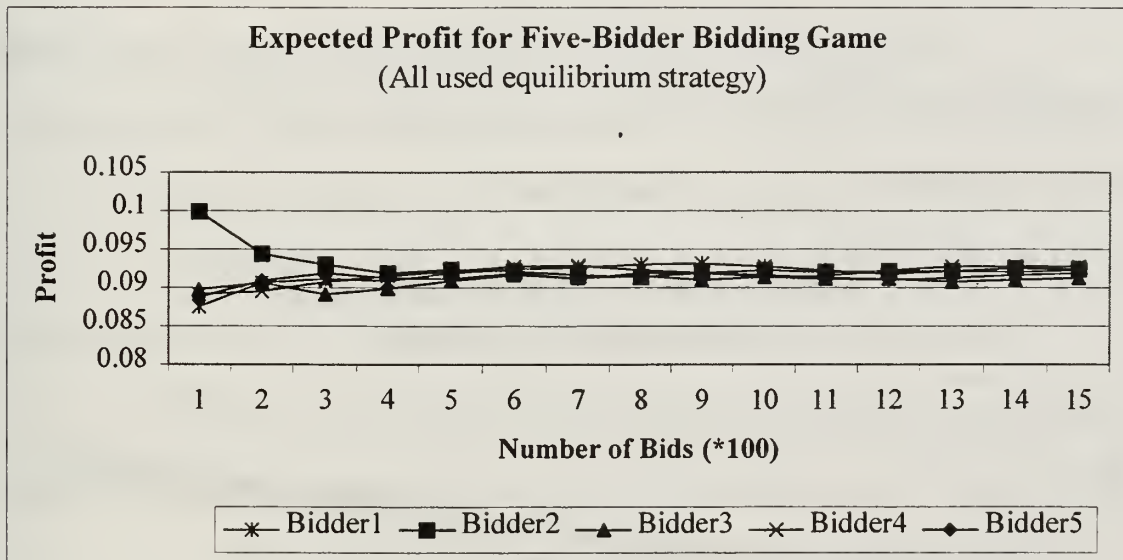


Figure 74

Average expected profit graph is shown in Figure 75 and overall simulation results obtained are shown in Tables 53 and 54 in Appendix C. One more time, the average expected profits reinforced the game equilibrium. The average expected

profit for the bidders approached approximately 0.018 as the number of games approached 1500.

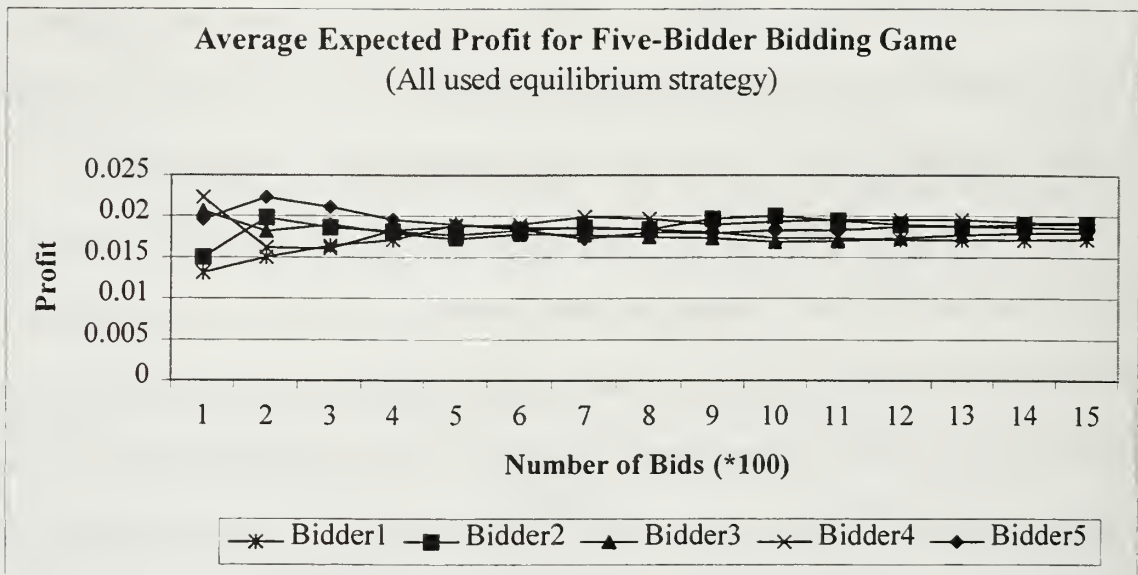


Figure 75

The experiment resulted in very close total profits, ranging from 25.7489 to 28.8431, as seen in Table 54. Total profit graph is shown in Figure 76. The number of games won by bidders was also very close to each other, ranging from 279 to 311.

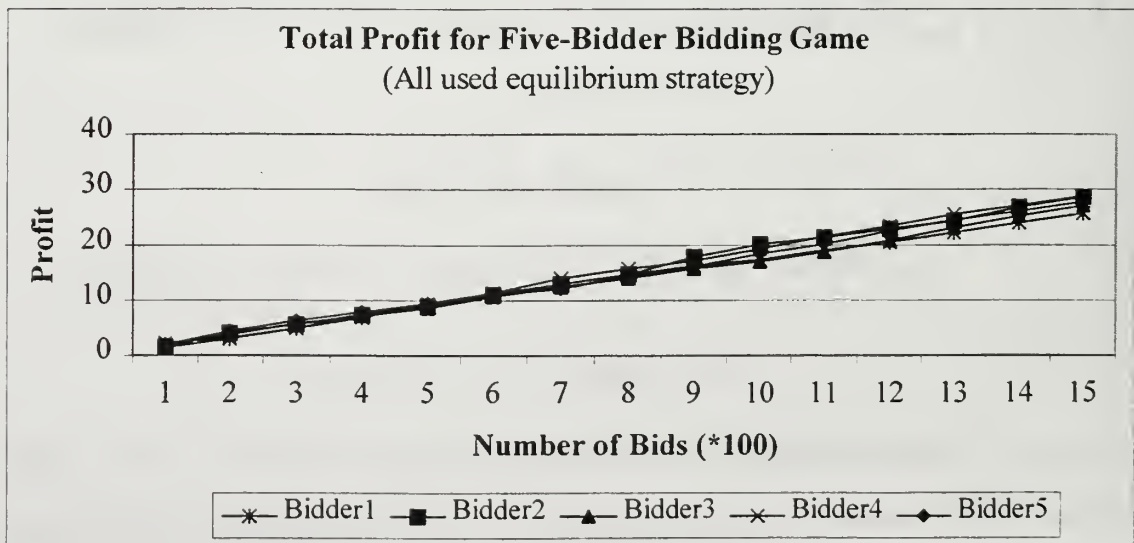


Figure 76

The second experiment, where bidder1 underbid by 0.05 and others used the equilibrium strategy, resulted in following:

Bidder1 made a lower expected profit, 0.04, than others did, which was 0.093, as the number of games approached 1500. The expected profit graph is shown in Figure 77.

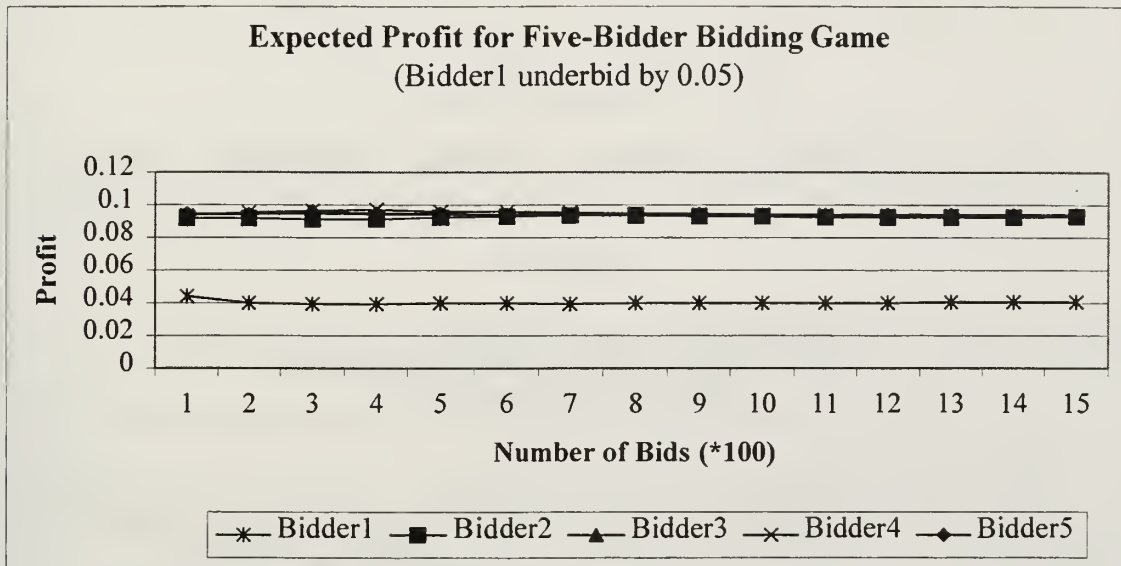


Figure 77

The average profit for bidder1 approached 0.0109 while others did between 0.0167 and 0.0178. The average profit curves are shown in Figure 78.

Bidder1 won 402 games while others won the games almost equally, ranging from 269 to 286. Total profit for bidder1 was also lower than the others', 16.3539 as opposed to 25.1509-26.7629, even though bidder1 won more games than the others did. The total profit curves are graphically shown in Figure 79.

Overall results of the experiment are outlined in Tables 55 and 56 in Appendix C.

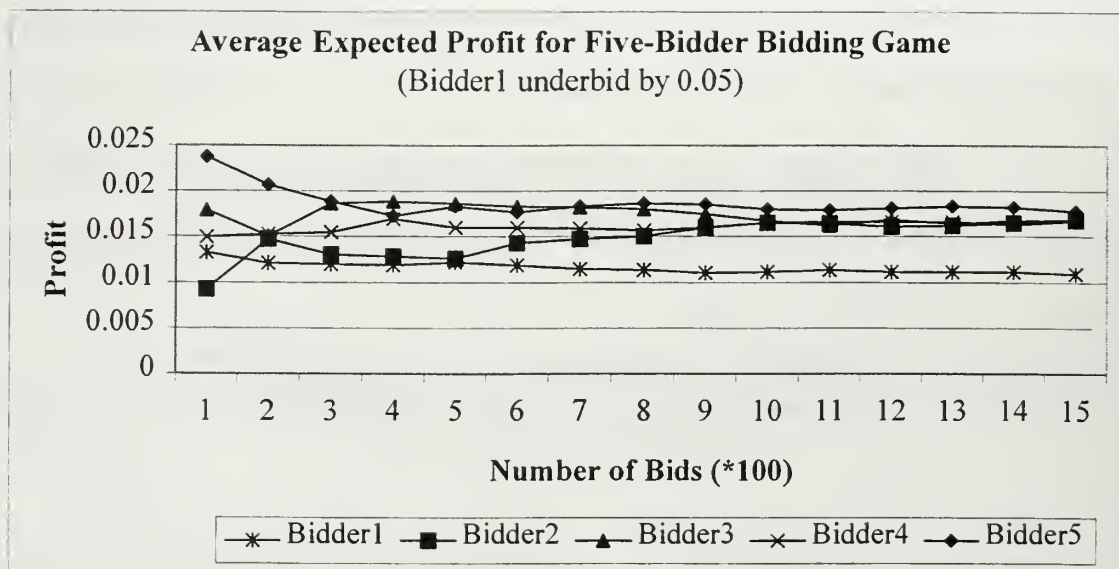


Figure 78

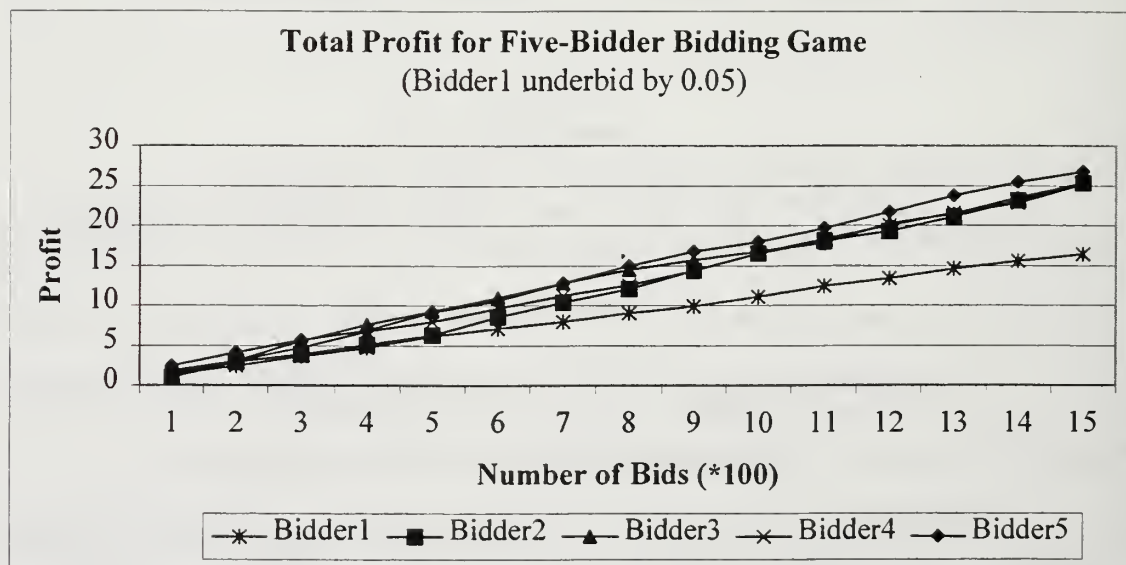


Figure 79

In the third scenario, both bidders “1” and “2” underbid by 0.05 while the rest of the bidders used equilibrium strategy. Overall results of the experiment are shown in Tables 57 and 58 in Appendix C. As for the expected profits, bidders “1” and “2” made lower profits, approximately 0.041, than the rest did, 0.093, as shown in Figure 80:

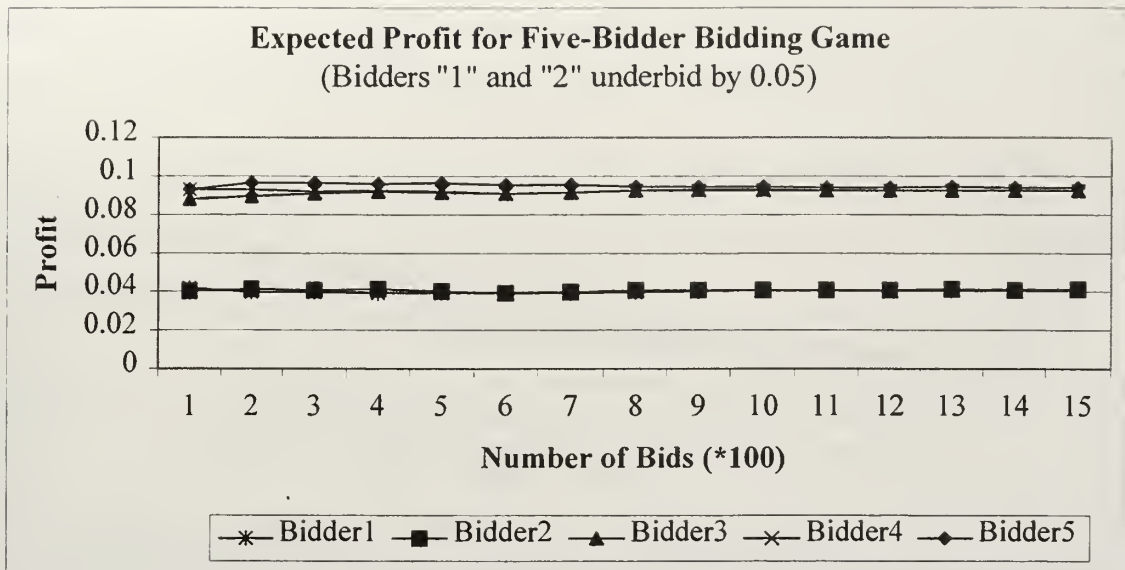


Figure 80

The average profit scheme resulted the same; bidders "1" and "2" made 0.009 while others did between 0.01487 and 0.01724 as seen in Figure 81:

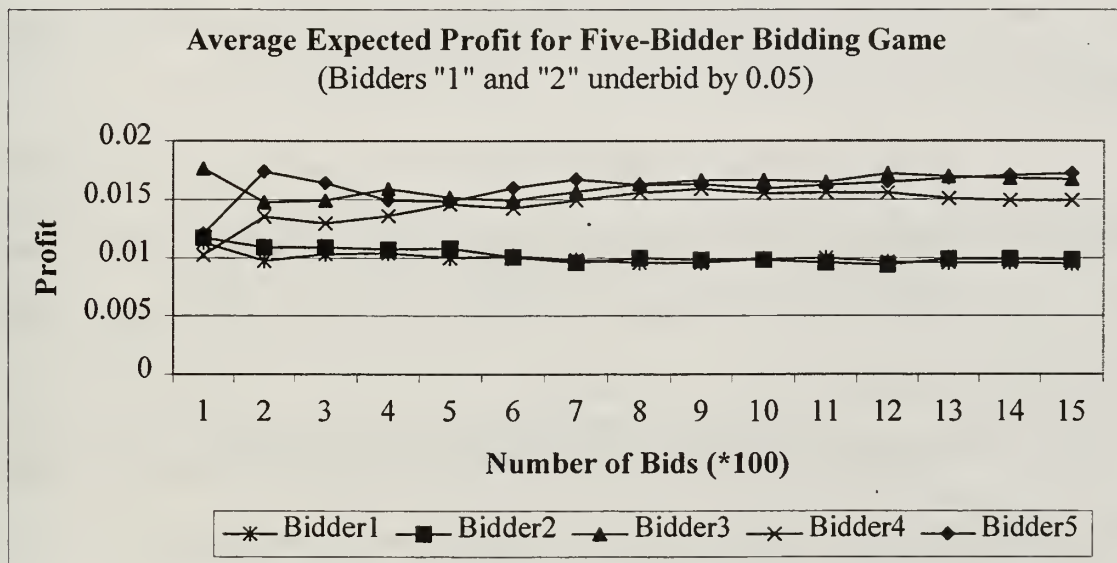


Figure81

Total profit for bidders "1" and "2" also were lower than the others' profits. They made 14.2425 and 14.7406 respectively, while others profit ranged from 22.3112 to 25.8637. Total profit curves for bidders are shown in Figure 82:

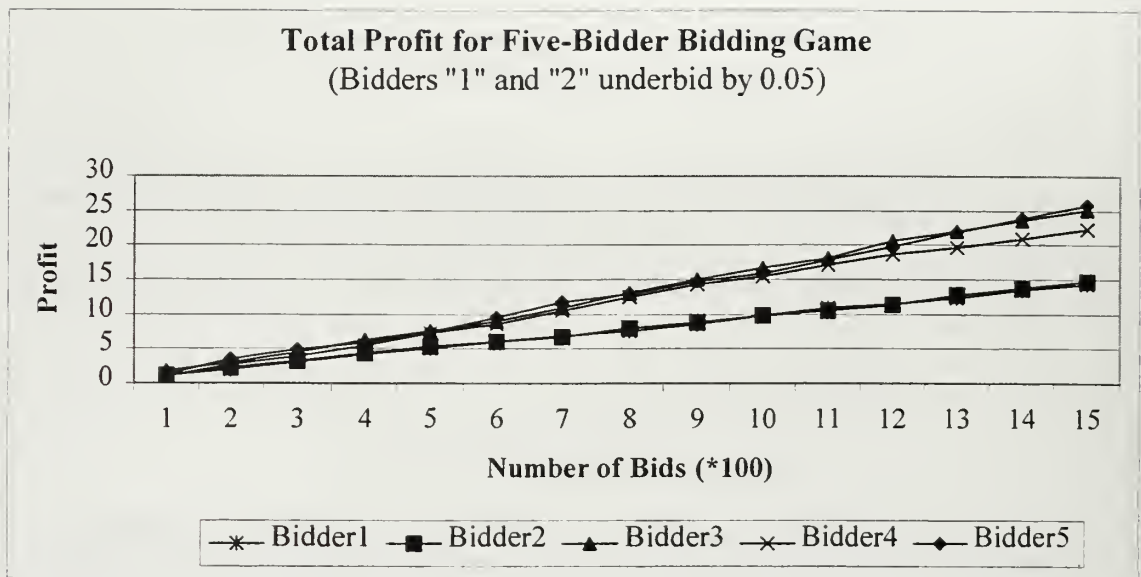


Figure 82

Bidders "1" and "2" won 313 and 329 games out of 1500 games. The rest won the games 282-293 times. As it was the case in previous experiments, underbidding bidders made lower profits, although they won more games than the equilibrium strategy bidders.

In the last scenario, bidders "1" and "2" underbid by 0.05 and 0.075 respectively while others used equilibrium strategy. The underbidding bidders made lower profits of 0.041 and 0.015 respectively while others made approximately 0.093. The expected profit graph is shown in Figure 83.

The average profit for bidders "1" and "2" were 0.00997 and 0.0041 while others' approached 0.015 in 2500 games as seen in Figure 84. The equilibrium strategy bidders made higher expected and average profit than underbidding bidders did.

Bidders "1" and "2" won 582 and 674 games while the others won very close number of games ranging from 402 to 424 as seen in Table 59 in Appendix C. Despite bidders "1" and "2" won much more games than others, they made lower total

profits in 2500 games. The lowest bidder, bidder2, made the lowest total profit, 10.194. The second lowest total profit was bidder2's, 24.156. Finally, equilibrium strategy bidders made very close total profits ranging from 36.97 to 39.499. Total profit curves are shown in Figure 85 and overall results of the experiment are in Tables 59 and 60 in Appendix C.

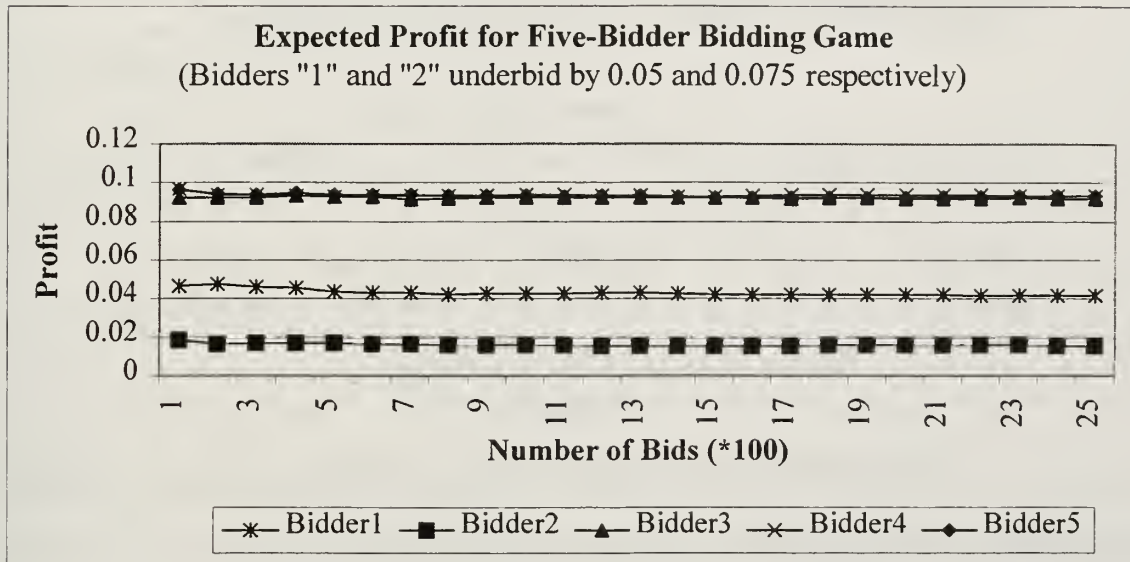


Figure 83

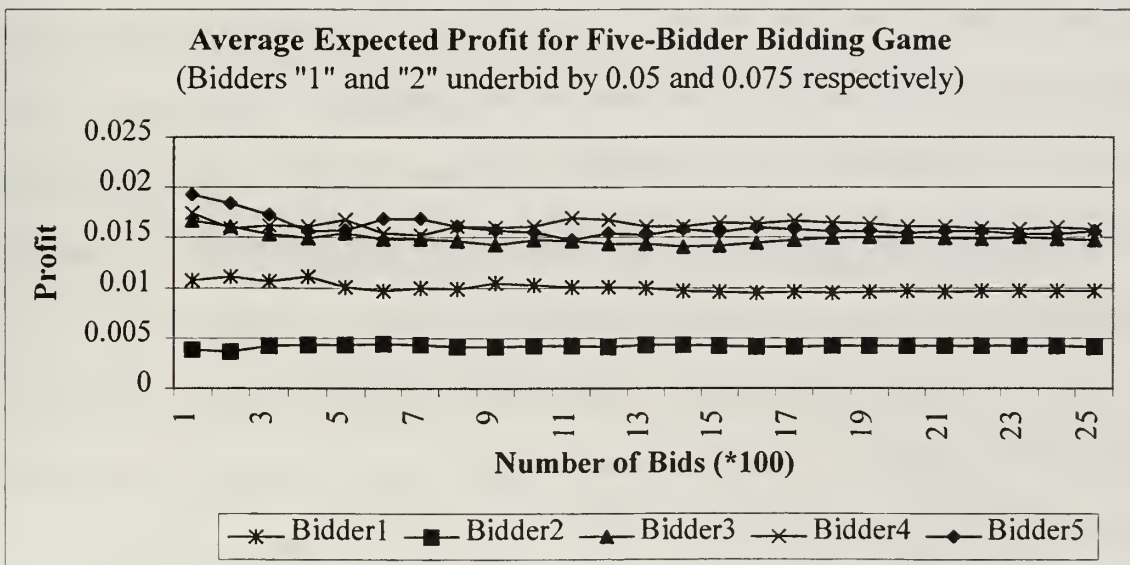


Figure 84

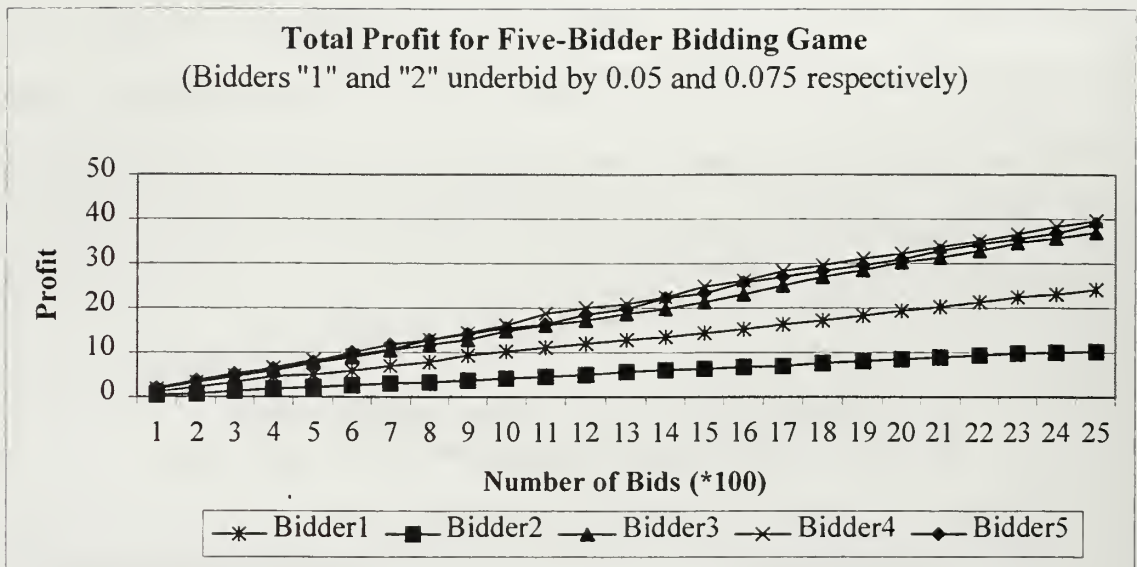


Figure 85

e. Experimentation With Ten Bidders

The simulation was conducted with ten bidders according to triangular cost distribution within interval $[0, 2]$ with a mode value of 2.

In these series of experiments, four different scenarios used to simulate the bidding process. In the first scenario, all bidders used the equilibrium strategy. In the second one, bidder1 underbid while others used the equilibrium strategy again. In the third one, both bidders "1" and "2" underbid by the same amount while all others used the equilibrium strategy. Finally, in the last scenario, bidders "1", "2", "3" and "4" underbid by different amounts while the rest used the equilibrium strategy.

The first experiment of which all used equilibrium strategy resulted in a well-defined equilibrium. All bidders' expected profit approached approximately 0.048 as the number of games approached 1500. The graphical representation of the expected profit curves is shown in Figure 86. As can be seen in the overall results of the

experiment in Tables 61 through 64 in Appendix C, introducing more bidders to the game reduced the expected profit from bidding more.

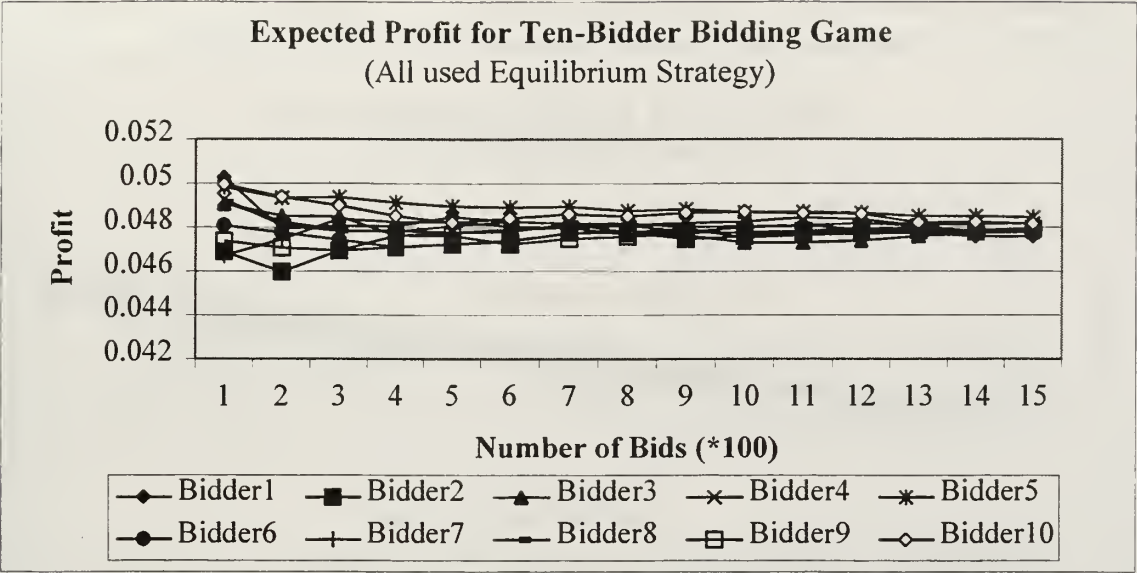


Figure 86

The average expected profit graph is shown in Figure 87. The average expected profit graph also reinforces the equilibrium of the game. In this simulation, bidders' average expected profit approached 0.005 in 1500 games.

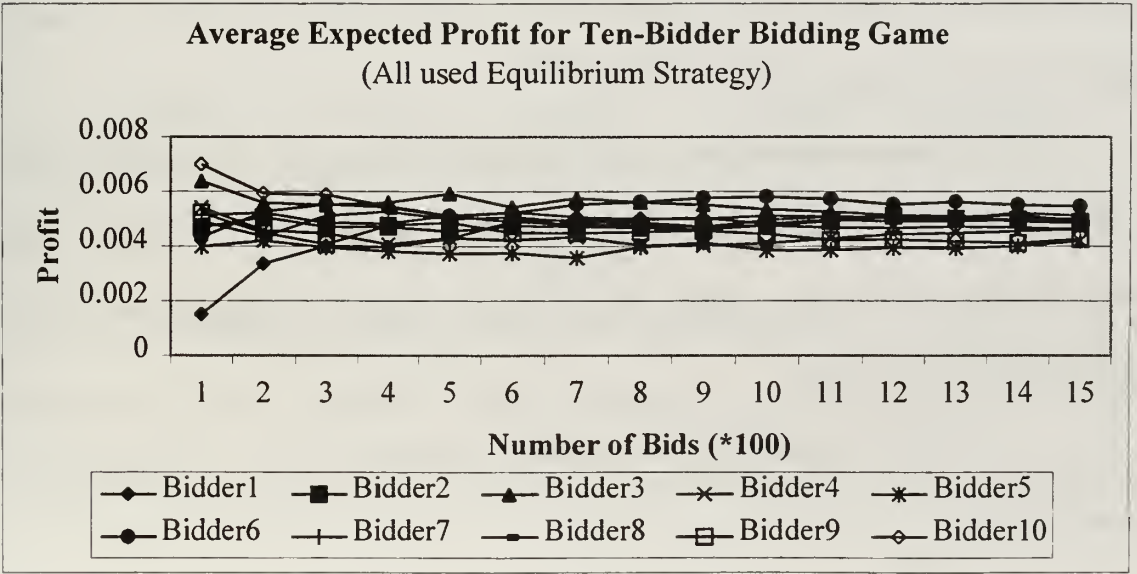


Figure 87

As seen in Table 62, each bidder won the games approximately equal number of times. Overall, bidders won in a range of 130 to 161 games each. Total profit for bidders as seen in Figure 88, were almost the same, ranging between 6.2965 and 7.66.

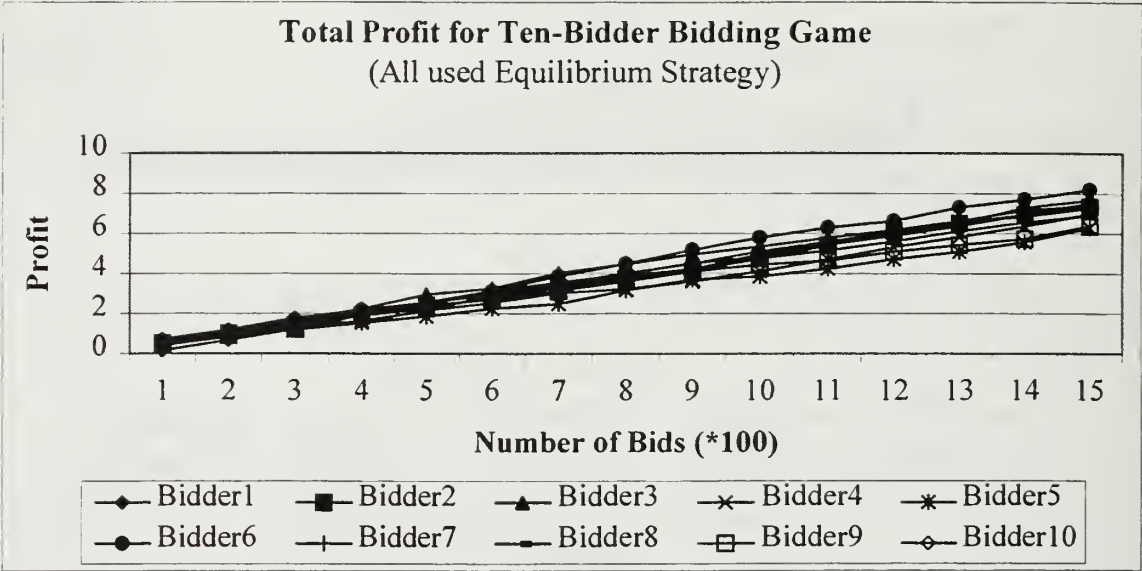


Figure 88

In the next experiment, bidder1 underbid by 0.025 while others bid according to the equilibrium strategy. As a result of 2500 games, bidder1’s expected profit decreased to 0.0225 while other bidders’ remained at 0.048. The expected profit graph is shown in Figure 89.

Average expected profits for the bidders also reinforced the same scheme as the expected profit results had shown. Bidder1’s average profit was 0.0026 while others were almost the same, ranging from 0.0043 to 0.0052. Average expected profit graph is shown in Figure 90.

Overall results of the experiment are shown in Tables 65 through 68 in Appendix C.

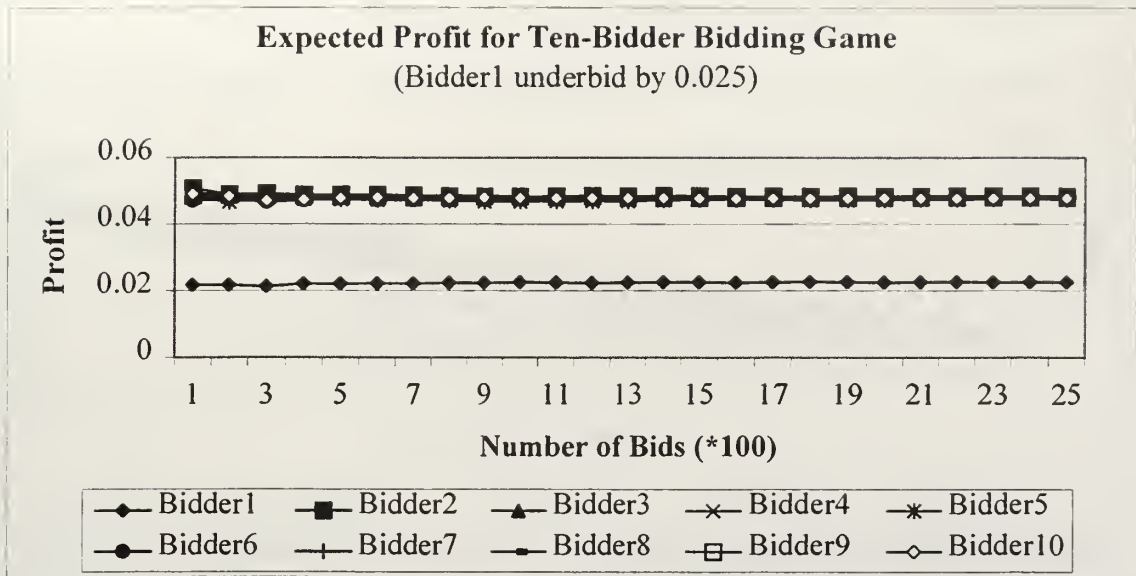


Figure 89

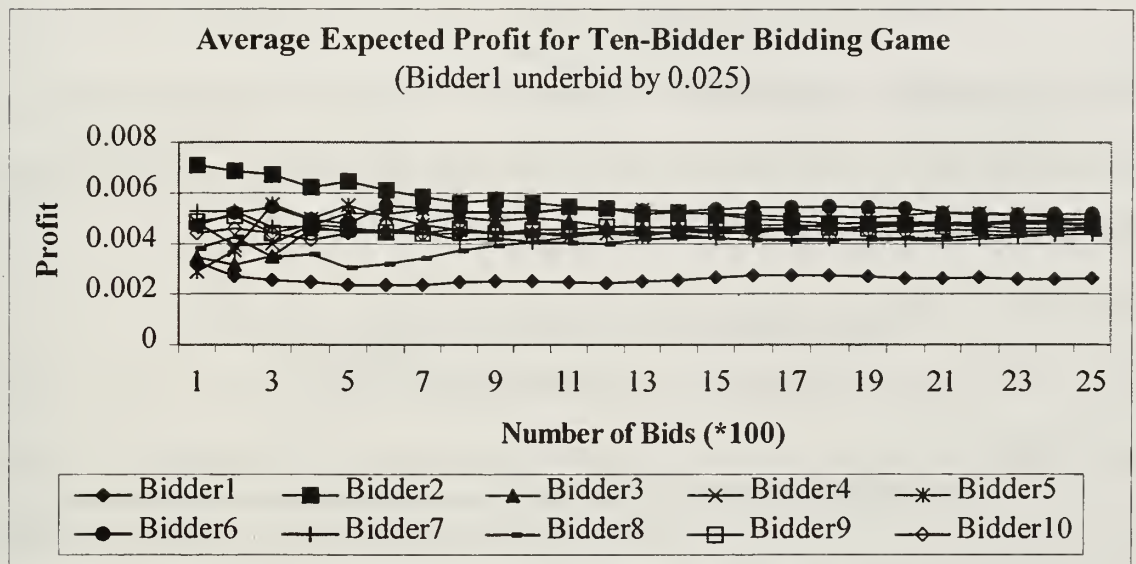


Figure 90

Bidder1 won 289 of 2500 games while others won in a range of 226 to 270 games as shown in Table 66. Although bidder1 won more games than the others won, its total profit was 6.5026 which is much lower than the others' profits ranging from 10.859 to 12.876. Total profit graph for this experiment is shown in Figure 91.

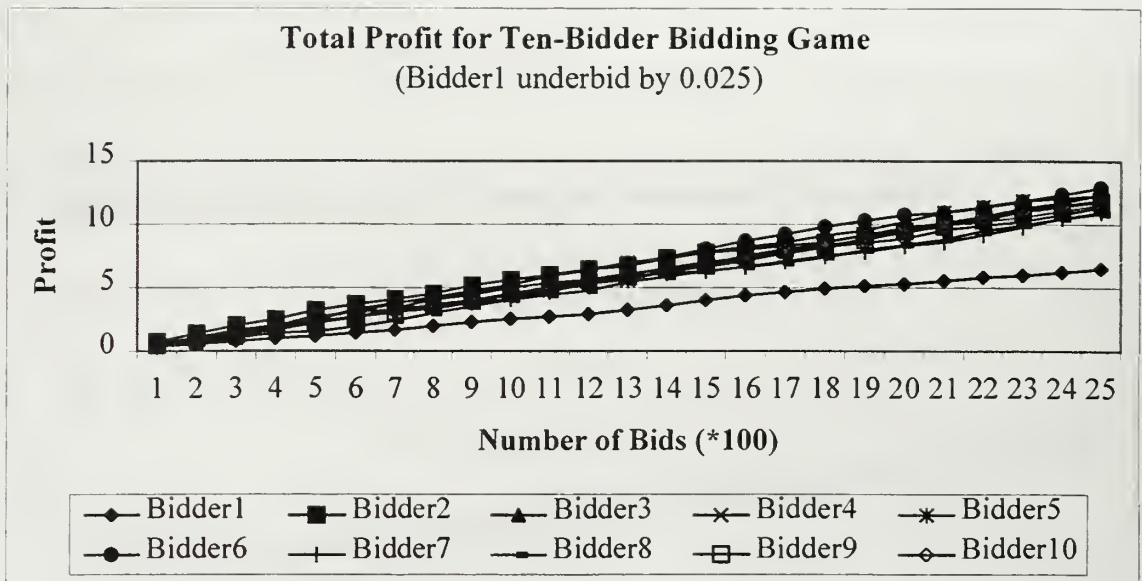


Figure 91

In the third experiment, bidders "1" and "2" underbid by 0.025 while others bid according to the equilibrium strategy. As a result of 2500 games, bidders "1" and "2" made expected profits, 0.0225 and 0.023, while other bidders' remained at 0.048 again. The expected profit graph is shown in Figure 92:

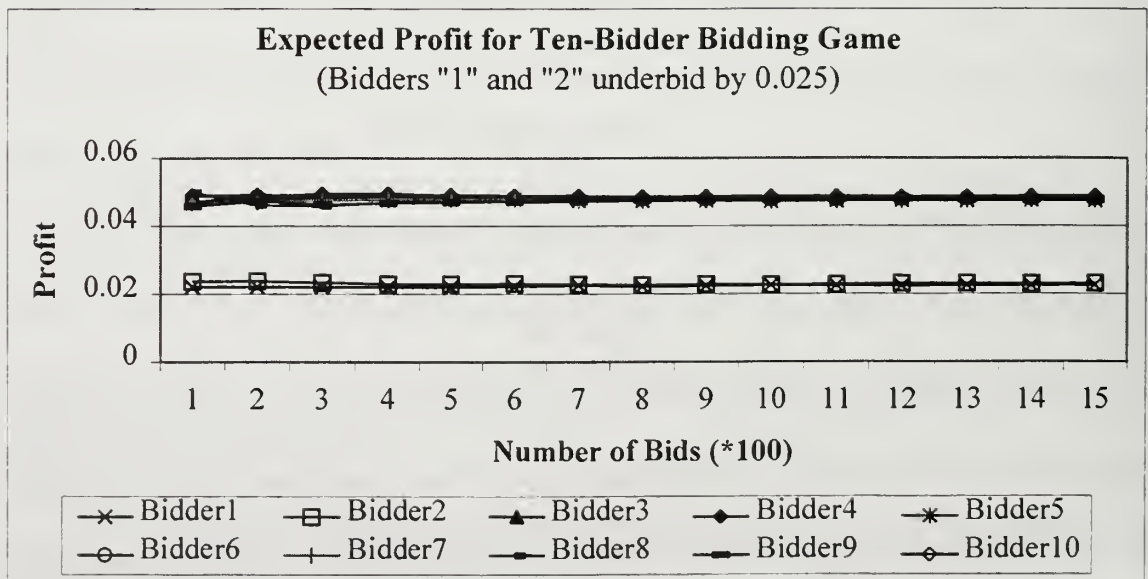


Figure 92

Average expected profits for the bidders also resulted in the same trend as the expected profits did. Underbidding bidders' average profits were 0.00277 and 0.00248, while others were almost the same, ranging from 0.0043 to 0.0054, as graphically shown in Figure 93.

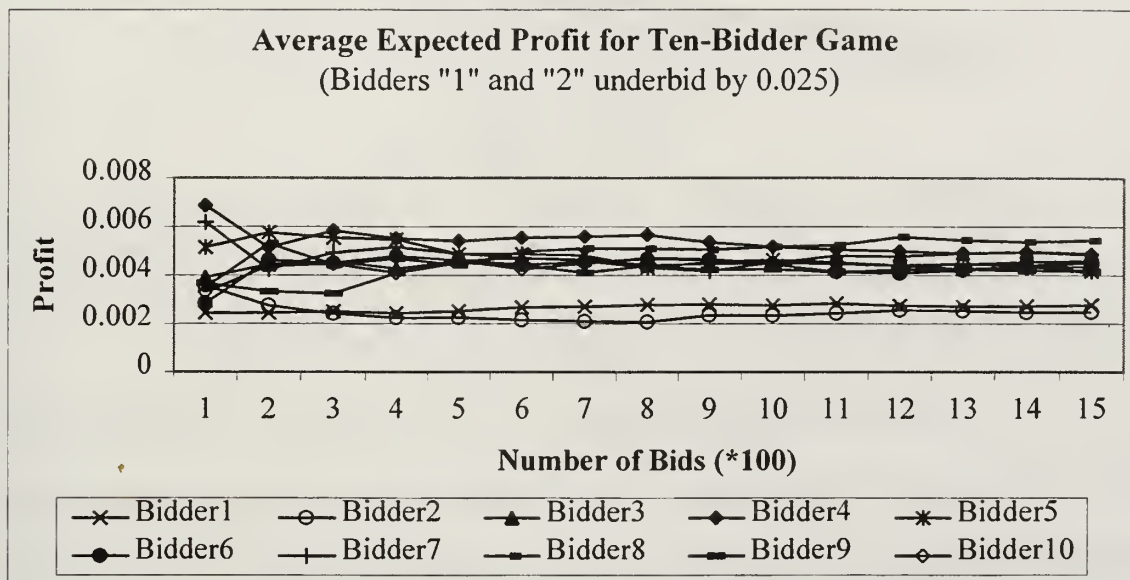


Figure 93

Bidders "1" and "2" won 184 and 161 games respectively while others won in a range of 131 to 170 games. Total profits for underbidding bidders were 4.1538 and 3.7136, which are much lower than the others' profits ranging from 6.234 to 8.1608. Total profit graph for this experiment is shown in Figure 94.

Overall results of the experiment are shown in Tables 69 through 72 in Appendix C.

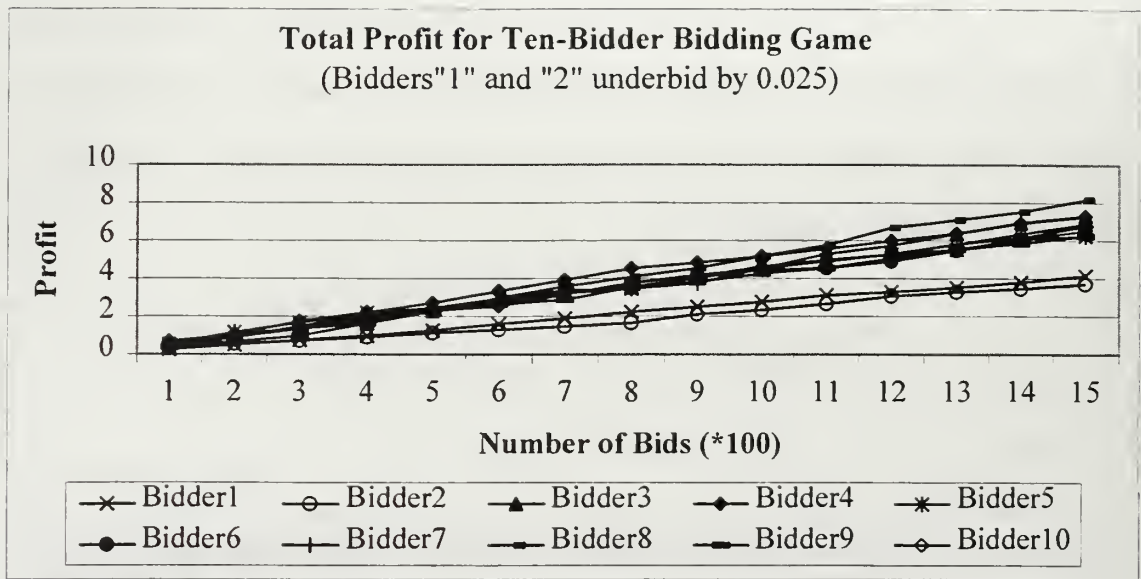


Figure 94

In the last experiment, bidders "1", "2", "3" and "4" underbid by 0.03, 0.025, 0.02 and 0.015 respectively in 2000 games. The remaining six bidders used equilibrium strategy. The underbidding bidders made lower profits, 0.0074, 0.0129, 0.0174 and 0.0225, than the equilibrium strategy bidders did, which was 0.048 again. The expected profit curves are shown in Figure 95.

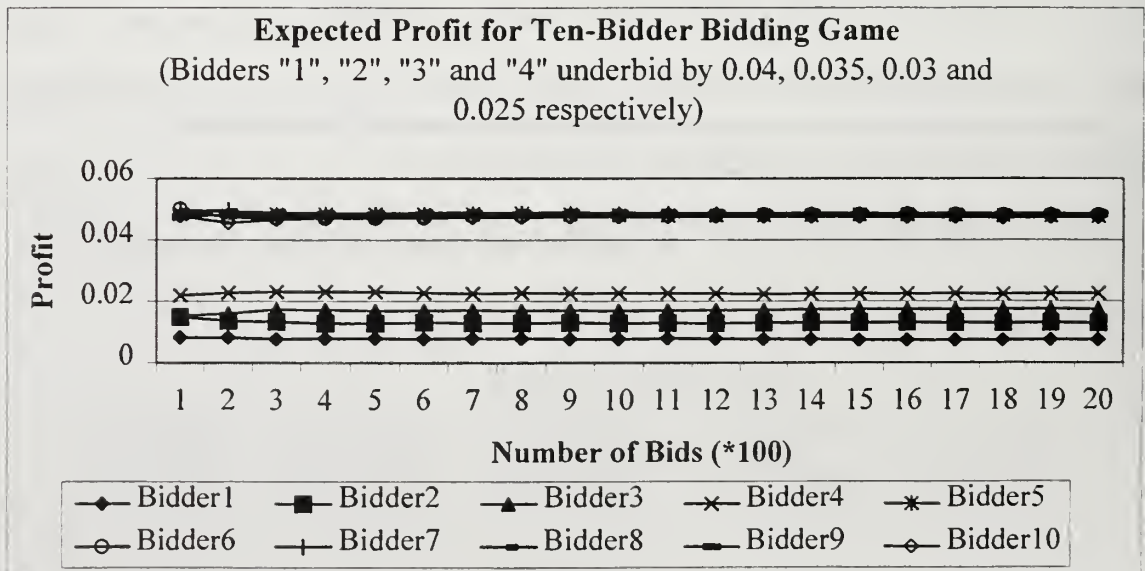


Figure 95

Average profit results were almost identical to similar previous experiments. The equilibrium strategy bidders made higher average profits as seen in Figure 96. The underbidding bidders average profits were 0.001, 0.0018, 0.0021 and 0.0023 while equilibrium strategy bidders' ranged from 0.0036 to 0.0043.

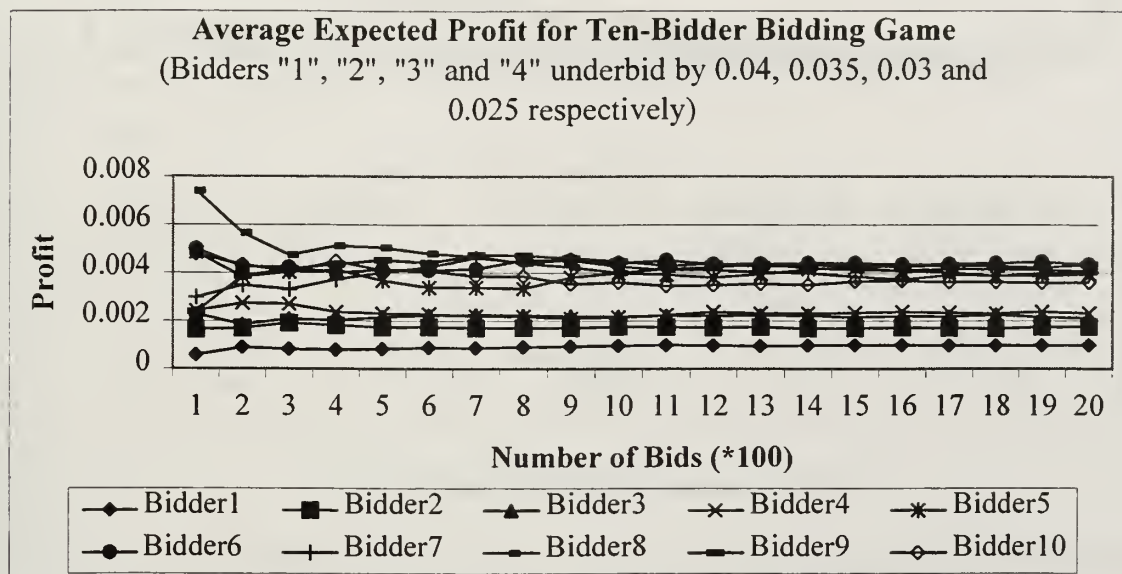


Figure 96

The underbidding bidders won more games than the others did. They won the games in a range of 206 to 273 times while equilibrium strategy bidders won in a range of 152 to 180 times. Despite this result, the equilibrium bidders made higher total profits than underbidding bidders as seen in Table 75 in Appendix C. According to this, underbidding bidders made profits; 1.9694, 3.5223, 4.2502 and 4.6414 respectively, while others did in a range of 7.1935 to 8.7005. The total profit curves are graphically shown in Figure 97.

Overall detailed results of the experiment are shown in Tables 73 through 76 in Appendix C.

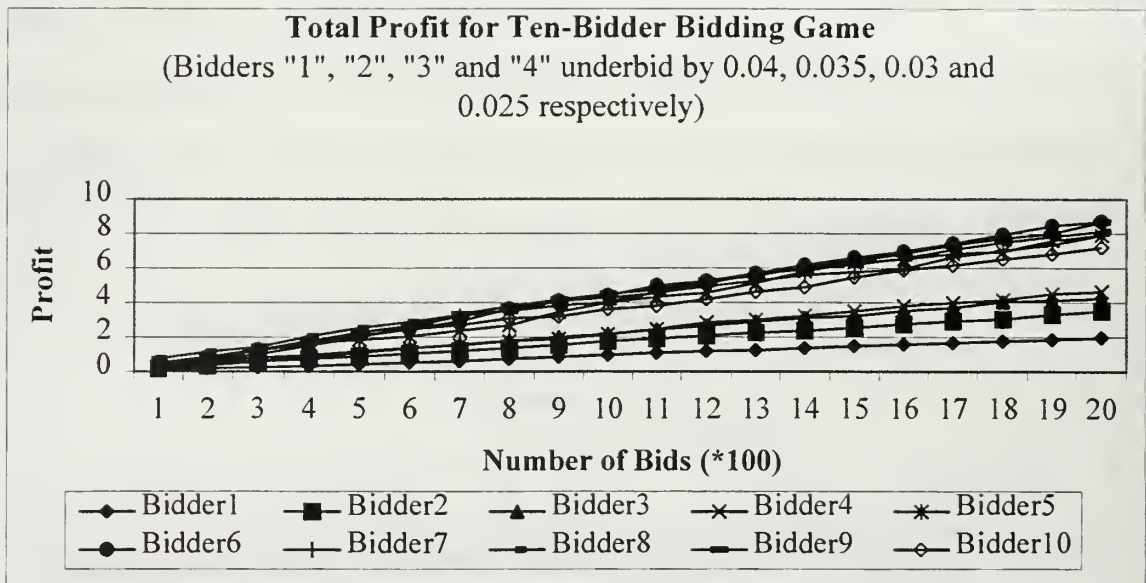


Figure 97

f. Experimentation With Fifteen Bidders

The simulation was conducted with fifteen bidders according to triangular cost distribution within interval $[0, 2]$ with a mode value of 2.

In these last series of experiments under triangular cost distribution, four different scenarios used to simulate the bidding process. In the first scenario, all bidders used equilibrium strategies. In the second one, bidder1 underbid while others used the equilibrium strategy again. Third experiment was conducted as bidders "1", "2" and "3" underbid by the same amount while all others used the equilibrium strategy. Finally, in the last scenario, bidders "1", "2", "3", "4" and "5" underbid by different amounts while the rest used the equilibrium strategy.

The first experiment of which all used equilibrium strategies resulted in a well-defined equilibrium. All bidders' expected profit approached 0.032 as the number of

games approached 1500. The graphical representation of the expected profit results is shown in Figure 98.

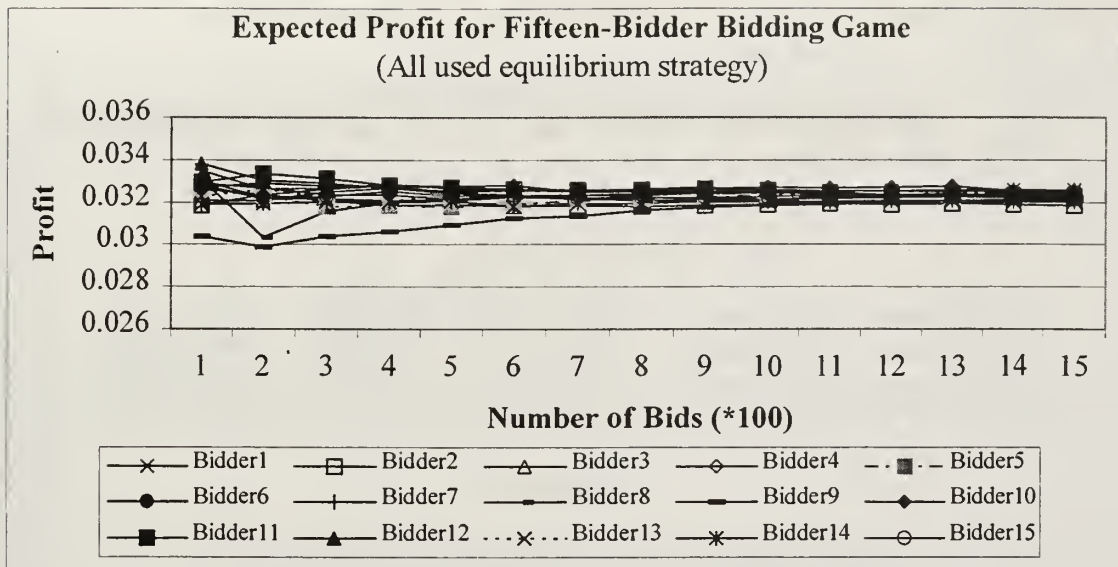


Figure 98

As it was the result of the previous experiments, introducing more bidders to the game reduced the expected profit from bidding more. The results of the experiment are outlined in Tables 77 through 80 in Appendix C.

The average expected profit graph is shown in Figure 99. The average expected profit graph also reinforces the equilibrium of the game. In this simulation, bidders' average expected profit approached approximately 0.002 while the number of games approached 1500.

As seen in Table 78, each bidder won the games approximately equal number of times. Overall, bidders won in a range of 82 to 139 games each. Total profit for bidders as seen in Figure 100, ranged between 2.6708 and 4.1422.

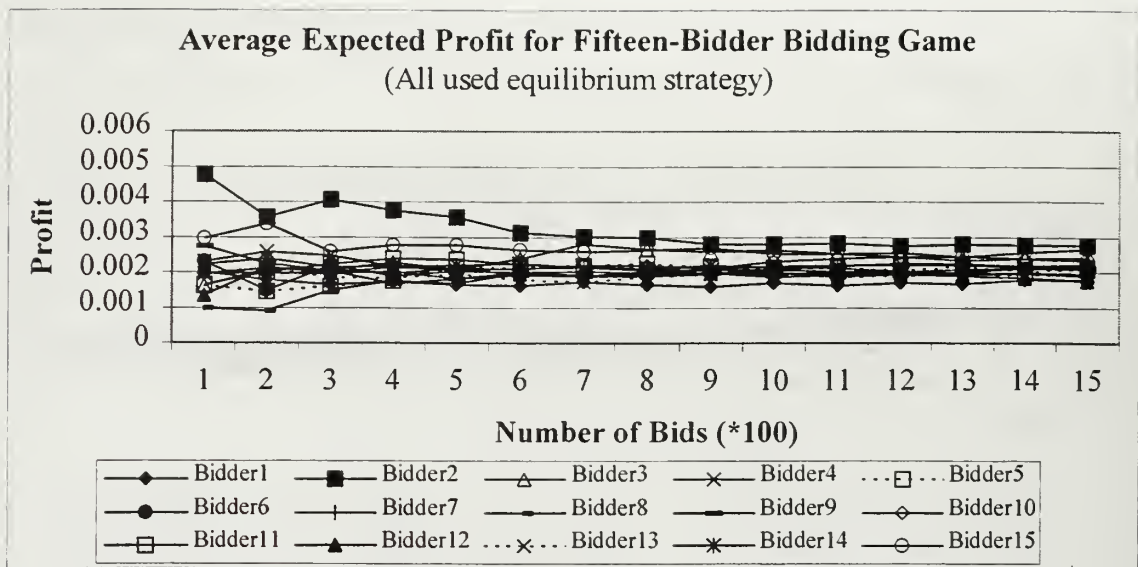


Figure 99

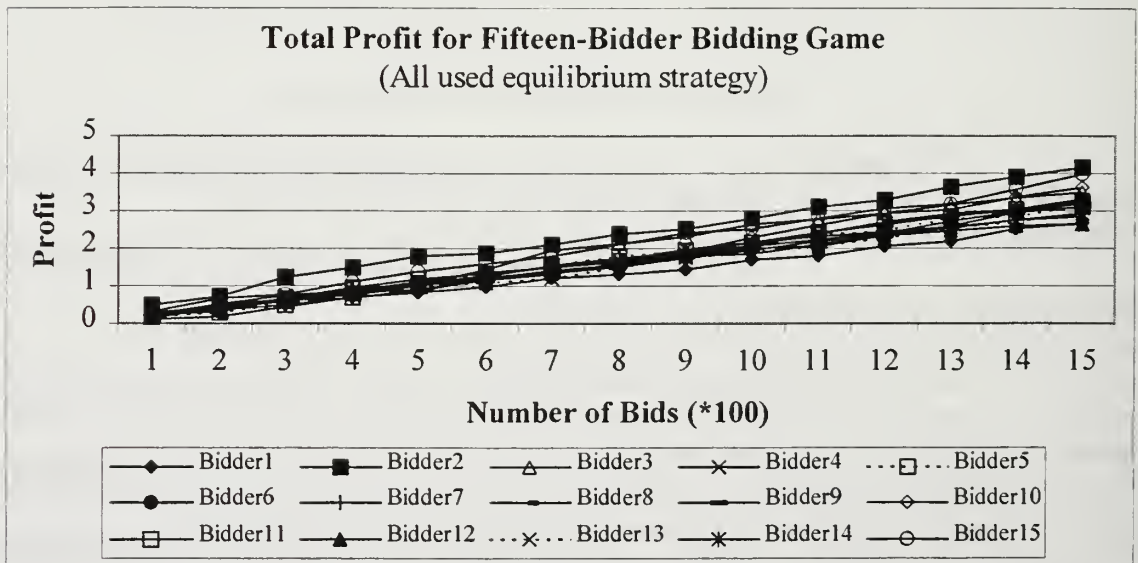


Figure 100

In the next experiment, bidder1 underbid by 0.015 while others bid according to the equilibrium strategy. As a result of 1500 games, bidder1's expected profit decreased to 0.01735 while the other bidders' remained at 0.032. The expected profit graph is shown in Figure 101.

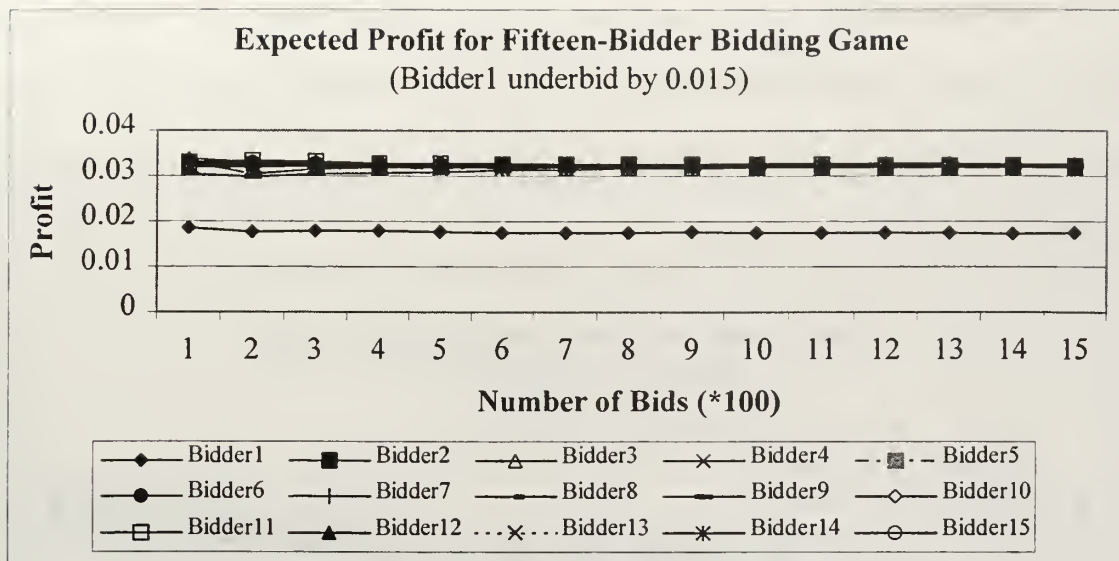


Figure 101

Average profit for the bidders also reinforced the same scheme as the equilibrium game had shown. Bidder1's average profit was 0.0011 while others ranged from 0.00174 to 0.00272, as graphically shown in Figure 102.

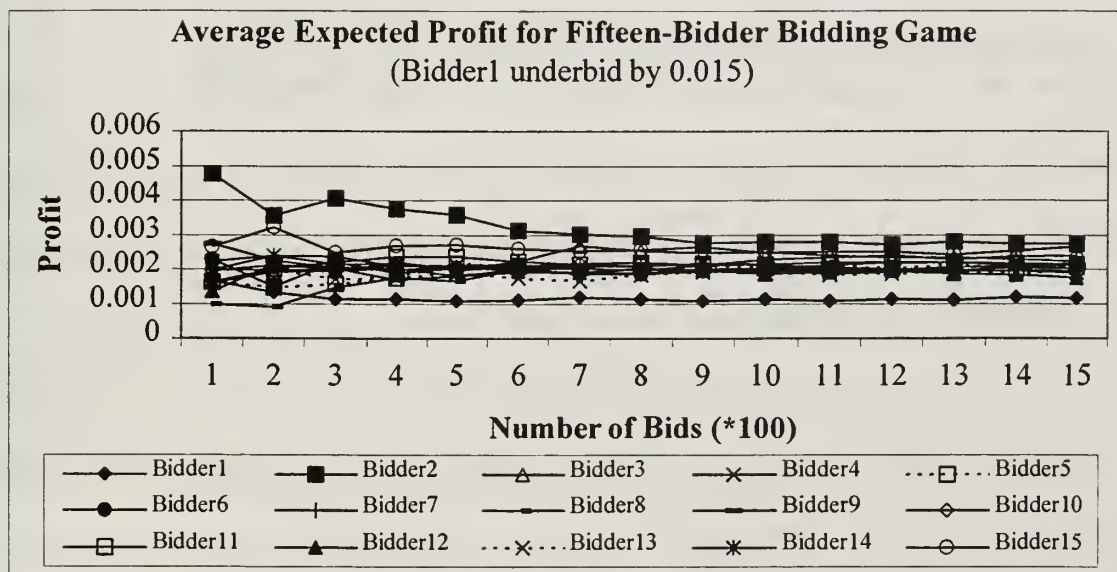


Figure 102

Bidder1 won 100 of 1500 games while others won in a range of 81 to 128 games as shown in Table 82 in Appendix C. Although bidder1 underbid, it won less

games than some of the others won. Total profit for bidder1 was 1.735, which is lower than the others' profits ranging from 2.6137 to 4.0855. Total profit graph for this experiment is shown in Figure 103 and detailed results of the experiment are shown in Tables 81 through 84 in Appendix C.

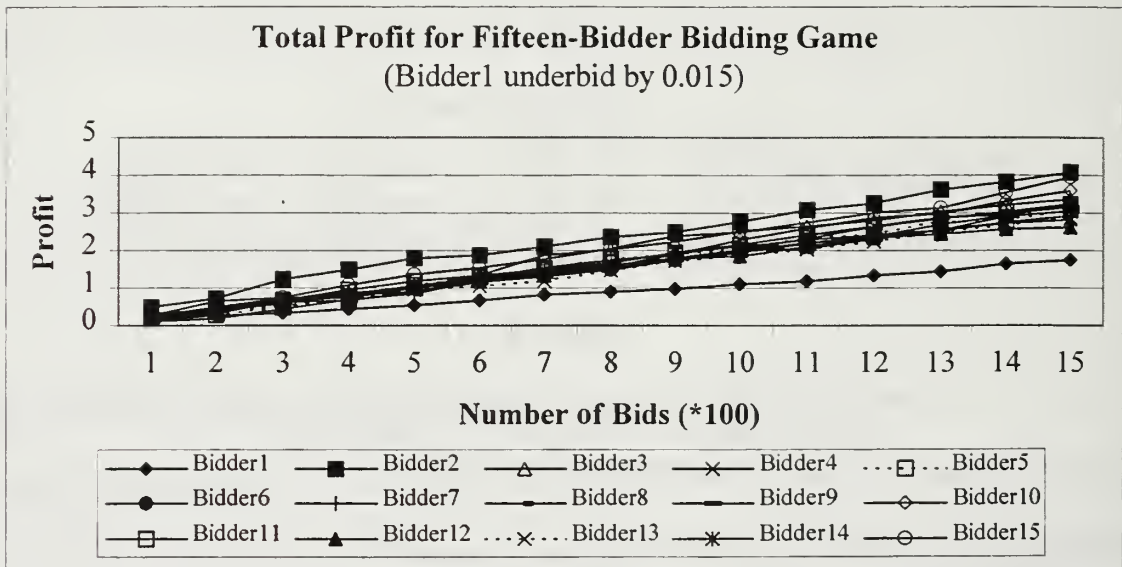


Figure 103

In the third experiment, bidders “1”, “2” and “3” underbid by 0.02 while others used equilibrium strategy in 2000 games. As a result of the experiment, underbidding bidders' expected profit approached 0.0125 while others' approached 0.032. The expected profit graph is shown in Figure 104.

Average expected profit results were parallel to the expected profit results. Underbidding bidders' average profits approached 0.001 while others' approached approximately 0.002 in 2000 games. This result along with all previous experiments showed that there is indeed a strong equilibrium in contract games and any underbidding bidder would get less profit than equilibrium strategy bidders. The average expected profit graph is shown in Figure 105.

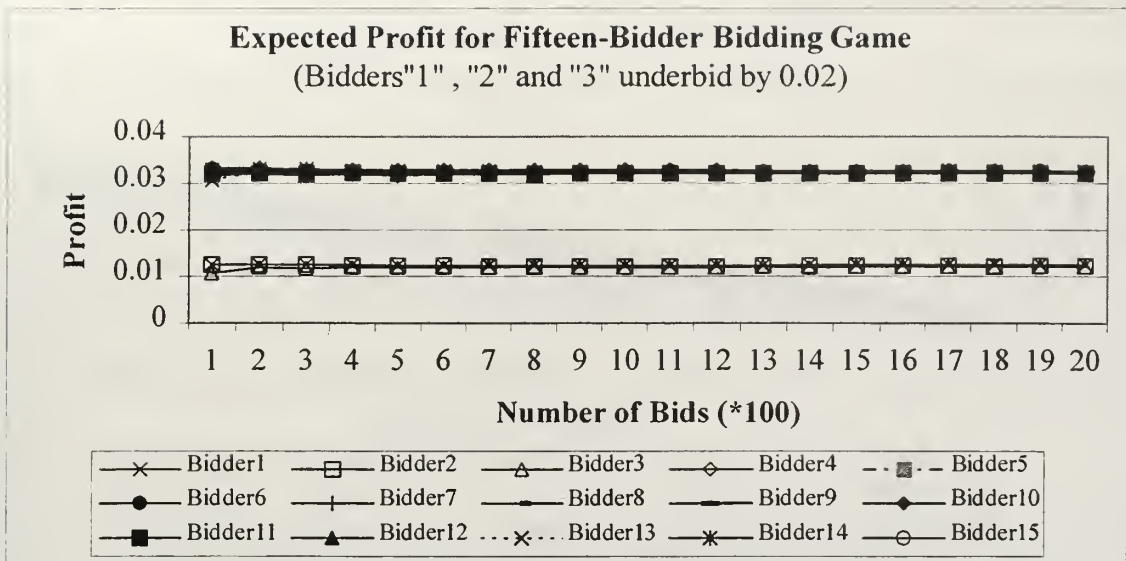


Figure 104

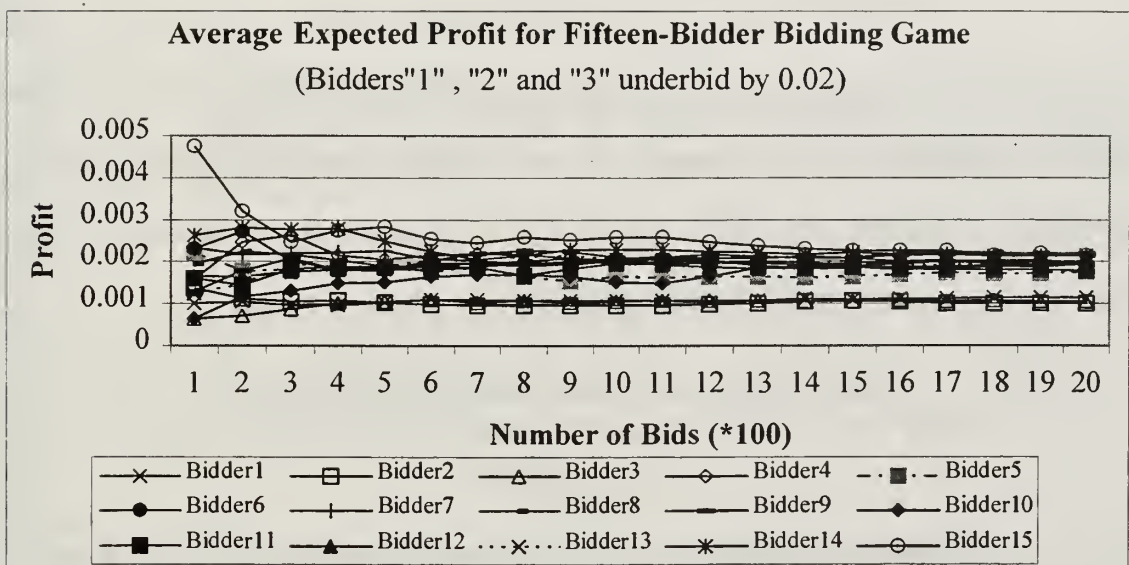


Figure 105

Total profits for underbidding bidders, ranging from 2.0129 to 2.277, were also lower than the equilibrium strategy bidders' total profits, ranging from 3.5074 to 4.3862. The underbidding bidders won 165 to 182 games as opposed to others won games of 108 to 135. Total profit curves are shown in Figure 106 and the results of the experiment are outlined in Tables 85 through 88 in Appendix C.

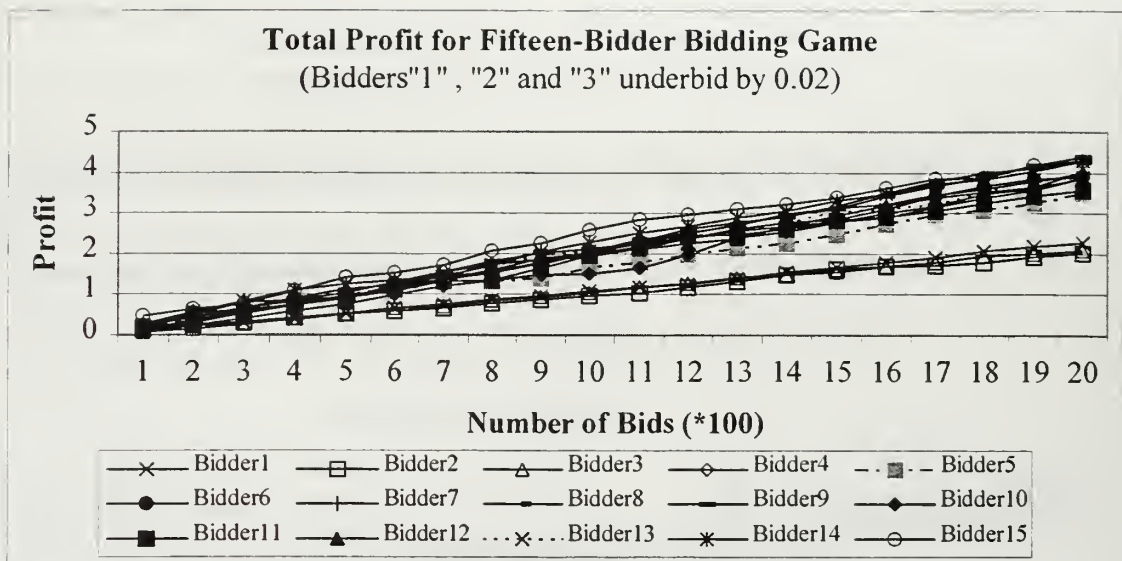


Figure 106

In the last experiment with fifteen bidders, bidders "1", "2", "3", "4" and "5" underbid by 0.025, 0.0225, 0.02, 0.0175 and 0.015 respectively, while others used equilibrium strategy in 1500 games. As a result of the experiment, underbidding bidders' expected profit approached 0.0074, 0.0095, 0.012, 0.014 and 0.017 respectively, while others' approached 0.032. The expected profit graph is shown in Figure 107.

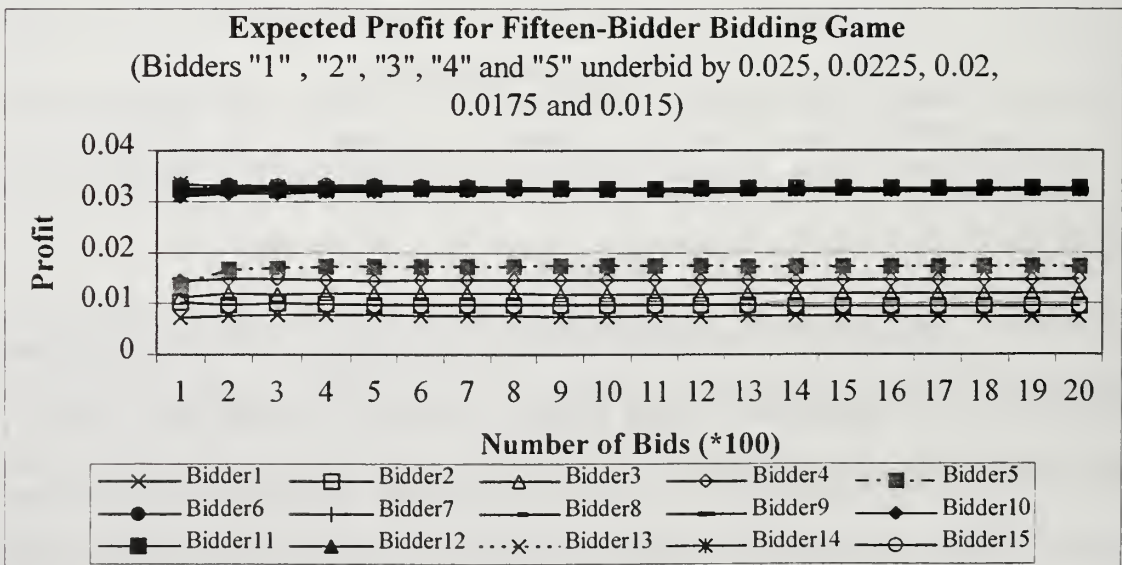


Figure 107

The average expected profit results were parallel to the expected profit results. Underbidding bidders' average profits approached 0.0006, 0.0009, 0.00091, 0.0011 and 0.0011 respectively while others' approached a range between 0.0017 and 0.0023 in 2000 games. This result along with all previous experiments showed that there is indeed a strong equilibrium in contract games. The deviant bidders lose profit even if they win more games than the equilibrium strategy bidders. The average expected profit graph for this experiment is shown in Figure 108.

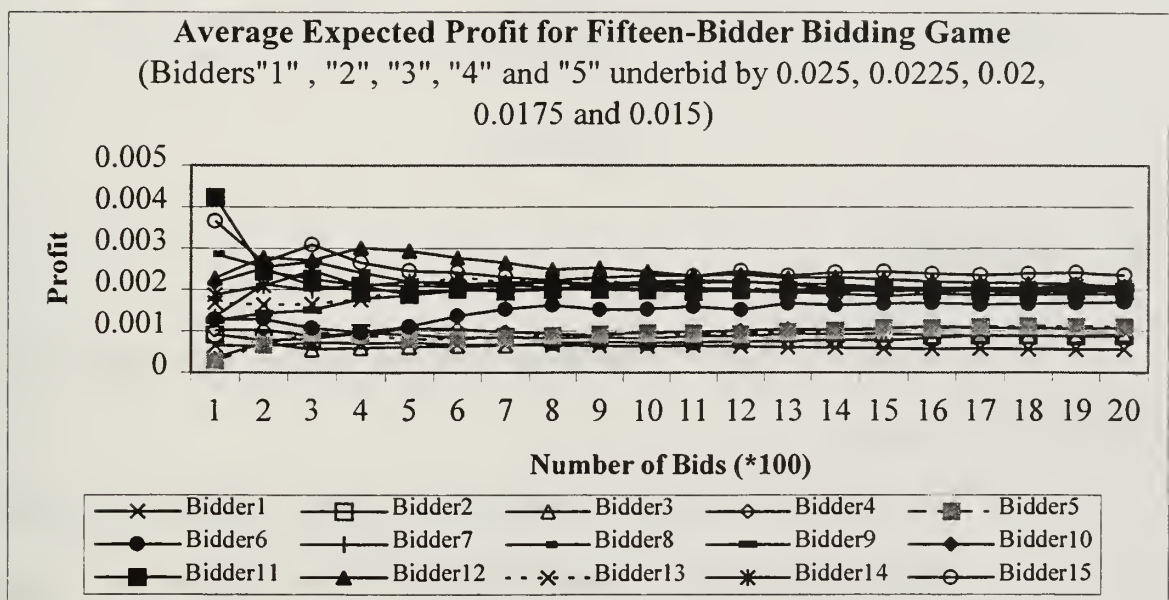


Figure 108

Total profits for underbidding bidders, ranging from 1.1038 to 2.2021, were also lower than the equilibrium strategy bidders' total profits, ranging from 3.4011 to 4.6884. The underbidding bidders won 127 to 186 games as opposed to others, who won the games 108 to 135 times. Total profit curves are shown in Figure 109 and results of the game are outlined in Tables 89 through 92 in Appendix C.

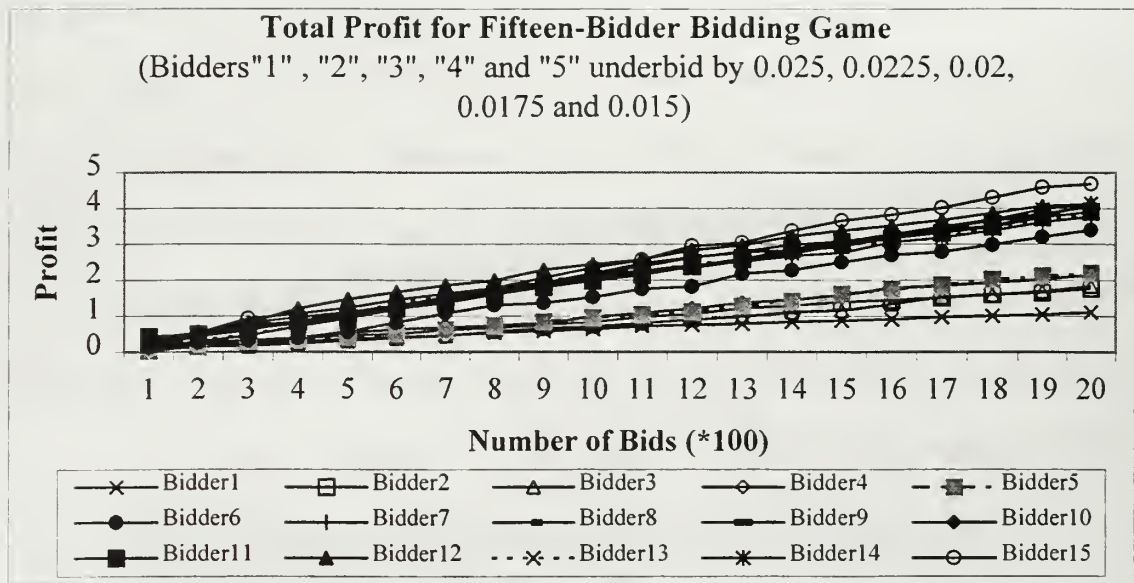


Figure 109

D. SUMMARY

After analyzing the results of experiments with both uniform and triangular cost distributions, we can say:

1. The equilibrium strategy maximizes the bidders expected profit and average expected profit. Underbidding bidders' both expected and average expected profits are lower than the equilibrium strategy bidders' profits.
2. If one of the bidders underbid, it reduces not only its total profit but also other bidders' total profits.
3. Underbidding bidders always won more games assuming they were responsive and responsible bidders.
4. Changing the cost distribution did not effect the general trend in simulations, but substantially altered the expected quantitative outcomes of the simulated FPSB in all simulations. The difference between the

expected profit outcomes resulted from the probability distributions' characteristics.

5. Adding additional competitors to the game does not change the general trend found in the games, but reduces both the expected and average expected profits. In fact, the number of competing bidders is one of the most influential factors in FPSB simulations.
6. Different scenarios used throughout the simulations showed that the more a bidder underbids the less profit it makes.

IV. CONCLUSION

This chapter of the thesis will review and discuss the research questions written in Chapter I. In answering these questions, the findings of the experiments will be the foundation for discussion and conclusions. Following this section, recommendations will be provided and areas for further research will also be identified.

A. ANSWERS TO RESEARCH QUESTIONS

The primary question for this research was: “*How do profit maximizing bidders choose their bids in a competitive environment?*” In answering this question, discussing and answering the subsidiary questions will be helpful, that is why, first the subsidiary questions will be answered.

1. Answers to Subsidiary Questions

a. Subsidiary Question 1

“*What are the equilibrium strategies of the bidders?*” As it was analyzed and discussed in Chapter II of the thesis, the bidding for contract game had been identified as a static non-cooperative game with incomplete information. The theory of contract games predicts that these games have an equilibrium. This thesis found a strong and well-defined equilibrium in the games during the simulation and experimentation as discussed in Chapter III.

This thesis used uniform and triangular production cost distributions for bidders. Then, equilibrium strategies, also called “bidding functions”, were developed in

Chapter III, under these two distributions. These strategies were used to develop the simulation programs and conduct the experimentation.

The first experiments were conducted under a uniform cost distribution with several different numbers of bidders and scenarios. As the bidders used the developed theoretical equilibrium strategy, the experiments resulted in very close outcomes that showed the existence of a well-defined equilibrium in these games. This is called symmetric Bayesian Nash equilibrium.

The resulting game equilibrium and the proven existence of equilibrium strategy provide the opportunity of predicting the behavior of the prospective suppliers. However, the reader should be careful about the probabilistic characteristics of the simulated games. The simulation cannot predict the actual behavior of the bidders in a particular bid, but it can approximate the general trend and expected average winning bid with a reasonable accuracy.

b. Subsidiary Question 2

“Do the bidders have a dominant strategy in FPSB?” An equilibrium strategy should maximize the bidders’ profits simultaneously. If a strategy maximizes the bidders’ profits, no bidder is willing to deviate from this strategy. This is the equilibrium of the game. This equilibrium condition was experimented with during the simulations and found that whatever the number of underbidding bidders in a game, deviations from the equilibrium strategy resulted in an expected loss for the underbidding bidders. In some of the experiments, scenarios were set up as if all the bidders did not use the equilibrium strategy and the result was they all made lower profits. The bidders choose

their bids (i.e., their strategies) based on their assumptions of the decisions of other bidders. That is, there is not a dominant strategy for the bidders in FPSB. The assumption for the information symmetry made it possible to analyze only the symmetric Nash equilibrium.

c. Subsidiary Question 3

“How does the number of bidders affect the outcome of the FPSB?”

Number of bidders was found to be one of the most influential factors affecting the game outcomes in FPSB.

Experiments were conducted with up to 15 bidders under both uniform and triangular cost distributions. The simulation programs in Appendices A and B were designed to experiment with up to 25 and 16 bidders respectively. As discussed throughout Chapter III of the Thesis, as the number of bidders increased, bidders' profits decreased. To be able to analyze this situation, a series of experiments were conducted with different numbers of bidders. Figure 110 shows the results of these experiments. As the Figure indicates, the bidders' expected profits decrease as the number of bidders increases from two to fifteen.

In conclusion, the number of participants in FPSB significantly affects the outcome by influencing the bidders' decisions in FPSB. The bidders consider it a potential factor affecting the probability of winning the bidding game. The higher the number of bids, the higher the probability of that any given bid will not be the lowest bid. Therefore, expected bid price decreases as the number of bidders increases, though, expected profit for the winners decreases.

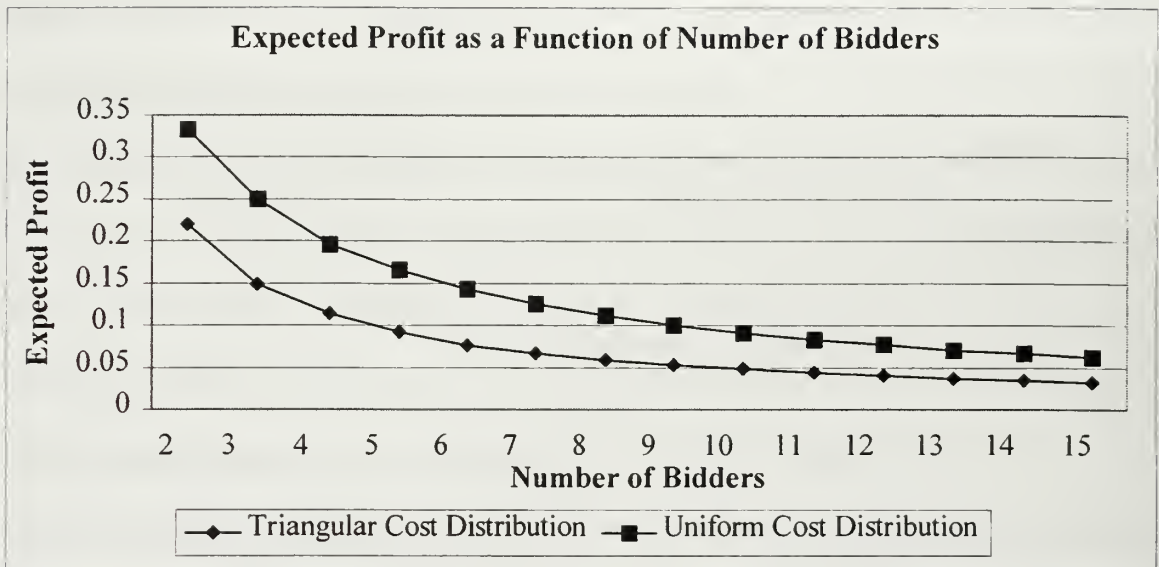


Figure 110

d. Subsidiary Question 4

“How does the cost distribution affect the outcome of the FPSB?”

Changing the production cost distribution did not affect the general pattern of the FPSB. However, it had a significant effect on the value of bids and the bidders' expected profits. To be able to compare the outcomes of the two simulations under these two distributions, another experiment was conducted under uniform distribution over the cost interval $[0, 2]$ with two bidders. The bidders' expected profit was 0.667 under uniform cost distribution when using equilibrium strategy. This is almost three times higher than the equilibrium strategy profit of 0.22 under triangular distribution over the same cost interval with mode $[1]$.

The change in the cost distribution has a significant effect on the FPSB outcomes. However, it is not just the cost distribution that affects the bidding outcomes; bidders' perceptions about the other bidders' production costs, influences their bids as

well. Because, as discussed in the first secondary question, bidders choose their bidding strategies according to their guess on the other participants probable behavior and production costs. This thesis assumed that the bidders' cost distributions are common knowledge among the bidders and identical to each other. It did not analyze the potential influence of bidders' cost assumptions on the other participants.

In reality, bidders do not have specific information about the other participants' cost distributions. They have some perceptions about it. Sometimes they develop their perceptions based on the signals received from other bidders. It is in the senders interest to distort this information, mainly in an increasing direction. The bidders have to be aware of this strategic misrepresentation.

d. Subsidiary Question 5

“How can the procuring agencies use the findings of “Game Theory” in their acquisition practice?” The simulations revealed that the equilibrium strategy is the strategy that maximizes the bidders' expected profits simultaneously. The suppliers will most likely use this strategy when formulating their bids in an actual bidding. Procuring agencies can use this finding to project the expected winning bid. The agencies can use these projections to forecast the expected spending on procurement. If the procuring agencies choose to negotiate with the offerors, they can develop negotiation positions and prepare the negotiators. The essence of these projections is forecasting the accurate expected cost range. In other words, this finding can help procuring agencies to develop competitive ranges for the bids.

Findings regarding the number of bidders and their effect on the expected bid can be used to establish the number of bidders to be invited. Although, the benefits of competition are known throughout the countries with free economy, there aren't many defense industry firms developing unique defense systems, such as aircraft, ships, missiles and so on. Therefore, the simulation of the FPSB can provide some insight through analysis of bidders behavior under changing situations.

2. Primary Research Question and Discussion

Bidders in a FPSB have some information available to them on which they base their bids. This thesis assumed symmetry of information among bidders. That is, all bidders' production cost distributions are common knowledge and are identical. However, the bidders' own production cost is private information. Another important factor in building the FPSB model was the number of bidders.

Bidders consider their own production costs, other bidders' production cost distribution and the number of competing bidders while preparing their bids. Since they have their own production cost information, they estimate their production cost for a given work while projecting their competitors' production costs based on experience, signals from them etc. Based on these assumptions, it can also be said, in a competitive world, bidders should know their potential competitors' bidding habits and production potentials.

This thesis applied two probability distributions on the production costs, uniform and triangular distributions. It is assumed that uniform cost distribution is applicable to the production of standard, non-complex items while triangular cost distribution is

applicable to more complex productions and where some firms have production advantages over the competitors.

It is furthermore assumed that, bidders are risk neutral and act to maximize their profits. It is this assumption that made it possible for us to construct the FPSB model and derive the equilibrium bidding functions. Bidders want to maximize their profits but their bid should be low enough to have a reasonable chance to win the award. This thesis found that if the bidders act rationally (i.e., bid to maximize their profits), they should use the equilibrium strategy. Simulation of FPSB also provided an opportunity for bidders to analyze tradeoffs between profit maximizing and probability of winning the award.

One of the results of the experiments with the FPSB was underbidding bidders won significantly more games than the equilibrium strategy bidders. This was the result of the artificially low bids they chose. However, these bidders made lower profits than the equilibrium strategy bidders. That is, equilibrium strategy bidders made higher profits in less number of contracts. Therefore, they can devote their time to other areas, which means they have more time to gain more profits.

First Price Sealed Bidding is an ancient market institution, which has been used for thousands of years. Bidders' behavior in FPSB is determined by several factors and perceptions. This thesis used the game theory approach in microeconomics to describe and analyze this behavior. However, there are several different approaches in game theory to construct different models or a model including many different approaches in one simulation. The game simulation developed in this thesis is one of the methods to explore the FPSB. However, simulation gives some insight to the reality and one should not confuse the simulation with reality. Data resulted from the simulation give an average

information about the real life. The model does not prescribe the bidders' behavior; it forecasts and describes the average behavior expected from the players over time.

B. AREAS FOR FURTHER RESEARCH

There are several areas for further research related to this thesis. Some are as follows:

1. The second price sealed bidding is one of the top areas for further research. Simulation of the second price sealed bidding technique, and experimentation would probably provide more insights to the auctioning theory. It can also be said that, since second price sealed bidding is being experimented to be an alternative for FPSB because of inefficiencies described as “winner’s curse” and “buyer’s angst”[Ref. 13], analysis of this technique would surely be a valuable research.

2. The competitive range is a measure established by the buyer to influence the bidders' behavior. Further research would analyze the effect of this measure on the FPSB.

APPENDIX A

SIMULATION PROGRAM WITH UNIFORM COST DISTRIBUTION

Sub Uniform-Simulation ()

' Uniform-Simulation Macro

' Macro recorded 8/28/97 by Suat Tozendenmir

'Input for experiment

Sheets("sheet1").Select

Range("a1").Select

ActiveCell.FormulaR1C1 = "Write # of Bidders (n) [2,25]"

Range("A2").Select

ActiveCell.FormulaR1C1 = "Write Lower Limit of Uniform Cost Dist. (k)"

Range("A3").Select

ActiveCell.FormulaR1C1 = "Write Upper Limit of Uniform Cost Dist. (h)"

Columns("B:B").Select

'Page design

Range("A1:B3").Select

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.Weight = xlThin

.ColorIndex = xlAutomatic

End With

With Selection.Borders(xlEdgeTop)

.LineStyle = xlContinuous

.Weight = xlThin

.ColorIndex = xlAutomatic

End With

With Selection.Borders(xlEdgeBottom)

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.Weight = xlThin

.ColorIndex = xlAutomatic

End With

With Selection.Borders(xlEdgeRight)

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.Weight = xlThin

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End With

With Selection.Borders(xlInsideVertical)

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End With

With Selection.Borders(xlInsideHorizontal)

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End With

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.Weight = xlMedium

.ColorIndex = xlAutomatic

End With

With Selection.Borders(xlEdgeBottom)

.LineStyle = xlContinuous

.Weight = xlMedium

.ColorIndex = xlAutomatic

End With

With Selection.Borders(xlEdgeRight)

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.Weight = xlMedium

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End With

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.ColorIndex = xlAutomatic

End With

With Selection.Borders(xlInsideHorizontal)

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.Weight = xlThin

.ColorIndex = xlAutomatic

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Range("E5").Select

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Range("F5").Select

ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-4]>=5, ""Cost5"", "" "")"

Range("G5").Select

ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-5]>=6, ""Cost6"", "" "")"

Range("H5").Select

ActiveCell.FormulaR1C1 = "=IF(R[-4]C[-6]>=7, ""Cost7"", "" "")"

Range("I5").Select

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Range("J5").Select

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Range("V5").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=21, ""Cost21"", "" "")"

Range("W5").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=22, ""Cost22"", "" "")"

Range("X5").Select

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'Assigning payoff variables

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For I = 1 To 1500 ' NUMBER OF EXPERIEMENTS TO BE DONE

'Calculation of costs

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Range("C6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=2, ((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2), "" "")"

Range("D6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=3, ((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2), "" "")"

Range("E6").Select

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=4,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("F6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=5,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("G6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=6,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("H6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=7,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("I6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=8,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("J6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=9,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("K6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=10,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("L6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=11,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

Range("M6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=12,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""""")"

Range("N6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=13,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""""")"

Range("O6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=14,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""""")"

Range("P6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=15,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""""")"

Range("Q6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=16,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""""")"

Range("R6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=17,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""""")"

Range("S6").Select

ActiveCell.FormulaR1C1 = "=IF(R1C2>=18,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""""")"

Range("T6").Select

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=19,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("U6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=20,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("V6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=21,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("W6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=22,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("X6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=23,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("Y6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=24,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

```
Range("Z6").Select
```

```
ActiveCell.FormulaR1C1 = "=IF(R1C2>=25,((Sheet1!R3C2-Sheet1!R2C2)*RAND()+Sheet1!R2C2),""")"
```

'Copying costs to another row

```
Sheets("sheet1").Select
```

```
Range("b7").Select
```



```

Selection.EntireRow.Insert
Range("b6:z6").Select
Selection.Copy
Range("b7").Select
Selection.PasteSpecial Paste:=xlValues
Application.CutCopyMode = False
Range("b6:z6").Select
Selection.Clear

```

'Calculation of bids

```

Sheets("sheet2").Select

    Range("A2").Select

ActiveCell.FormulaR1C1 = _
    "=IF(Sheet1!R1C2>=2,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"

Range("B2").Select

ActiveCell.FormulaR1C1 = _
    "=IF(Sheet1!R1C2>=2,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"

Range("C2").Select

ActiveCell.FormulaR1C1 = _
    "=IF(Sheet1!R1C2>=3,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"

Range("D2").Select

```

```

ActiveCell.FormulaR1C1 = _

    "=IF(Sheet1!R1C2>=4,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"

Range("E2").Select

ActiveCell.FormulaR1C1 = _

    "=IF(Sheet1!R1C2>=5,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"

Range("F2").Select

ActiveCell.FormulaR1C1 = _

    "=IF(Sheet1!R1C2>=6,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"

Range("G2").Select

ActiveCell.FormulaR1C1 = _

    "=IF(Sheet1!R1C2>=7,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"

Range("H2").Select

ActiveCell.FormulaR1C1 = _

    "=IF(Sheet1!R1C2>=8,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"

Range("I2").Select

ActiveCell.FormulaR1C1 = _

    "=IF(Sheet1!R1C2>=9,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"" "")"

```

Range("J2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=10,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("K2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=11,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("L2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=12,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("M2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=13,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("N2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=14,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("O2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=15,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("P2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=16,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("Q2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=17,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("R2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=18,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("S2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=19,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("T2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=20,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

Range("U2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet1!R1C2>=21,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"

```
Range("V2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet1!R1C2>=22,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-  
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"
```

```
Range("W2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet1!R1C2>=23,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-  
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"
```

```
Range("X2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet1!R1C2>=24,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-  
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"
```

```
Range("Y2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet1!R1C2>=25,((Sheet1!R3C2-Sheet1!R2C2)+(Sheet1!R1C2-  
1)*Sheet1!R[5]C[1])/Sheet1!R1C2,"")"
```

'Copying bids to another row

```
Sheets("Sheet2").Select
```

```
Range("a3").Select
```

```
Selection.EntireRow.Insert
```

```
Sheets("sheet2").Select
```

```
Range("a2:y2").Select
```

```
Selection.Copy
```



```
Sheets("Sheet2").Select
```

```
Range("a3").Select
```

```
Selection.PasteSpecial Paste:=xlValues
```

```
Application.CutCopyMode = False
```

```
Sheets("Sheet2").Select
```

```
Range("a2:y2").Select
```

```
Selection.Clear
```

'Selection of winner and its payoff

```
Sheets("sheet3").Select
```

```
Range("A2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C1=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C1-
```

```
Sheet1!R[5]C[1],""""")"
```

```
Range("b2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C2=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C2-
```

```
Sheet1!R[5]C[1],""""")"
```

```
Range("c2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C3=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C3-
```

```
Sheet1!R[5]C[1],""""")"
```

```
Range("d2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

"=IF(Sheet2!R3C4=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C4-
Sheet1!R[5]C[1],""""")"

Range("e2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C5=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C5-
Sheet1!R[5]C[1],""""")"

Range("f2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C6=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C6-
Sheet1!R[5]C[1],""""")"

Range("g2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C7=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C7-
Sheet1!R[5]C[1],""""")"

Range("h2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C8=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C8-
Sheet1!R[5]C[1],""""")"

Range("i2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C9=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C9-
Sheet1!R[5]C[1],""""")"

Range("j2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C10=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C10-Sheet1!R[5]C[1],""""")"

Range("k2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C11=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C11-Sheet1!R[5]C[1],""""")"

Range("l2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C12=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C12-Sheet1!R[5]C[1],""""")"

Range("m2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C13=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C13-Sheet1!R[5]C[1],""""")"

Range("n2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C14=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C14-Sheet1!R[5]C[1],""""")"

Range("o2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C15=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C15-Sheet1!R[5]C[1],""""")"

Range("p2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C16=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C16-Sheet1!R[5]C[1],""""")"

Range("q2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C17=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C17-Sheet1!R[5]C[1],""""")"

Range("r2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C18=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C18-Sheet1!R[5]C[1],""""")"

Range("s2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C19=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C19-Sheet1!R[5]C[1],""""")"

Range("t2").Select

ActiveCell.FormulaR1C1 = _

"=IF(Sheet2!R3C20=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C20-Sheet1!R[5]C[1],""""")"

Range("u2").Select

ActiveCell.FormulaR1C1 = _

```
"=IF(Sheet2!R3C21=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C21-  
Sheet1!R[5]C[1],""""")"
```

```
Range("v2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C22=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C22-  
Sheet1!R[5]C[1],""""")"
```

```
Range("w2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C23=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C23-  
Sheet1!R[5]C[1],""""")"
```

```
Range("x2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C24=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C24-  
Sheet1!R[5]C[1],""""")"
```

```
Range("y2").Select
```

```
ActiveCell.FormulaR1C1 = _
```

```
"=IF(Sheet2!R3C25=MIN(Sheet2!R3C1:R2C25),Sheet2!R3C25-  
Sheet1!R[5]C[1],""""")"
```

'Copying winner's payoff to another row

```
Sheets("Sheet3").Select
```

```
Range("a3").Select
```

```
Selection.EntireRow.Insert
```

```
Sheets("sheet3").Select
```



```
Range("a2:y2").Select  
Selection.Copy  
Sheets("Sheet3").Select  
Range("a3").Select  
Selection.PasteSpecial Paste:=xlValues  
Application.CutCopyMode = False  
Sheets("Sheet3").Select  
Range("a2:y2").Select  
Selection.Clear  
Next  
Beep  
End Sub
```


APPENDIX B

SIMULATION PROGRAM WITH TRIANGULAR COST DISTRIBUTION

Sub Triangular-Simulation()

' Triangular-Simulation Macro

' Macro recorded 9/1/97 by Suat Tozendemir

Application.ScreenUpdating = False

'Assigning variables

Dim h, m, k, n, num, rand1, rand2, rand3, rand4, rand5, rand6, rand7, rand8, rand9,
cost1, cost2, cost3, cost4, cost5, cost6, cost7, cost8, cost9, cost10, cost11, cost12, cost13,
cost14, cost15, cost16 As Variant

Dim bid1, bid1a, bid1b, bid1c, bid2, bid2a, bid2b, bid3, bid3a, bid3b, bid4, bid4a,
bid4b, bid5, bid5a, bid5b, bid6, bid6a, bid6b, bid7, bid7a, bid7b, bid8, bid8a, bid8b, bid9,
bid9a, bid9b, bid10, bid10a, bid10b, bid11, bid11a, bid11b, bid12, bid12a, bid12b, bid13,
bid13a, bid13b, bid14, bid14a, bid14b, bid15, bid15a, bid15b, bid16, bid16a, bid16b As
Variant

Dim payoff1, payoff2, payoff3, payoff4, payoff5, payoff6, payoff7, payoff8, payoff9,
payoff10, payoff11, payoff12, payoff13, payoff14, payoff15, payoff16 As Variant

'giving initial values to variables

Sheets("sheet2").Select

Range("a1").Select

ActiveCell.FormulaR1C1 = "k"

k = 0 'lower limit of cost range

Range("b1").Select

ActiveCell.Value = k

Range("c1").Select

ActiveCell.FormulaR1C1 = "m"

m = 1 'mode of the cost distribution

Range("d1").Select

ActiveCell.Value = m

Range("e1").Select

ActiveCell.FormulaR1C1 = "h"

h = 2 'the higher limit of the cost range

Range("f1").Select

ActiveCell.Value = h

bid1a = 0

bid1b = 0

bid1c = 0

Range("g1").Select

ActiveCell.FormulaR1C1 = "n"

n = 2 'number of bidders

Range("h1").Select

ActiveCell.Value = n

'start loop enter the number of required after "To"

num = 0

```

For num = 1 To 3
    If n >= 2 Then
        Sheets("sheet2").Select
        Range("a4").Select
        ActiveCell.FormulaR1C1 = " cost1"
        Range("b4").Select
        ActiveCell.FormulaR1C1 = " cost2"
        Sheets("sheet3").Select
        Range("a4").Select
        ActiveCell.FormulaR1C1 = " bid1"
        Range("b4").Select
        ActiveCell.FormulaR1C1 = " bid2"
        Sheets("sheet4").Select
        Range("a4").Select
        ActiveCell.FormulaR1C1 = " payoff1"
        Range("b4").Select
        ActiveCell.FormulaR1C1 = " payoff2"

```

'computation of cost1

```

rand1 = Rnd()
If rand1 < ((m - k) / (h - k)) Then
    cost1 = ((rand1 * (h - k) * (m - k)) ^ 0.5) + 1
Else
    cost1 = h - ((1 - rand1) * (h - k) * (h - m)) ^ 0.5

```


End If

'cost1 to sheet2

Sheets("sheet2").Select

Range("a3").Select

ActiveCell.Value = cost1

'computation of cost2

rand2 = Rnd()

If rand2 < ((m - k) / (h - k)) Then

cost2 = ((rand2 * (h - k) * (m - k)) ^ 0.5) + 1

Else

cost2 = h - ((1 - rand2) * (h - k) * (h - m)) ^ 0.5

End If

'cost2 to sheet2

Sheets("sheet2").Select

Range("b3").Select

ActiveCell.Value = cost2

'computation of bid1

bid1a = (n - 1) + 0.5

If cost1 < m Then

bid1b = n * k + (n - 1) * cost1

bid1c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost1 * (n - 1) * k)

bid1 = (bid1b + (bid1b ^ 2 + 2 * bid1c) ^ 0.5) / (2 * bid1a)

Else

$$\text{bid1} = (h + (2 * (n - 1) * \text{cost1})) / (2 * \text{bid1a})$$

End If

'bid1 to sheet2

Sheets("sheet3").Select

Range("a3").Select

ActiveCell.Value = bid1

'computation of bid2

If cost2 < m Then

$$\text{bid2b} = n * k + (n - 1) * \text{cost2}$$

$$\text{bid2c} = \text{bid1a} * (h - k) * (m - k) - (k^2) - (2 * \text{cost2} * (n - 1) * k)$$

$$\text{bid2} = (\text{bid2b} + (\text{bid2b}^2 + 2 * \text{bid2c})^{0.5}) / (2 * \text{bid1a})$$

Else

$$\text{bid2} = (h + (2 * (n - 1) * \text{cost2})) / (2 * \text{bid1a})$$

End If

'bid2 to sheet2

Sheets("sheet3").Select

Range("b3").Select

ActiveCell.Value = bid2

End If

If n >= 3 Then GoTo row10:

'defining the winner and its payoff

If bid1 < bid2 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

Else

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

row10:

Sheets("sheet2").Select

Range("c4").Select

ActiveCell.FormulaR1C1 = " cost3"

Sheets("sheet3").Select

Range("c4").Select

ActiveCell.FormulaR1C1 = " bid3"

Sheets("sheet4").Select

Range("c4").Select

ActiveCell.FormulaR1C1 = " payoff3"

'computation of cost3

rand3 = Rnd()

If rand3 < ((m - k) / (h - k)) Then

cost3 = ((rand3 * (h - k) * (m - k)) ^ 0.5) + 1

Else

cost3 = h - ((1 - rand3) * (h - k) * (h - m)) ^ 0.5

End If

'cost3 to sheet2

Sheets("sheet2").Select

Range("c3").Select

ActiveCell.Value = cost3

'computation of bid3

If cost3 < m Then

bid3b = n * k + (n - 1) * cost3

bid3c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost3 * (n - 1) * k)

bid3 = (bid3b + (bid3b ^ 2 + 2 * bid3c) ^ 0.5) / (2 * bid1a)

Else

bid3 = (h + (2 * (n - 1) * cost3)) / (2 * bid1a)

End If

'bid3 to sheet2

Sheets("sheet3").Select

Range("c3").Select

ActiveCell.Value = bid3

If n >= 4 Then GoTo row11:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

Else

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

row11:

```
Sheets("sheet2").Select
```

```
Range("d4").Select
```

```
ActiveCell.FormulaR1C1 = " cost4"
```

```
Sheets("sheet3").Select
```

```
Range("d4").Select
```

```
ActiveCell.FormulaR1C1 = " bid4"
```

```
Sheets("sheet4").Select
```

```
Range("d4").Select
```

```
ActiveCell.FormulaR1C1 = " payoff4"
```

```
'computation of cost4
```

```
rand4 = Rnd()
```

```
If rand4 < ((m - k) / (h - k)) Then
```

```
    cost4 = ((rand4 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
    cost4 = h - ((1 - rand4) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost4 to sheet2
```

```
Sheets("sheet2").Select
```

```
Range("d3").Select
```

```
ActiveCell.Value = cost4
```

```
'computation of bid4
```

```
If cost4 < m Then
```


$\text{bid4b} = n * k + (n - 1) * \text{cost4}$

$\text{bid4c} = \text{bid1a} * (h - k) * (m - k) - (k^2) - (2 * \text{cost4} * (n - 1) * k)$

$\text{bid4} = (\text{bid4b} + (\text{bid4b}^2 + 2 * \text{bid4c})^{0.5}) / (2 * \text{bid1a})$

Else

$\text{bid4} = (h + (2 * (n - 1) * \text{cost4})) / (2 * \text{bid1a})$

End If

'bid4 to sheet3

Sheets("sheet3").Select

Range("d3").Select

ActiveCell.Value = bid4

If n >= 5 Then GoTo row12:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

```

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 Then

    payoff3 = bid3 - cost3

    Sheets("sheet4").Select

    Range("c3").Select

    ActiveCell.Value = payoff3

    GoTo row100:

Else

    payoff4 = bid4 - cost4

    Sheets("sheet4").Select

    Range("d3").Select

    ActiveCell.Value = payoff4

    GoTo row100:

End If

row12:

    Sheets("sheet2").Select

    Range("e4").Select

    ActiveCell.FormulaR1C1 = " cost5"

    Sheets("sheet3").Select

    Range("e4").Select

    ActiveCell.FormulaR1C1 = " bid5"

```

Sheets("sheet4").Select

Range("e4").Select

ActiveCell.FormulaR1C1 = " payoff5"

'computation of cost5

rand5 = Rnd()

If rand5 < ((m - k) / (h - k)) Then

cost5 = ((rand5 * (h - k) * (m - k)) ^ 0.5) + 1

Else

cost5 = h - ((1 - rand5) * (h - k) * (h - m)) ^ 0.5

End If

'cost5 to sheet2

Sheets("sheet2").Select

Range("e3").Select

ActiveCell.Value = cost5

'computation of bid5

If cost5 < m Then

bid5b = n * k + (n - 1) * cost5

bid5c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost5 * (n - 1) * k)

bid5 = (bid5b + (bid5b ^ 2 + 2 * bid5c) ^ 0.5) / (2 * bid1a)

Else

bid5 = (h + (2 * (n - 1) * cost5)) / (2 * bid1a)

End If

'bid5 to sheet3

Sheets("sheet3").Select

Range("e3").Select

ActiveCell.Value = bid5

If n >= 6 Then GoTo row13:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

Else

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

row13:

Sheets("sheet2").Select

Range("f4").Select

ActiveCell.FormulaR1C1 = " cost6"

Sheets("sheet3").Select

Range("f4").Select

ActiveCell.FormulaR1C1 = " bid6"

Sheets("sheet4").Select

Range("f4").Select

ActiveCell.FormulaR1C1 = " payoff6"

'computation of cost6

rand6 = Rnd()

If rand6 < ((m - k) / (h - k)) Then

cost6 = ((rand6 * (h - k) * (m - k)) ^ 0.5) + 1

Else

cost6 = h - ((1 - rand6) * (h - k) * (h - m)) ^ 0.5

End If

'cost6 to sheet2

Sheets("sheet2").Select

Range("f3").Select

ActiveCell.Value = cost6

'computation of bid6

If cost6 < m Then

bid6b = n * k + (n - 1) * cost6

bid6c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost6 * (n - 1) * k)

bid6 = (bid6b + (bid6b ^ 2 + 2 * bid6c) ^ 0.5) / (2 * bid1a)

Else

bid6 = (h + (2 * (n - 1) * cost6)) / (2 * bid1a)

End If

'bid6 to sheet3

Sheets("sheet3").Select

Range("f3").Select

ActiveCell.Value = bid6

If n >= 7 Then GoTo row14:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 Then

payoff3 = bid3 - cost3

```

Sheets("sheet4").Select
Range("c3").Select
ActiveCell.Value = payoff3
GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 Then

    payoff4 = bid4 - cost4
    Sheets("sheet4").Select
    Range("d3").Select
    ActiveCell.Value = payoff4
    GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 Then

    payoff5 = bid5 - cost5
    Sheets("sheet4").Select
    Range("e3").Select
    ActiveCell.Value = payoff5

Else

    payoff6 = bid6 - cost6
    Sheets("sheet4").Select
    Range("f3").Select
    ActiveCell.Value = payoff6
    GoTo row100:

```

End If

row14:

Sheets("sheet2").Select

Range("g4").Select

ActiveCell.FormulaR1C1 = " cost7"

Sheets("sheet3").Select

Range("g4").Select

ActiveCell.FormulaR1C1 = " bid7"

Sheets("sheet4").Select

Range("g4").Select

ActiveCell.FormulaR1C1 = " payoff7"

'computation of cost7

rand7 = Rnd()

If rand7 < ((m - k) / (h - k)) Then

cost7 = ((rand7 * (h - k) * (m - k)) ^ 0.5) + 1

Else

cost7 = h - ((1 - rand7) * (h - k) * (h - m)) ^ 0.5

End If

'cost7 to sheet2

Sheets("sheet2").Select

Range("g3").Select

ActiveCell.Value = cost7

'computation of bid7

If cost7 < m Then

bid7b = n * k + (n - 1) * cost7

bid7c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost7 * (n - 1) * k)

bid7 = (bid7b + (bid7b ^ 2 + 2 * bid7c) ^ 0.5) / (2 * bid1a)

Else

bid7 = (h + (2 * (n - 1) * cost7)) / (2 * bid1a)

End If

'bid7 to sheet3

Sheets("sheet3").Select

Range("g3").Select

ActiveCell.Value = bid7

If n >= 8 Then GoTo row15:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And

bid1 < bid7 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

Else

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

row15:

Sheets("sheet2").Select


```
Range("h4").Select
```

```
ActiveCell.FormulaR1C1 = " cost8"
```

```
Sheets("sheet3").Select
```

```
Range("h4").Select
```

```
ActiveCell.FormulaR1C1 = " bid8"
```

```
Sheets("sheet4").Select
```

```
Range("h4").Select
```

```
ActiveCell.FormulaR1C1 = " payoff8"
```

```
'computation of cost8
```

```
rand8 = Rnd()
```

```
If rand8 < ((m - k) / (h - k)) Then
```

```
cost8 = ((rand8 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
cost8 = h - ((1 - rand8) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost8 to sheet2
```

```
Sheets("sheet2").Select
```

```
Range("h3").Select
```

```
ActiveCell.Value = cost8
```

```
'computation of bid8
```

```
If cost8 < m Then
```

```
bid8b = n * k + (n - 1) * cost8
```

$$\text{bid8c} = \text{bid1a} * (\text{h} - \text{k}) * (\text{m} - \text{k}) - (\text{k}^2) - (2 * \text{cost8} * (\text{n} - 1) * \text{k})$$

$$\text{bid8} = (\text{bid8b} + (\text{bid8b}^2 + 2 * \text{bid8c})^{0.5}) / (2 * \text{bid1a})$$

Else

$$\text{bid8} = (\text{h} + (2 * (\text{n} - 1) * \text{cost8})) / (2 * \text{bid1a})$$

End If

'bid8 to sheet3

Sheets("sheet3").Select

Range("h3").Select

ActiveCell.Value = bid8

If n >= 9 Then GoTo row16:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And
bid1 < bid7 And bid1 < bid8 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 Then

payoff5 = bid5 - cost5

```

Sheets("sheet4").Select
Range("e3").Select
ActiveCell.Value = payoff5
GoTo row100:
End If
If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 Then
    payoff6 = bid6 - cost6
    Sheets("sheet4").Select
    Range("f3").Select
    ActiveCell.Value = payoff6
    GoTo row100:
End If
If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 Then
    payoff7 = bid7 - cost7
    Sheets("sheet4").Select
    Range("g3").Select
    ActiveCell.Value = payoff7
    GoTo row100:
Else
    payoff8 = bid8 - cost8
    Sheets("sheet4").Select

```

```
Range("h3").Select
```

```
ActiveCell.Value = payoff8
```

```
GoTo row100:
```

```
End If
```

```
row16:
```

```
Sheets("sheet2").Select
```

```
Range("i4").Select
```

```
ActiveCell.FormulaR1C1 = " cost9"
```

```
Sheets("sheet3").Select
```

```
Range("i4").Select
```

```
ActiveCell.FormulaR1C1 = " bid9"
```

```
Sheets("sheet4").Select
```

```
Range("i4").Select
```

```
ActiveCell.FormulaR1C1 = " payoff9"
```

```
'computation of cost9
```

```
rand9 = Rnd()
```

```
If rand9 < ((m - k) / (h - k)) Then
```

```
cost9 = ((rand9 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
cost9 = h - ((1 - rand9) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost9 to sheet2
```

```
Sheets("sheet2").Select
```

Range("i3").Select

ActiveCell.Value = cost9

'computation of bid9

If cost9 < m Then

bid9b = n * k + (n - 1) * cost9

bid9c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost9 * (n - 1) * k)

bid9 = (bid9b + (bid9b ^ 2 + 2 * bid9c) ^ 0.5) / (2 * bid1a)

Else

bid9 = (h + (2 * (n - 1) * cost9)) / (2 * bid1a)

End If

'bid9 to sheet3

Sheets("sheet3").Select

Range("i3").Select

ActiveCell.Value = bid9

If n >= 10 Then GoTo row17:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And

bid1 < bid7 And bid1 < bid8 And bid1 < bid9 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

Else

payoff9 = bid9 - cost9

Sheets("sheet4").Select

Range("i3").Select

ActiveCell.Value = payoff9

GoTo row100:

End If

row17:

Sheets("sheet2").Select

Range("j4").Select

ActiveCell.FormulaR1C1 = " cost10"

Sheets("sheet3").Select

Range("j4").Select

ActiveCell.FormulaR1C1 = " bid10"

Sheets("sheet4").Select

Range("j4").Select

ActiveCell.FormulaR1C1 = " payoff10"

'computation of cost10

rand10 = Rnd() .

If rand10 < ((m - k) / (h - k)) Then

cost10 = ((rand10 * (h - k) * (m - k)) ^ 0.5) + 1

Else

cost10 = h - ((1 - rand10) * (h - k) * (h - m)) ^ 0.5

End If

'cost10 to sheet2

Sheets("sheet2").Select

Range("j3").Select

ActiveCell.Value = cost10

'computation of bid10

If cost10 < m Then

bid10b = n * k + (n - 1) * cost10

bid10c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost10 * (n - 1) * k)

bid10 = (bid10b + (bid10b ^ 2 + 2 * bid10c) ^ 0.5) / (2 * bid1a)

Else

$$\text{bid10} = (\text{h} + (2 * (\text{n} - 1) * \text{cost10})) / (2 * \text{bid1a})$$

End If

'bid10 to sheet3

Sheets("sheet3").Select

Range("j3").Select

ActiveCell.Value = bid10

If n >= 11 Then GoTo row18:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And
bid1 < bid7 And bid1 < bid8 And bid1 < bid9 And bid1 < bid10 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 And bid2 < bid10 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 And bid3 < bid10 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 And bid4 < bid10 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 And bid5 < bid10 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 And bid6 < bid10 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 And bid7 < bid10 Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 And bid8 < bid10 Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

End If

If bid9 < bid1 And bid9 < bid2 And bid9 < bid3 And bid9 < bid4 And bid9 < bid5 And
bid9 < bid6 And bid9 < bid7 And bid9 < bid8 And bid9 < bid10 Then

payoff9 = bid9 - cost9

Sheets("sheet4").Select

Range("i3").Select

ActiveCell.Value = payoff9

GoTo row100:

Else

payoff10 = bid10 - cost10

Sheets("sheet4").Select

Range("j3").Select

ActiveCell.Value = payoff10

End If

row18:

Sheets("sheet2").Select

Range("k4").Select

ActiveCell.FormulaR1C1 = " cost11"

Sheets("sheet3").Select

Range("k4").Select

ActiveCell.FormulaR1C1 = " bid11"

```
Sheets("sheet4").Select
```

```
Range("k4").Select
```

```
ActiveCell.FormulaR1C1 = " payoff11"
```

```
'computation of cost11
```

```
rand11 = Rnd()
```

```
If rand11 < ((m - k) / (h - k)) Then
```

```
cost11 = ((rand11 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
cost11 = h - ((1 - rand11) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost11 to sheet2
```

```
Sheets("sheet2").Select
```

```
Range("k3").Select
```

```
ActiveCell.Value = cost11
```

```
'computation of bid11
```

```
If cost11 < m Then
```

```
bid11b = n * k + (n - 1) * cost11
```

```
bid11c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost11 * (n - 1) * k)
```

```
bid11 = (bid11b + (bid11b ^ 2 + 2 * bid11c) ^ 0.5) / (2 * bid1a)
```

```
Else
```

```
bid11 = (h + (2 * (n - 1) * cost11)) / (2 * bid1a)
```

```
End If
```

'bid11 to sheet3

Sheets("sheet3").Select

Range("k3").Select

ActiveCell.Value = bid11

If n >= 12 Then GoTo row19:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And
bid1 < bid7 And bid1 < bid8 And bid1 < bid9 And bid1 < bid10 And bid1 < bid11 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 And bid2 < bid10 And bid2 < bid11 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 And bid3 < bid10 And bid3 < bid11 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 And bid4 < bid10 And bid4 < bid11 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 And bid5 < bid10 And bid5 < bid11 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 And bid6 < bid10 And bid6 < bid11 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 And bid7 < bid10 And bid7 < bid11 Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 And bid8 < bid10 And bid8 < bid11 Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

End If

If bid9 < bid1 And bid9 < bid2 And bid9 < bid3 And bid9 < bid4 And bid9 < bid5 And
bid9 < bid6 And bid9 < bid7 And bid9 < bid8 And bid9 < bid10 And bid9 < bid11 Then

payoff9 = bid9 - cost9

Sheets("sheet4").Select

Range("i3").Select

ActiveCell.Value = payoff9

GoTo row100:

End If

If bid10 < bid1 And bid10 < bid2 And bid10 < bid3 And bid10 < bid4 And bid10 < bid5
And bid10 < bid6 And bid10 < bid7 And bid10 < bid8 And bid10 < bid9 And bid10 <
bid11 Then

payoff10 = bid10 - cost10

Sheets("sheet4").Select

Range("j3").Select

ActiveCell.Value = payoff10

GoTo row100:

Else

payoff11 = bid11 - cost11

Sheets("sheet4").Select

Range("k3").Select

ActiveCell.Value = payoff11

```

GoTo row100:

End If

row19:

    Sheets("sheet2").Select

    Range("l4").Select

    ActiveCell.FormulaR1C1 = " cost12"

    Sheets("sheet3").Select

    Range("l4").Select

    ActiveCell.FormulaR1C1 = " bid12"

    Sheets("sheet4").Select

    Range("l4").Select

    ActiveCell.FormulaR1C1 = " payoff12"

'computation of cost12

rand12 = Rnd()

If rand12 < ((m - k) / (h - k)) Then

    cost12 = ((rand12 * (h - k) * (m - k)) ^ 0.5) + 1

Else

    cost12 = h - ((1 - rand12) * (h - k) * (h - m)) ^ 0.5

End If

'cost12 to sheet2

    Sheets("sheet2").Select

    Range("l3").Select

    ActiveCell.Value = cost12

```

'computation of bid12

If cost12 < m Then

bid12b = n * k + (n - 1) * cost12

bid12c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost12 * (n - 1) * k)

bid12 = (bid12b + (bid12b ^ 2 + 2 * bid12c) ^ 0.5) / (2 * bid1a)

Else

bid12 = (h + (2 * (n - 1) * cost12)) / (2 * bid1a)

End If

'bid12 to sheet3

Sheets("sheet3").Select

Range("l3").Select

ActiveCell.Value = bid12

If n >= 13 Then GoTo row20:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And
bid1 < bid7 And bid1 < bid8 And bid1 < bid9 And bid1 < bid10 And bid1 < bid11 And
bid1 < bid12 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 And bid2 < bid10 And bid2 < bid11 And
bid2 < bid12 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 And bid3 < bid10 And bid3 < bid11 And
bid3 < bid12 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 And bid4 < bid10 And bid4 < bid11 And
bid4 < bid12 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 And bid5 < bid10 And bid5 < bid11 And
bid5 < bid12 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 And bid6 < bid10 And bid6 < bid11 And
bid6 < bid12 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 And bid7 < bid10 And bid7 < bid11 And
bid7 < bid12 Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 And bid8 < bid10 And bid8 < bid11 And
bid8 < bid12 Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

End If

If bid9 < bid1 And bid9 < bid2 And bid9 < bid3 And bid9 < bid4 And bid9 < bid5 And
bid9 < bid6 And bid9 < bid7 And bid9 < bid8 And bid9 < bid10 And bid9 < bid11 And
bid9 < bid12 Then

payoff9 = bid9 - cost9

Sheets("sheet4").Select

Range("i3").Select

ActiveCell.Value = payoff9

GoTo row100:

End If

If bid10 < bid1 And bid10 < bid2 And bid10 < bid3 And bid10 < bid4 And bid10 < bid5
And bid10 < bid6 And bid10 < bid7 And bid10 < bid8 And bid10 < bid9 And bid10 <
bid11 And bid10 < bid12 Then

payoff10 = bid10 - cost10

Sheets("sheet4").Select

Range("j3").Select

ActiveCell.Value = payoff10

GoTo row100:

End If

If bid11 < bid1 And bid11 < bid2 And bid11 < bid3 And bid11 < bid4 And bid11 < bid5
And bid11 < bid6 And bid11 < bid7 And bid11 < bid8 And bid11 < bid9 And bid11 <
bid10 And bid11 < bid12 Then

payoff11 = bid11 - cost11

Sheets("sheet4").Select

Range("k3").Select

ActiveCell.Value = payoff11

GoTo row100:

Else

payoff12 = bid12 - cost12

```

Sheets("sheet4").Select
Range("l3").Select
ActiveCell.Value = payoff12
GoTo row100:

```

End If

row20:

```

Sheets("sheet2").Select
Range("m4").Select
ActiveCell.FormulaR1C1 = " cost13"
Sheets("sheet3").Select
Range("m4").Select
ActiveCell.FormulaR1C1 = " bid13"
Sheets("sheet4").Select
Range("m4").Select
ActiveCell.FormulaR1C1 = " payoff13"

```

'computation of cost13

```

rand13 = Rnd()
If rand13 < ((m - k) / (h - k)) Then
    cost13 = ((rand13 * (h - k) * (m - k)) ^ 0.5) + 1
Else
    cost13 = h - ((1 - rand13) * (h - k) * (h - m)) ^ 0.5
End If

```

'cost13 to sheet2

Sheets("sheet2").Select

Range("m3").Select

ActiveCell.Value = cost13

'computation of bid13

If cost13 < m Then

bid13b = n * k + (n - 1) * cost13

bid13c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost13 * (n - 1) * k)

bid13 = (bid13b + (bid13b ^ 2 + 2 * bid13c) ^ 0.5) / (2 * bid1a)

Else

bid13 = (h + (2 * (n - 1) * cost13)) / (2 * bid1a)

End If

'bid13 to sheet3

Sheets("sheet3").Select

Range("m3").Select

ActiveCell.Value = bid13

If n >= 14 Then GoTo row21:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And

bid1 < bid7 And bid1 < bid8 And bid1 < bid9 And bid1 < bid10 And bid1 < bid11 And

bid1 < bid12 And bid1 < bid13 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

```

Range("a3").Select
ActiveCell.Value = payoff1
GoTo row100:
End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 And bid2 < bid10 And bid2 < bid11 And
bid2 < bid12 And bid2 < bid13 Then
    payoff2 = bid2 - cost2
    Sheets("sheet4").Select
    Range("b3").Select
    ActiveCell.Value = payoff2
    GoTo row100:
End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 And bid3 < bid10 And bid3 < bid11 And
bid3 < bid12 And bid3 < bid13 Then
    payoff3 = bid3 - cost3
    Sheets("sheet4").Select
    Range("c3").Select
    ActiveCell.Value = payoff3
    GoTo row100:
End If

```

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 And bid4 < bid10 And bid4 < bid11 And
bid4 < bid12 And bid4 < bid13 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 And bid5 < bid10 And bid5 < bid11 And
bid5 < bid12 And bid5 < bid13 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 And bid6 < bid10 And bid6 < bid11 And
bid6 < bid12 And bid6 < bid13 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 And bid7 < bid10 And bid7 < bid11 And
bid7 < bid12 And bid7 < bid13 Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 And bid8 < bid10 And bid8 < bid11 And
bid8 < bid12 And bid8 < bid13 Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

End If

If bid9 < bid1 And bid9 < bid2 And bid9 < bid3 And bid9 < bid4 And bid9 < bid5 And
bid9 < bid6 And bid9 < bid7 And bid9 < bid8 And bid9 < bid10 And bid9 < bid11 And
bid9 < bid12 And bid9 < bid13 Then

payoff9 = bid9 - cost9

Sheets("sheet4").Select

Range("i3").Select

ActiveCell.Value = payoff9

GoTo row100:

End If

If bid10 < bid1 And bid10 < bid2 And bid10 < bid3 And bid10 < bid4 And bid10 < bid5
And bid10 < bid6 And bid10 < bid7 And bid10 < bid8 And bid10 < bid9 And bid10 <
bid11 And bid10 < bid12 And bid10 < bid13 Then

payoff10 = bid10 - cost10

Sheets("sheet4").Select

Range("j3").Select

ActiveCell.Value = payoff10

GoTo row100:

End If

If bid11 < bid1 And bid11 < bid2 And bid11 < bid3 And bid11 < bid4 And bid11 < bid5
And bid11 < bid6 And bid11 < bid7 And bid11 < bid8 And bid11 < bid9 And bid11 <
bid10 And bid11 < bid12 And bid11 < bid13 Then

payoff11 = bid11 - cost11

Sheets("sheet4").Select

```

Range("k3").Select
ActiveCell.Value = payoff11
GoTo row100:

End If

If bid12 < bid1 And bid12 < bid2 And bid12 < bid3 And bid12 < bid4 And bid12 < bid5
And bid12 < bid6 And bid12 < bid7 And bid12 < bid8 And bid12 < bid9 And bid12 <
bid10 And bid12 < bid11 And bid12 < bid13 Then

    payoff12 = bid12 - cost12

    Sheets("sheet4").Select
    Range("l3").Select
    ActiveCell.Value = payoff12
    GoTo row100:

Else

    payoff13 = bid13 - cost13

    Sheets("sheet4").Select
    Range("m3").Select
    ActiveCell.Value = payoff13

End If

row21:

    Sheets("sheet2").Select
    Range("n4").Select
    ActiveCell.FormulaR1C1 = " cost14"
    Sheets("sheet3").Select

```

```
Range("n4").Select
```

```
ActiveCell.FormulaR1C1 = " bid14"
```

```
Sheets("sheet4").Select
```

```
Range("n4").Select
```

```
ActiveCell.FormulaR1C1 = " payoff14"
```

```
'computation of cost14
```

```
rand14 = Rnd()
```

```
If rand14 < ((m - k) / (h - k)) Then
```

```
cost14 = ((rand14 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
cost14 = h - ((1 - rand14) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost14 to sheet2
```

```
Sheets("sheet2").Select
```

```
Range("n3").Select
```

```
ActiveCell.Value = cost14
```

```
'computation of bid14
```

```
If cost14 < m Then
```

```
bid14b = n * k + (n - 1) * cost14
```

```
bid14c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost14 * (n - 1) * k)
```

```
bid14 = (bid14b + (bid14b ^ 2 + 2 * bid14c) ^ 0.5) / (2 * bid1a)
```

```
Else
```

$$\text{bid14} = (\text{h} + (2 * (\text{n} - 1) * \text{cost14})) / (2 * \text{bid1a})$$

End If

'bid14 to sheet3

Sheets("sheet3").Select

Range("n3").Select

ActiveCell.Value = bid14

If n >= 15 Then GoTo row22:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And
bid1 < bid7 And bid1 < bid8 And bid1 < bid9 And bid1 < bid10 And bid1 < bid11 And
bid1 < bid12 And bid1 < bid13 And bid1 < bid14 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 And bid2 < bid10 And bid2 < bid11 And
bid2 < bid12 And bid2 < bid13 And bid2 < bid14 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 And bid3 < bid10 And bid3 < bid11 And
bid3 < bid12 And bid3 < bid13 And bid3 < bid14 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 And bid4 < bid10 And bid4 < bid11 And
bid4 < bid12 And bid4 < bid13 And bid4 < bid14 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 And bid5 < bid10 And bid5 < bid11 And
bid5 < bid12 And bid5 < bid13 And bid5 < bid14 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 And bid6 < bid10 And bid6 < bid11 And
bid6 < bid12 And bid6 < bid13 And bid6 < bid14 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 And bid7 < bid10 And bid7 < bid11 And
bid7 < bid12 And bid7 < bid13 And bid7 < bid14 Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 And bid8 < bid10 And bid8 < bid11 And
bid8 < bid12 And bid8 < bid13 And bid8 < bid14 Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

End If

If bid9 < bid1 And bid9 < bid2 And bid9 < bid3 And bid9 < bid4 And bid9 < bid5 And
bid9 < bid6 And bid9 < bid7 And bid9 < bid8 And bid9 < bid10 And bid9 < bid11 And
bid9 < bid12 And bid9 < bid13 And bid9 < bid14 Then

payoff9 = bid9 - cost9

Sheets("sheet4").Select

Range("i3").Select

ActiveCell.Value = payoff9

GoTo row100:

End If

If bid10 < bid1 And bid10 < bid2 And bid10 < bid3 And bid10 < bid4 And bid10 < bid5
And bid10 < bid6 And bid10 < bid7 And bid10 < bid8 And bid10 < bid9 And bid10 <
bid11 And bid10 < bid12 And bid10 < bid13 And bid10 < bid14 Then

payoff10 = bid10 - cost10

Sheets("sheet4").Select

Range("j3").Select

ActiveCell.Value = payoff10

GoTo row100:

End If

If bid11 < bid1 And bid11 < bid2 And bid11 < bid3 And bid11 < bid4 And bid11 < bid5

And bid11 < bid6 And bid11 < bid7 And bid11 < bid8 And bid11 < bid9 And bid11 <

bid10 And bid11 < bid12 And bid11 < bid13 And bid11 < bid14 Then

payoff11 = bid11 - cost11

Sheets("sheet4").Select

Range("k3").Select

ActiveCell.Value = payoff11

GoTo row100:

End If

If bid12 < bid1 And bid12 < bid2 And bid12 < bid3 And bid12 < bid4 And bid12 < bid5

And bid12 < bid6 And bid12 < bid7 And bid12 < bid8 And bid12 < bid9 And bid12 <

bid10 And bid12 < bid11 And bid12 < bid13 And bid12 < bid14 Then

payoff12 = bid12 - cost12

Sheets("sheet4").Select

Range("l3").Select

ActiveCell.Value = payoff12

GoTo row100:

End If

If bid13 < bid1 And bid13 < bid2 And bid13 < bid3 And bid13 < bid4 And bid13 < bid5
And bid13 < bid6 And bid13 < bid7 And bid13 < bid8 And bid13 < bid9 And bid13 <
bid10 And bid13 < bid11 And bid13 < bid12 And bid13 < bid14 Then

payoff13 = bid13 - cost13

Sheets("sheet4").Select

Range("m3").Select

ActiveCell.Value = payoff13

GoTo row100:

Else

payoff14 = bid14 - cost14

Sheets("sheet4").Select

Range("n3").Select

ActiveCell.Value = payoff14

GoTo row100:

End If

row22:

Sheets("sheet2").Select

Range("o4").Select

ActiveCell.FormulaR1C1 = " cost15"

Sheets("sheet3").Select

Range("o4").Select

ActiveCell.FormulaR1C1 = " bid15"

Sheets("sheet4").Select

```
Range("o4").Select
```

```
ActiveCell.FormulaR1C1 = " payoff15"
```

```
'computation of cost15
```

```
rand15 = Rnd()
```

```
If rand15 < ((m - k) / (h - k)) Then
```

```
cost15 = ((rand15 * (h - k) * (m - k)) ^ 0.5) + 1
```

```
Else
```

```
cost15 = h - ((1 - rand15) * (h - k) * (h - m)) ^ 0.5
```

```
End If
```

```
'cost15 to sheet2
```

```
Sheets("sheet2").Select
```

```
Range("o3").Select
```

```
ActiveCell.Value = cost15
```

```
'computation of bid15
```

```
If cost15 < m Then
```

```
bid15b = n * k + (n - 1) * cost15
```

```
bid15c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost15 * (n - 1) * k)
```

```
bid15 = (bid15b + (bid15b ^ 2 + 2 * bid15c) ^ 0.5) / (2 * bid1a)
```

```
Else
```

```
bid15 = (h + (2 * (n - 1) * cost15)) / (2 * bid1a)
```

```
End If
```

```
'bid15 to sheet3
```

Sheets("sheet3").Select

Range("o3").Select

ActiveCell.Value = bid15

If n >= 16 Then GoTo row23:

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And
bid1 < bid7 And bid1 < bid8 And bid1 < bid9 And bid1 < bid10 And bid1 < bid11 And
bid1 < bid12 And bid1 < bid13 And bid1 < bid14 And bid1 < bid15 Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 And bid2 < bid10 And bid2 < bid11 And
bid2 < bid12 And bid2 < bid13 And bid2 < bid14 And bid2 < bid15 Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 And bid3 < bid10 And bid3 < bid11 And
bid3 < bid12 And bid3 < bid13 And bid3 < bid14 And bid3 < bid15 Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 And bid4 < bid10 And bid4 < bid11 And
bid4 < bid12 And bid4 < bid13 And bid4 < bid14 And bid4 < bid15 Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 And bid5 < bid10 And bid5 < bid11 And
bid5 < bid12 And bid5 < bid13 And bid5 < bid14 And bid5 < bid15 Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 And bid6 < bid10 And bid6 < bid11 And
bid6 < bid12 And bid6 < bid13 And bid6 < bid14 And bid6 < bid15 Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 And bid7 < bid10 And bid7 < bid11 And
bid7 < bid12 And bid7 < bid13 And bid7 < bid14 And bid7 < bid15 Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 And bid8 < bid10 And bid8 < bid11 And
bid8 < bid12 And bid8 < bid13 And bid8 < bid14 And bid8 < bid15 Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

End If

If bid9 < bid1 And bid9 < bid2 And bid9 < bid3 And bid9 < bid4 And bid9 < bid5 And
bid9 < bid6 And bid9 < bid7 And bid9 < bid8 And bid9 < bid10 And bid9 < bid11 And
bid9 < bid12 And bid9 < bid13 And bid9 < bid14 And bid9 < bid15 Then

payoff9 = bid9 - cost9

Sheets("sheet4").Select

Range("i3").Select

ActiveCell.Value = payoff9

GoTo row100:

End If

If bid10 < bid1 And bid10 < bid2 And bid10 < bid3 And bid10 < bid4 And bid10 < bid5
And bid10 < bid6 And bid10 < bid7 And bid10 < bid8 And bid10 < bid9 And bid10 <
bid11 And bid10 < bid12 And bid10 < bid13 And bid10 < bid14 And bid10 < bid15 Then

payoff10 = bid10 - cost10

Sheets("sheet4").Select

Range("j3").Select

ActiveCell.Value = payoff10

GoTo row100:

End If

If bid11 < bid1 And bid11 < bid2 And bid11 < bid3 And bid11 < bid4 And bid11 < bid5
And bid11 < bid6 And bid11 < bid7 And bid11 < bid8 And bid11 < bid9 And bid11 <
bid10 And bid11 < bid12 And bid11 < bid13 And bid11 < bid14 And bid11 < bid15 Then

payoff11 = bid11 - cost11

Sheets("sheet4").Select

Range("k3").Select

ActiveCell.Value = payoff11

GoTo row100:

End If

If bid12 < bid1 And bid12 < bid2 And bid12 < bid3 And bid12 < bid4 And bid12 < bid5
And bid12 < bid6 And bid12 < bid7 And bid12 < bid8 And bid12 < bid9 And bid12 <
bid10 And bid12 < bid11 And bid12 < bid13 And bid12 < bid14 And bid12 < bid15 Then

payoff12 = bid12 - cost12

Sheets("sheet4").Select

Range("l3").Select

ActiveCell.Value = payoff12

GoTo row100:

End If

```

If bid13 < bid1 And bid13 < bid2 And bid13 < bid3 And bid13 < bid4 And bid13 < bid5
And bid13 < bid6 And bid13 < bid7 And bid13 < bid8 And bid13 < bid9 And bid13 <
bid10 And bid13 < bid11 And bid13 < bid12 And bid13 < bid14 And bid13 < bid15 Then

    payoff13 = bid13 - cost13

    Sheets("sheet4").Select

    Range("m3").Select

    ActiveCell.Value = payoff13

    GoTo row100:

End If

If bid14 < bid1 And bid14 < bid2 And bid14 < bid3 And bid14 < bid4 And bid14 < bid5
And bid14 < bid6 And bid14 < bid7 And bid14 < bid8 And bid14 < bid9 And bid14 <
bid10 And bid14 < bid11 And bid14 < bid12 And bid14 < bid13 And bid14 < bid15 Then

    payoff14 = bid14 - cost14

    Sheets("sheet4").Select

    Range("n3").Select

    ActiveCell.Value = payoff14

    GoTo row100:

Else

    payoff15 = bid15 - cost15

    Sheets("sheet4").Select

    Range("o3").Select

    ActiveCell.Value = payoff15

    GoTo row100:

```

End If

row23:

Sheets("sheet2").Select

Range("p4").Select

ActiveCell.FormulaR1C1 = " cost16"

Sheets("sheet3").Select

Range("p4").Select

ActiveCell.FormulaR1C1 = " bid16"

Sheets("sheet4").Select

Range("p4").Select

ActiveCell.FormulaR1C1 = " payoff16"

'computation of cost16

rand16 = Rnd()

If rand16 < ((m - k) / (h - k)) Then

cost16 = ((rand16 * (h - k) * (m - k)) ^ 0.5) + 1

Else

cost16 = h - ((1 - rand16) * (h - k) * (h - m)) ^ 0.5

End If

'cost16 to sheet2

Sheets("sheet2").Select

Range("p3").Select

ActiveCell.Value = cost16

'computation of bid16

If cost16 < m Then

bid16b = n * k + (n - 1) * cost16

bid16c = bid1a * (h - k) * (m - k) - (k ^ 2) - (2 * cost16 * (n - 1) * k)

bid16 = (bid16b + (bid16b ^ 2 + 2 * bid16c) ^ 0.5) / (2 * bid1a)

Else

bid16 = (h + (2 * (n - 1) * cost16)) / (2 * bid1a)

End If

'bid16 to sheet3

Sheets("sheet3").Select

Range("p3").Select

ActiveCell.Value = bid16

'Defining the winner and it's payoff

If bid1 < bid2 And bid1 < bid3 And bid1 < bid4 And bid1 < bid5 And bid1 < bid6 And

bid1 < bid7 And bid1 < bid8 And bid1 < bid9 And bid1 < bid10 And bid1 < bid11 And

bid1 < bid12 And bid1 < bid13 And bid1 < bid14 And bid1 < bid15 And bid1 < bid16

Then

payoff1 = bid1 - cost1

Sheets("sheet4").Select

Range("a3").Select

ActiveCell.Value = payoff1

GoTo row100:

End If

If bid2 < bid1 And bid2 < bid3 And bid2 < bid4 And bid2 < bid5 And bid2 < bid6 And
bid2 < bid7 And bid2 < bid8 And bid2 < bid9 And bid2 < bid10 And bid2 < bid11 And
bid2 < bid12 And bid2 < bid13 And bid2 < bid14 And bid2 < bid15 And bid2 < bid16

Then

payoff2 = bid2 - cost2

Sheets("sheet4").Select

Range("b3").Select

ActiveCell.Value = payoff2

GoTo row100:

End If

If bid3 < bid1 And bid3 < bid2 And bid3 < bid4 And bid3 < bid5 And bid3 < bid6 And
bid3 < bid7 And bid3 < bid8 And bid3 < bid9 And bid3 < bid10 And bid3 < bid11 And
bid3 < bid12 And bid3 < bid13 And bid3 < bid14 And bid3 < bid15 And bid3 < bid16

Then

payoff3 = bid3 - cost3

Sheets("sheet4").Select

Range("c3").Select

ActiveCell.Value = payoff3

GoTo row100:

End If

If bid4 < bid1 And bid4 < bid2 And bid4 < bid3 And bid4 < bid5 And bid4 < bid6 And
bid4 < bid7 And bid4 < bid8 And bid4 < bid9 And bid4 < bid10 And bid4 < bid11 And
bid4 < bid12 And bid4 < bid13 And bid4 < bid14 And bid4 < bid15 And bid4 < bid16

Then

payoff4 = bid4 - cost4

Sheets("sheet4").Select

Range("d3").Select

ActiveCell.Value = payoff4

GoTo row100:

End If

If bid5 < bid1 And bid5 < bid2 And bid5 < bid3 And bid5 < bid4 And bid5 < bid6 And
bid5 < bid7 And bid5 < bid8 And bid5 < bid9 And bid5 < bid10 And bid5 < bid11 And
bid5 < bid12 And bid5 < bid13 And bid5 < bid14 And bid5 < bid15 And bid5 < bid16

Then

payoff5 = bid5 - cost5

Sheets("sheet4").Select

Range("e3").Select

ActiveCell.Value = payoff5

GoTo row100:

End If

If bid6 < bid1 And bid6 < bid2 And bid6 < bid3 And bid6 < bid4 And bid6 < bid5 And
bid6 < bid7 And bid6 < bid8 And bid6 < bid9 And bid6 < bid10 And bid6 < bid11 And
bid6 < bid12 And bid6 < bid13 And bid6 < bid14 And bid6 < bid15 And bid6 < bid16

Then

payoff6 = bid6 - cost6

Sheets("sheet4").Select

Range("f3").Select

ActiveCell.Value = payoff6

GoTo row100:

End If

If bid7 < bid1 And bid7 < bid2 And bid7 < bid3 And bid7 < bid4 And bid7 < bid5 And
bid7 < bid6 And bid7 < bid8 And bid7 < bid9 And bid7 < bid10 And bid7 < bid11 And
bid7 < bid12 And bid7 < bid13 And bid7 < bid14 And bid7 < bid15 And bid7 < bid16

Then

payoff7 = bid7 - cost7

Sheets("sheet4").Select

Range("g3").Select

ActiveCell.Value = payoff7

GoTo row100:

End If

If bid8 < bid1 And bid8 < bid2 And bid8 < bid3 And bid8 < bid4 And bid8 < bid5 And
bid8 < bid6 And bid8 < bid7 And bid8 < bid9 And bid8 < bid10 And bid8 < bid11 And
bid8 < bid12 And bid8 < bid13 And bid8 < bid14 And bid8 < bid15 And bid8 < bid16

Then

payoff8 = bid8 - cost8

Sheets("sheet4").Select

Range("h3").Select

ActiveCell.Value = payoff8

GoTo row100:

End If

If bid9 < bid1 And bid9 < bid2 And bid9 < bid3 And bid9 < bid4 And bid9 < bid5 And
bid9 < bid6 And bid9 < bid7 And bid9 < bid8 And bid9 < bid10 And bid9 < bid11 And
bid9 < bid12 And bid9 < bid13 And bid9 < bid14 And bid9 < bid15 And bid9 < bid16

Then

 payoff9 = bid9 - cost9

 Sheets("sheet4").Select

 Range("i3").Select

 ActiveCell.Value = payoff9

 GoTo row100:

End If

If bid10 < bid1 And bid10 < bid2 And bid10 < bid3 And bid10 < bid4 And bid10 < bid5
And bid10 < bid6 And bid10 < bid7 And bid10 < bid8 And bid10 < bid9 And bid10 <
bid11 And bid10 < bid12 And bid10 < bid13 And bid10 < bid14 And bid10 < bid15 And
bid10 < bid16 Then

 payoff10 = bid10 - cost10

 Sheets("sheet4").Select

 Range("j3").Select

 ActiveCell.Value = payoff10

 GoTo row100:

End If

If bid11 < bid1 And bid11 < bid2 And bid11 < bid3 And bid11 < bid4 And bid11 < bid5
And bid11 < bid6 And bid11 < bid7 And bid11 < bid8 And bid11 < bid9 And bid11 <

bid10 And bid11 < bid12 And bid11 < bid13 And bid11 < bid14 And bid11 < bid15 And
bid11 < bid16 Then

payoff11 = bid11 - cost11

Sheets("sheet4").Select

Range("k3").Select

ActiveCell.Value = payoff11

GoTo row100:

End If

If bid12 < bid1 And bid12 < bid2 And bid12 < bid3 And bid12 < bid4 And bid12 < bid5
And bid12 < bid6 And bid12 < bid7 And bid12 < bid8 And bid12 < bid9 And bid12 <
bid10 And bid12 < bid11 And bid12 < bid13 And bid12 < bid14 And bid12 < bid15 And
bid12 < bid16 Then

payoff12 = bid12 - cost12

Sheets("sheet4").Select

Range("l3").Select

ActiveCell.Value = payoff12

GoTo row100:

End If

If bid13 < bid1 And bid13 < bid2 And bid13 < bid3 And bid13 < bid4 And bid13 < bid5
And bid13 < bid6 And bid13 < bid7 And bid13 < bid8 And bid13 < bid9 And bid13 <
bid10 And bid13 < bid11 And bid13 < bid12 And bid13 < bid14 And bid13 < bid15 And
bid13 < bid16 Then

payoff13 = bid13 - cost13

```

Sheets("sheet4").Select
Range("m3").Select
ActiveCell.Value = payoff13
GoTo row100:

End If

If bid14 < bid1 And bid14 < bid2 And bid14 < bid3 And bid14 < bid4 And bid14 < bid5
And bid14 < bid6 And bid14 < bid7 And bid14 < bid8 And bid14 < bid9 And bid14 <
bid10 And bid14 < bid11 And bid14 < bid12 And bid14 < bid13 And bid14 < bid15 And
bid14 < bid16 Then

    payoff14 = bid14 - cost14

    Sheets("sheet4").Select
    Range("n3").Select
    ActiveCell.Value = payoff14
    GoTo row100:

End If

If bid15 < bid1 And bid15 < bid2 And bid15 < bid3 And bid15 < bid4 And bid15 < bid5
And bid15 < bid6 And bid15 < bid7 And bid15 < bid8 And bid15 < bid9 And bid15 <
bid10 And bid15 < bid11 And bid15 < bid12 And bid15 < bid13 And bid15 < bid14 And
bid15 < bid16 Then

    payoff15 = bid15 - cost15

    Sheets("sheet4").Select
    Range("o3").Select
    ActiveCell.Value = payoff15

```


GoTo row100:

Else

payoff16 = bid16 - cost16

Sheets("sheet4").Select

Range("p3").Select

ActiveCell.Value = payoff16

GoTo row100:

End If

row100: 'collecting the results of the game in a new row

Sheets("sheet2").Select

Range("a4").Select

Selection.EntireRow.Insert

Range("a3:x3").Select

Selection.Copy

Range("a6").Select

Selection.PasteSpecial Paste:=xlValues, operation:=xlNone

Application.CutCopyMode = False

Range("a3:x3").Select

Selection.Clear

Sheets("sheet3").Select

Range("a4").Select

Selection.EntireRow.Insert

Range("a3:x3").Select

```

Selection.Copy

Range("a6").Select

Selection.PasteSpecial Paste:=xlValues, operation:=xlNone

Application.CutCopyMode = False

Range("a3:x3").Select

Selection.Clear

Sheets("sheet4").Select

Range("a4").Select

Selection.EntireRow.Insert

Range("a3:x3").Select

Selection.Copy

Range("a6").Select

Selection.PasteSpecial Paste:=xlValues, operation:=xlNone

Application.CutCopyMode = False

Range("a3:x3").Select

Selection.Clear

Next

'end signal

For I = 1 To 3 'loop 3 times.

Beep 'sound a tone.

Next

End Sub

```


APPENDIX C

RESULTS OF THE COMPUTER SIMULATIONS

A. UNIFORM COST DISTRIBUTION

1. Two-Bidder Bidding Game

Scenario 1. Both bidders used equilibrium strategy.

# of games	Expected Profit		# of Games Won		Total Profit		Ave. Exp. Profit	
	Bidder1	Bidder2	Bidder1	Bidder2	Bidder1	Bidder2	Bidder1	Bidder2
100	0.34764	0.34379	44	56	15.2961	19.2524	0.15296	0.19252
200	0.3365	0.32693	92	108	30.9579	35.308	0.15479	0.17654
300	0.33524	0.3373	144	156	48.2748	52.6193	0.16092	0.1754
400	0.33433	0.33199	202	198	67.5337	65.7344	0.16883	0.16434
500	0.33716	0.33486	239	261	80.5803	87.3994	0.16116	0.1748
600	0.33828	0.33584	289	311	97.7642	104.447	0.16294	0.17408
700	0.33779	0.33627	334	366	112.823	123.077	0.16118	0.17582
800	0.33656	0.3337	385	415	129.575	138.486	0.16197	0.17311
900	0.33654	0.336	435	465	146.394	156.242	0.16266	0.1736
1000	0.33346	0.33578	480	520	160.061	174.604	0.16006	0.1746
1100	0.32966	0.33415	521	579	171.751	193.471	0.15614	0.17588
1200	0.33086	0.33339	576	624	190.578	208.036	0.15881	0.17336
1300	0.33128	0.33411	637	663	211.025	221.512	0.16233	0.17039
1400	0.33158	0.33237	682	718	226.135	238.644	0.16153	0.17046
1500	0.33274	0.33028	731	769	243.236	253.983	0.16216	0.16932
1600	0.33395	0.33071	780	820	260.48	271.182	0.1628	0.16949
1700	0.33452	0.3313	826	874	276.315	289.555	0.16254	0.17033
1800	0.33299	0.33111	872	928	290.369	307.267	0.16132	0.1707
1900	0.33349	0.33235	928	972	309.479	323.041	0.16288	0.17002
2000	0.33326	0.33288	982	1018	327.263	338.872	0.16363	0.16944

Table 1

Scenario 2. Bidder1 underbid by 0.2 while bidder2 used equilibrium strategy.

# of games	Expected Profit		# of Games Won		Total Profit		Ave. Exp. Profit	
	Bidder1	Bidder2	Bidder1	Bidder2	Bidder1	Bidder2	Bidder1	Bidder2
100	0.08862	0.38873	77	23	6.824	8.94082	0.06824	0.08941
200	0.08651	0.40146	159	41	13.7548	16.46	0.06877	0.0823
300	0.087	0.39823	239	61	20.792	24.2921	0.06931	0.08097
400	0.08602	0.39419	318	82	27.3553	32.3235	0.06839	0.08081
500	0.08643	0.39843	396	104	34.2254	41.4365	0.06845	0.08287
600	0.08903	0.39825	468	132	41.6643	52.5689	0.06944	0.08761
700	0.08518	0.39929	553	147	47.1067	58.6958	0.0673	0.08385
800	0.08239	0.40089	637	163	52.4801	65.3448	0.0656	0.08168
900	0.08197	0.40151	724	176	59.3467	70.6651	0.06594	0.07852
1000	0.08459	0.4002	807	193	68.2648	77.2383	0.06826	0.07724
1100	0.08464	0.39952	885	215	74.9079	85.8959	0.0681	0.07809
1200	0.08703	0.3999	971	229	84.5068	91.5763	0.07042	0.07631
1300	0.08366	0.39966	1052	248	88.0134	99.1149	0.0677	0.07624
1400	0.08614	0.40073	1139	261	98.114	104.59	0.07008	0.07471
1500	0.0856	0.40128	1221	279	104.514	111.956	0.06968	0.07464

Table2

Scenario 3. Both bidders did not use equilibrium strategy and underbid by 0.1.

# of games	Expected Profit		# of Games Won		Total Profit		Ave. Exp. Profit	
	Bidder1	Bidder2	Bidder1	Bidder2	Bidder1	Bidder2	Bidder1	Bidder2
100	0.2245	0.2025	50	50	11.227	10.126	0.1123	0.1013
200	0.2272	0.1958	98	102	22.263	19.971	0.1113	0.0999
300	0.2296	0.2018	148	152	33.981	30.68	0.1133	0.1023
400	0.2295	0.2168	190	210	43.604	45.53	0.109	0.1138
500	0.2319	0.2194	244	256	56.58	56.175	0.1132	0.1124
600	0.226	0.2202	299	301	67.579	66.281	0.1126	0.1105
700	0.2274	0.2215	350	350	79.593	77.532	0.1137	0.1108
800	0.2276	0.223	405	395	92.184	88.093	0.1152	0.1101
900	0.2297	0.2274	452	448	103.84	101.85	0.1154	0.1132
1000	0.2301	0.2277	501	499	115.28	113.64	0.1153	0.1136
1100	0.231	0.2308	546	554	126.12	127.86	0.1147	0.1162
1200	0.2299	0.2309	590	610	135.61	140.84	0.113	0.1174
1300	0.2318	0.233	640	660	148.35	153.79	0.1141	0.1183
1400	0.2302	0.2349	695	705	159.96	165.63	0.1143	0.1183
1500	0.2306	0.2316	742	758	171.08	175.54	0.1141	0.117

Table 3

2. Three-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

# of games	Expected Profit			Number of Games Won			Total Profit			Ave. Expected Profit		
	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3
100	0.2498	0.251	0.2405	37	23	40	9.2416	5.7734	9.6207	0.0924	0.0577	0.0962
200	0.251	0.2509	0.2536	70	55	75	17.568	13.798	19.018	0.0878	0.069	0.0951
300	0.2532	0.2517	0.2537	104	92	104	26.33	23.156	26.382	0.0878	0.0772	0.0879
400	0.2517	0.2509	0.2504	135	125	140	33.98	31.361	35.06	0.0849	0.0784	0.0876
500	0.2549	0.254	0.2459	164	156	180	41.81	39.63	44.266	0.0836	0.0793	0.0885
600	0.2558	0.2517	0.2465	196	185	219	50.13	46.572	53.98	0.0836	0.0776	0.09
700	0.2554	0.2488	0.2476	226	224	250	57.727	55.721	61.905	0.0825	0.0796	0.0884
800	0.2537	0.2488	0.2472	265	253	282	67.22	62.946	69.719	0.084	0.0787	0.0871
900	0.2539	0.249	0.2465	306	288	306	77.703	71.719	75.414	0.0863	0.0797	0.0838
1000	0.2533	0.249	0.2483	338	321	341	85.628	79.944	84.678	0.0856	0.0799	0.0847
1100	0.2541	0.2481	0.2492	372	355	373	94.537	88.082	92.951	0.0859	0.0801	0.0845
1200	0.2541	0.2485	0.2485	412	383	405	104.68	95.189	100.66	0.0872	0.0793	0.0839
1300	0.252	0.2492	0.2498	451	413	436	113.67	102.91	108.93	0.0874	0.0792	0.0838
1400	0.2519	0.2501	0.2494	490	449	461	123.44	112.32	114.97	0.0882	0.0802	0.0821
1500	0.2513	0.2498	0.2499	524	486	490	131.7	121.42	122.45	0.0878	0.0809	0.0816
1600	0.2514	0.2482	0.2507	557	519	524	140.05	128.81	131.39	0.0875	0.0805	0.0821
1700	0.2515	0.2487	0.2501	582	562	556	146.39	139.76	139.06	0.0861	0.0822	0.0818
1800	0.251	0.2493	0.251	612	597	591	153.6	148.85	148.36	0.0853	0.0827	0.0824
1900	0.2501	0.25	0.2514	643	623	634	160.82	155.77	159.37	0.0846	0.082	0.0839
2000	0.25	0.2494	0.2513	683	652	665	170.72	162.6	167.13	0.0854	0.0813	0.0836

Table 4

Scenario 2. Bidder1 underbid by 0.1 while others used equilibrium strategy.

# of games	Expected Profit			Number of Games Won			Total Profit			Average Expected Profit		
	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3
100	0.1293	0.2497	0.2543	42	26	32	5.4323	6.4931	8.1382	0.0543	0.0649	0.0814
200	0.1261	0.249	0.2563	89	57	54	11.222	14.193	13.839	0.0561	0.071	0.0692
300	0.1293	0.2494	0.2545	128	90	82	16.546	22.446	20.867	0.0552	0.0748	0.0696
400	0.1319	0.2519	0.2542	172	116	112	22.694	29.216	28.47	0.0567	0.073	0.0712
500	0.1311	0.2512	0.2537	213	149	138	27.914	37.433	35.004	0.0558	0.0749	0.07
600	0.1318	0.2512	0.2553	260	175	165	34.275	43.959	42.12	0.0571	0.0733	0.0702
700	0.1307	0.2507	0.2584	302	201	197	39.477	50.383	50.914	0.0564	0.072	0.0727
800	0.1301	0.2501	0.2579	352	228	220	45.786	57.023	56.729	0.0572	0.0713	0.0709
900	0.1307	0.254	0.2586	402	256	242	52.557	65.016	62.585	0.0584	0.0722	0.0695
1000	0.1292	0.2565	0.2588	439	285	276	56.707	73.111	71.431	0.0567	0.0731	0.0714
1100	0.1302	0.2579	0.2598	492	307	301	64.076	79.169	78.189	0.0583	0.072	0.0711
1200	0.1278	0.2572	0.2586	544	331	325	69.55	85.141	84.057	0.058	0.071	0.07
1300	0.1294	0.2568	0.2589	583	363	354	75.466	93.212	91.633	0.0581	0.0717	0.0705
1400	0.1296	0.2559	0.2584	632	386	382	81.933	98.773	98.723	0.0585	0.0706	0.0705
1500	0.1302	0.2578	0.2595	680	412	408	88.502	106.2	105.87	0.059	0.0708	0.0706

Table 5

Scenario 3. Bidders “1” and “2” underbid by 0.1 and 0.15 respectively, while others used equilibrium strategy.

# of games	Expected Profit			Number of Games Won			Total Profit			Average Expected Profit		
	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3
100	0.1208	0.0815	0.2891	32	51	17	3.8649	4.1544	4.9149	0.0386	0.0415	0.0491
200	0.141	0.0976	0.2812	72	92	36	10.151	8.9808	10.124	0.0508	0.0449	0.0506
300	0.1425	0.0953	0.2715	112	134	54	15.962	12.774	14.659	0.0532	0.0426	0.0489
400	0.1435	0.0925	0.2671	149	185	66	21.381	17.106	17.63	0.0535	0.0428	0.0441
500	0.1457	0.092	0.2652	184	232	84	26.8	21.35	22.273	0.0536	0.0427	0.0445
600	0.1463	0.0912	0.2656	218	284	98	31.892	25.888	26.029	0.0532	0.0431	0.0434
700	0.1438	0.0904	0.2689	260	319	121	37.38	28.827	32.535	0.0534	0.0412	0.0465
800	0.1445	0.0894	0.2654	295	361	144	42.628	32.269	38.212	0.0533	0.0403	0.0478
900	0.1437	0.0898	0.2643	333	404	163	47.861	36.282	43.083	0.0532	0.0403	0.0479
1000	0.144	0.0893	0.2634	369	457	174	53.121	40.823	45.84	0.0531	0.0408	0.0458
1100	0.1447	0.0882	0.2643	402	510	188	58.189	44.973	49.68	0.0529	0.0409	0.0452
1200	0.1456	0.087	0.2656	441	555	204	64.193	48.277	54.175	0.0535	0.0402	0.0451
1300	0.1456	0.0875	0.2641	476	600	224	69.319	52.518	59.157	0.0533	0.0404	0.0455
1400	0.1444	0.0866	0.2637	508	649	243	73.347	56.211	64.072	0.0524	0.0402	0.0458
1500	0.1441	0.0857	0.2645	541	703	256	77.961	60.255	67.722	0.052	0.0402	0.0451

Table 6

3. Five-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

# of games	Expected Profit					Number of Games Won				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	0.16454	0.16666	0.16219	0.16679	0.14863	23	24	18	20	15
200	0.16428	0.16866	0.16528	0.16976	0.16478	42	44	29	44	41
300	0.16326	0.16577	0.16625	0.16859	0.16719	60	71	48	69	52
400	0.16242	0.16581	0.16628	0.17011	0.16849	78	95	63	92	72
500	0.16003	0.16653	0.16856	0.17027	0.16734	94	114	91	108	93
600	0.16297	0.16639	0.16682	0.16953	0.16519	116	134	109	128	113
700	0.16416	0.16618	0.16642	0.16798	0.16598	131	157	123	156	133
800	0.16533	0.16724	0.16768	0.16829	0.16704	142	176	143	179	160
900	0.16544	0.16813	0.16729	0.16763	0.16794	155	202	167	198	178
1000	0.16481	0.16728	0.16799	0.16717	0.16784	181	220	186	219	194
1100	0.16429	0.16714	0.1677	0.16772	0.16637	202	237	205	239	217
1200	0.16452	0.16698	0.1681	0.16658	0.16665	222	254	229	259	236
1300	0.16376	0.16536	0.16885	0.16611	0.16682	239	277	249	277	258
1400	0.16377	0.16549	0.1686	0.16555	0.16746	257	296	275	290	282
1500	0.16384	0.16578	0.16823	0.16564	0.16781	277	313	294	310	306

Table 7

# of games	Total Profit					Average Expected Profit				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	3.78435	3.99975	2.91939	3.33584	2.22947	0.03784	0.04	0.02919	0.03336	0.02229
200	6.8999	7.42123	4.79326	7.46951	6.75599	0.0345	0.03711	0.02397	0.03735	0.03378
300	9.79589	11.7695	7.98008	11.633	8.69383	0.03265	0.03923	0.0266	0.03878	0.02898
400	12.6686	15.7516	10.4759	15.6505	12.1314	0.03167	0.03938	0.02619	0.03913	0.03033
500	15.0424	18.9843	15.3387	18.3891	15.5623	0.03008	0.03797	0.03068	0.03678	0.03112
600	18.9044	22.2961	18.1839	21.6998	18.6661	0.03151	0.03716	0.03031	0.03617	0.03111
700	21.5047	26.0895	20.4691	26.2051	22.0758	0.03072	0.03727	0.02924	0.03744	0.03154
800	23.4768	29.4339	23.978	30.1234	26.7261	0.02935	0.03679	0.02997	0.03765	0.03341
900	25.6432	33.9621	27.9375	33.1909	29.8928	0.02849	0.03774	0.03104	0.03688	0.03321
1000	29.8303	36.8021	31.2462	36.6109	32.5607	0.02983	0.0368	0.03125	0.03661	0.03256
1100	33.1857	39.6125	34.379	40.0854	36.1022	0.03017	0.03601	0.03125	0.03644	0.03282
1200	36.523	42.4118	38.4945	43.1446	39.3303	0.03044	0.03534	0.03208	0.03595	0.03278
1300	39.1395	45.8051	42.0424	46.0134	43.039	0.03011	0.03523	0.03234	0.03539	0.03311
1400	42.0897	48.9861	46.3654	48.009	47.2229	0.03006	0.03499	0.03312	0.03429	0.03373
1500	45.383	51.889	49.4582	51.3496	51.3498	0.03026	0.03459	0.03297	0.03423	0.03423

Table 8

Scenario 2. Bidder1 underbid by 0.09 while others used equilibrium strategy.

# of games	Expected Profit					Number of Games Won				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	0.06662	0.17163	0.17003	0.17628	0.16915	28	24	9	24	15
200	0.06989	0.17244	0.16765	0.17623	0.17016	60	43	35	38	24
300	0.07082	0.17034	0.16696	0.17415	0.16936	103	56	47	54	40
400	0.07035	0.16842	0.16934	0.17014	0.16941	131	68	62	81	58
500	0.07073	0.16872	0.16992	0.16733	0.16788	159	93	79	100	69
600	0.07168	0.16837	0.16862	0.16918	0.16675	194	107	98	116	85
700	0.07025	0.16775	0.16787	0.16809	0.16647	226	122	112	137	103
800	0.07072	0.16788	0.16867	0.16895	0.16595	249	141	132	154	124
900	0.07063	0.16812	0.1685	0.16932	0.16621	281	161	146	171	141
1000	0.07025	0.16908	0.16885	0.16975	0.16656	309	181	162	192	156
1100	0.07056	0.16932	0.16916	0.16919	0.16647	333	201	175	216	175
1200	0.07043	0.16962	0.16974	0.16872	0.16617	359	219	189	242	191
1300	0.07031	0.16934	0.1697	0.16821	0.16634	390	235	206	256	213
1400	0.07078	0.16962	0.16917	0.16864	0.16708	416	251	232	275	226
1500	0.07034	0.17009	0.16929	0.16867	0.16774	446	272	249	292	241

Table 9

# of games	Total Profit					Average Expected Profit				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	1.86533	4.11905	1.5303	4.23083	2.53728	0.01865	0.04119	0.0153	0.04231	0.02537
200	4.19321	7.415	5.86764	6.69662	4.08382	0.02097	0.03708	0.02934	0.03348	0.02042
300	7.29441	9.53898	7.84725	9.4039	6.77447	0.02431	0.0318	0.02616	0.03135	0.02258
400	9.21626	11.4524	10.4992	13.7811	9.82578	0.02304	0.02863	0.02625	0.03445	0.02456
500	11.2464	15.6913	13.4236	16.7329	11.5839	0.02249	0.03138	0.02685	0.03347	0.02317
600	13.9068	18.0155	16.525	19.6243	14.1736	0.02318	0.03003	0.02754	0.03271	0.02362
700	15.8768	20.4652	18.8009	23.029	17.1465	0.02268	0.02924	0.02686	0.0329	0.02449
800	17.6103	23.6715	22.2644	26.0181	20.5779	0.02201	0.02959	0.02783	0.03252	0.02572
900	19.8471	27.0666	24.6006	28.9534	23.4349	0.02205	0.03007	0.02733	0.03217	0.02604
1000	21.7079	30.6035	27.3544	32.5928	25.9828	0.02171	0.0306	0.02735	0.03259	0.02598
1100	23.4981	34.0336	29.6034	36.5454	29.1322	0.02136	0.03094	0.02691	0.03322	0.02648
1200	25.2851	37.1476	32.0818	40.8302	31.7392	0.02107	0.03096	0.02673	0.03403	0.02645
1300	27.4214	39.7955	34.9583	43.0621	35.43	0.02109	0.03061	0.02689	0.03312	0.02725
1400	29.4465	42.5757	39.2472	46.3758	37.7599	0.02103	0.03041	0.02803	0.03313	0.02697
1500	31.3722	46.2656	42.152	49.2516	40.4257	0.02091	0.03084	0.0281	0.03283	0.02695

Table 10

Scenario 3. Bidders “1” and “2” underbid by 0.02 while others used

equilibrium strategy.

# of games	Expected Profit					Number of Games Won				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	0.0401	0.0465	0.1724	0.1725	0.1584	33	31	14	13	9
200	0.0379	0.0416	0.1713	0.1719	0.1691	69	63	27	26	15
300	0.0392	0.0424	0.1743	0.1675	0.1695	103	93	45	38	21
400	0.0361	0.0409	0.1715	0.167	0.1704	133	123	60	50	34
500	0.0347	0.0412	0.1721	0.168	0.1668	164	147	76	66	47
600	0.0359	0.0405	0.1727	0.1691	0.1688	194	174	89	84	59
700	0.0359	0.0413	0.1727	0.1695	0.1675	222	198	102	100	78
800	0.0362	0.0402	0.1719	0.1677	0.1684	254	224	116	112	94
900	0.0377	0.0407	0.1726	0.1668	0.1693	280	259	132	126	103
1000	0.0382	0.0409	0.1735	0.1659	0.1692	312	282	149	142	115
1100	0.0386	0.0414	0.1737	0.1667	0.1696	339	315	159	155	132
1200	0.0382	0.0405	0.1734	0.1668	0.1695	365	348	175	171	141
1300	0.0377	0.0405	0.1739	0.1668	0.1695	390	378	186	191	155
1400	0.0374	0.0408	0.174	0.1672	0.1705	416	404	195	206	179
1500	0.0378	0.0407	0.1739	0.1676	0.1704	449	436	207	219	189

Table 11

# of games	Total Profit					Average Expected Profit				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	1.3238	1.4415	2.4138	2.2427	1.426	0.0132	0.0144	0.0241	0.0224	0.0143
200	2.617	2.6196	4.6238	4.4683	2.5366	0.0131	0.0131	0.0231	0.0223	0.0127
300	4.0397	3.943	7.8443	6.3651	3.5598	0.0135	0.0131	0.0261	0.0212	0.0119
400	4.7984	5.0304	10.289	8.3486	5.7927	0.012	0.0126	0.0257	0.0209	0.0145
500	5.6849	6.0596	13.08	11.086	7.8381	0.0114	0.0121	0.0262	0.0222	0.0157
600	6.9729	7.0388	15.372	14.205	9.96	0.0116	0.0117	0.0256	0.0237	0.0166
700	7.9641	8.1723	17.619	16.945	13.062	0.0114	0.0117	0.0252	0.0242	0.0187
800	9.1951	8.9943	19.939	18.781	15.834	0.0115	0.0112	0.0249	0.0235	0.0198
900	10.543	10.552	22.78	21.021	17.438	0.0117	0.0117	0.0253	0.0234	0.0194
1000	11.928	11.535	25.846	23.552	19.452	0.0119	0.0115	0.0258	0.0236	0.0195
1100	13.089	13.04	27.614	25.843	22.39	0.0119	0.0119	0.0251	0.0235	0.0204
1200	13.945	14.103	30.352	28.516	23.898	0.0116	0.0118	0.0253	0.0238	0.0199
1300	14.705	15.319	32.348	31.851	26.28	0.0113	0.0118	0.0249	0.0245	0.0202
1400	15.56	16.483	33.92	34.439	30.528	0.0111	0.0118	0.0242	0.0246	0.0218
1500	16.992	17.737	36.004	36.711	32.21	0.0113	0.0118	0.024	0.0245	0.0215

Table 12

Scenario 4. Bidders “1” and “2” underbid by 0.12 and 0.1 while others used equilibrium strategy.

# of games	Expected Profit					Number of Games Won				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	0.03643	0.05744	0.16193	0.17741	0.17095	27	26	12	15	20
200	0.03835	0.05811	0.15915	0.17442	0.17133	67	52	23	28	30
300	0.03658	0.05538	0.16932	0.17086	0.16876	100	73	40	40	47
400	0.03551	0.05884	0.16984	0.17093	0.17146	126	102	55	56	61
500	0.037	0.05921	0.17139	0.16949	0.16925	162	126	71	66	75
600	0.03765	0.05914	0.16879	0.17093	0.16926	192	151	86	82	89
700	0.03853	0.05805	0.16947	0.1708	0.1674	222	175	96	100	107
800	0.03876	0.05844	0.16828	0.17028	0.16738	254	206	108	112	120
900	0.03837	0.05973	0.1689	0.17135	0.16791	286	233	123	125	133
1000	0.03819	0.06077	0.16959	0.17109	0.16783	312	263	143	137	145
1100	0.03838	0.06081	0.16839	0.17055	0.16755	336	293	158	156	157
1200	0.03836	0.06074	0.16743	0.16996	0.16707	364	329	170	166	171
1300	0.03828	0.06148	0.16815	0.17076	0.1684	389	360	183	181	187
1400	0.03794	0.06174	0.16842	0.17058	0.1691	420	388	198	193	201
1500	0.03781	0.06149	0.16858	0.16997	0.16899	448	414	211	209	218

Table 13

# of games	Total Profit					Average Expected Profit				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	0.98357	1.49353	1.94317	2.66117	3.41902	0.00984	0.01494	0.01943	0.02661	0.03419
200	2.56972	3.02163	3.66044	4.88385	5.13989	0.01285	0.01511	0.0183	0.02442	0.0257
300	3.65787	4.04251	6.77262	6.83429	7.93155	0.01219	0.01348	0.02258	0.02278	0.02644
400	4.47372	6.00132	9.34147	9.57231	10.4589	0.01118	0.015	0.02335	0.02393	0.02615
500	5.99464	7.46092	12.1687	11.1864	12.6938	0.01199	0.01492	0.02434	0.02237	0.02539
600	7.22792	8.93027	14.5162	14.0162	15.0637	0.01205	0.01488	0.02419	0.02336	0.02511
700	8.55258	10.1589	16.2691	17.0796	17.9118	0.01222	0.01451	0.02324	0.0244	0.02559
800	9.84624	12.0397	18.1744	19.0711	20.0857	0.01231	0.01505	0.02272	0.02384	0.02511
900	10.9743	13.9162	20.7746	21.4188	22.3324	0.01219	0.01546	0.02308	0.0238	0.02481
1000	11.9163	15.9819	24.251	23.439	24.335	0.01192	0.01598	0.02425	0.02344	0.02434
1100	12.894	17.8186	26.6053	26.6054	26.3049	0.01172	0.0162	0.02419	0.02419	0.02391
1200	13.964	19.9833	28.4625	28.2132	28.5691	0.01164	0.01665	0.02372	0.02351	0.02381
1300	14.8911	22.1316	30.7718	30.9067	31.4907	0.01145	0.01702	0.02367	0.02377	0.02422
1400	15.9334	23.9558	33.3479	32.9222	33.9893	0.01138	0.01711	0.02382	0.02352	0.02428
1500	16.9386	25.4564	35.57	35.5231	36.8405	0.01129	0.01697	0.02371	0.02368	0.02456

Table 14

4. Ten-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

# of games	Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.09112	0.09513	0.09277	0.08994	0.09006	0.09359	0.09605	0.08883	0.09575	0.09416
200	0.09293	0.09383	0.09253	0.08893	0.08917	0.09301	0.09231	0.0906	0.09409	0.09204
300	0.09073	0.09231	0.09183	0.09026	0.09108	0.09319	0.09291	0.08972	0.09291	0.09213
400	0.09113	0.09248	0.09146	0.0912	0.09054	0.09162	0.09195	0.09003	0.09274	0.09095
500	0.09044	0.0911	0.09131	0.09097	0.09059	0.09203	0.09134	0.09072	0.09238	0.09069
600	0.09031	0.091	0.09096	0.09122	0.09134	0.09126	0.09172	0.09106	0.09246	0.09055
700	0.09064	0.09087	0.09094	0.09137	0.09112	0.09169	0.09185	0.09104	0.09196	0.08951
800	0.09087	0.08979	0.09078	0.09138	0.0908	0.09167	0.09219	0.09123	0.09209	0.08976
900	0.09029	0.09	0.09076	0.09113	0.09112	0.09112	0.09196	0.09171	0.09178	0.08987
1000	0.09053	0.09067	0.09086	0.09085	0.09075	0.09142	0.09207	0.09149	0.0918	0.09006
1100	0.09063	0.09107	0.091	0.091	0.09098	0.091	0.09228	0.09176	0.09185	0.09006
1200	0.0908	0.0906	0.09101	0.09054	0.09089	0.09087	0.09198	0.09151	0.0916	0.0904
1300	0.09109	0.09057	0.09092	0.09065	0.09045	0.09077	0.092	0.09169	0.0915	0.09062
1400	0.09062	0.09083	0.09073	0.09086	0.09036	0.09075	0.09192	0.09169	0.09138	0.09093
1500	0.09072	0.0909	0.09068	0.0907	0.09014	0.09103	0.09188	0.09135	0.09148	0.09093

Table 15

# of games	Number of Games Won									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	4	7	15	13	10	12	12	9	9	9
200	20	18	33	18	15	21	23	20	18	14
300	26	28	44	29	23	31	38	25	32	24
400	39	36	50	35	31	46	50	40	41	32
500	49	46	53	45	36	58	62	59	53	39
600	58	52	64	57	47	68	72	67	63	52
700	65	60	76	71	53	78	80	78	77	62
800	74	68	90	79	59	88	90	89	89	74
900	83	75	104	92	66	100	102	99	99	80
1000	100	86	112	98	79	111	110	107	105	92
1100	108	100	117	106	90	124	115	121	118	101
1200	118	112	123	124	105	132	126	125	126	109
1300	128	121	134	130	117	145	138	136	135	116
1400	138	136	143	141	127	151	147	144	146	127
1500	146	146	151	147	140	162	157	156	156	139

Table 16

# of games	Total Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.36447	0.66594	1.39152	1.16916	0.90062	1.12302	1.15263	0.79944	0.86173	0.84748
200	1.85865	1.68889	3.05351	1.60077	1.33759	1.95321	2.12315	1.81191	1.69357	1.28859
300	2.359	2.58456	4.04047	2.6176	2.09487	2.889	3.53074	2.24301	2.97322	2.21122
400	3.55404	3.32931	4.57305	3.19207	2.80681	4.21455	4.59773	3.60111	3.80216	2.91044
500	4.43143	4.1904	4.83954	4.09343	3.26114	5.33768	5.66285	5.35238	4.89608	3.53674
600	5.23824	4.73194	5.82116	5.19937	4.29282	6.20598	6.60383	6.1012	5.82523	4.70863
700	5.89172	5.45218	6.91164	6.48759	4.82919	7.1518	7.34794	7.10141	7.08082	5.54992
800	6.72469	6.10603	8.17038	7.21896	5.35732	8.06681	8.29743	8.11988	8.19575	6.64225
900	7.49384	6.74991	9.43952	8.3841	6.01359	9.11161	9.38034	9.07973	9.08594	7.18936
1000	9.05273	7.79752	10.1761	8.90353	7.16935	10.1471	10.1281	9.78926	9.63874	8.28518
1100	9.78817	9.10718	10.6469	9.64636	8.18797	11.2845	10.6117	11.1035	10.8383	9.09649
1200	10.7149	10.1476	11.1938	11.2271	9.5439	11.9948	11.5901	11.439	11.5421	9.85373
1300	11.6596	10.9591	12.1827	11.7844	10.5822	13.1619	12.696	12.4697	12.3525	10.5117
1400	12.5058	12.3523	12.9742	12.8115	11.4753	13.7026	13.5125	13.2031	13.3417	11.5482
1500	13.2448	13.2716	13.6921	13.3329	12.6196	14.7461	14.4247	14.2506	14.2712	12.6389

Table 17

# of games	Average Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.00364	0.00666	0.01392	0.01169	0.00901	0.01123	0.01153	0.00799	0.00862	0.00847
200	0.00929	0.00844	0.01527	0.008	0.00669	0.00977	0.01062	0.00906	0.00847	0.00644
300	0.00786	0.00862	0.01347	0.00873	0.00698	0.00963	0.01177	0.00748	0.00991	0.00737
400	0.00889	0.00832	0.01143	0.00798	0.00702	0.01054	0.01149	0.009	0.00951	0.00728
500	0.00886	0.00838	0.00968	0.00819	0.00652	0.01068	0.01133	0.0107	0.00979	0.00707
600	0.00873	0.00789	0.0097	0.00867	0.00715	0.01034	0.01101	0.01017	0.00971	0.00785
700	0.00842	0.00779	0.00987	0.00927	0.0069	0.01022	0.0105	0.01014	0.01012	0.00793
800	0.00841	0.00763	0.01021	0.00902	0.0067	0.01008	0.01037	0.01015	0.01024	0.0083
900	0.00833	0.0075	0.01049	0.00932	0.00668	0.01012	0.01042	0.01009	0.0101	0.00799
1000	0.00905	0.0078	0.01018	0.0089	0.00717	0.01015	0.01013	0.00979	0.00964	0.00829
1100	0.0089	0.00828	0.00968	0.00877	0.00744	0.01026	0.00965	0.01009	0.00985	0.00827
1200	0.00893	0.00846	0.00933	0.00936	0.00795	0.01	0.00966	0.00953	0.00962	0.00821
1300	0.00897	0.00843	0.00937	0.00906	0.00814	0.01012	0.00977	0.00959	0.0095	0.00809
1400	0.00893	0.00882	0.00927	0.00915	0.0082	0.00979	0.00965	0.00943	0.00953	0.00825
1500	0.00883	0.00885	0.00913	0.00889	0.00841	0.00983	0.00962	0.0095	0.00951	0.00843

Table 18

Scenario 2. Bidder1 underbid by 0.0825 while others used equilibrium strategy.

# of games	Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.00514	0.09302	0.09079	0.08799	0.08817	0.09169	0.09593	0.09138	0.09197	0.09342
200	0.00556	0.09198	0.0918	0.08977	0.09045	0.09098	0.09227	0.09086	0.09054	0.09195
300	0.00626	0.0922	0.09096	0.09052	0.09039	0.09062	0.09317	0.09185	0.09212	0.09238
400	0.00646	0.09209	0.08904	0.0908	0.09035	0.09163	0.09308	0.09202	0.09194	0.09299
500	0.00691	0.0918	0.08943	0.08893	0.09127	0.09159	0.09222	0.09158	0.09157	0.09238
600	0.00679	0.09056	0.08903	0.08979	0.09046	0.09199	0.09158	0.09116	0.09181	0.09199
700	0.00669	0.09045	0.08957	0.09026	0.09115	0.09132	0.09133	0.09111	0.0913	0.09234
800	0.0066	0.0909	0.08945	0.0897	0.09075	0.09107	0.09126	0.09113	0.09191	0.09232
900	0.00631	0.09131	0.08964	0.08976	0.09122	0.09098	0.09108	0.09073	0.09185	0.0921
1000	0.0061	0.09141	0.08971	0.09028	0.09134	0.09065	0.09115	0.09087	0.09189	0.09166
1100	0.00613	0.09149	0.08945	0.09	0.09128	0.0911	0.09117	0.09083	0.09194	0.09138
1200	0.00585	0.09163	0.08972	0.08994	0.09123	0.09123	0.09147	0.09087	0.0916	0.09148
1300	0.00573	0.09173	0.08947	0.09007	0.09125	0.091	0.09143	0.09095	0.09159	0.09146
1400	0.0057	0.09175	0.08909	0.0903	0.09145	0.09108	0.09131	0.09108	0.0918	0.09147
1500	0.0057	0.0917	0.08923	0.09059	0.09109	0.09121	0.09135	0.09114	0.09185	0.09125

Table 19

# of games	Number of Games Won									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	16	6	14	12	8	8	5	16	8	7
200	39	20	18	16	15	18	19	21	13	21
300	67	27	23	23	30	23	27	27	24	29
400	89	33	31	29	39	35	36	35	36	37
500	107	41	44	40	46	45	47	44	41	45
600	127	54	52	47	58	48	54	54	50	56
700	144	66	64	55	67	58	61	58	62	65
800	164	76	69	68	76	65	72	64	74	72
900	184	88	77	79	82	77	80	71	84	78
1000	201	99	84	89	88	86	88	84	90	91
1100	218	106	92	100	99	95	97	93	98	102
1200	241	114	105	107	105	104	106	96	107	115
1300	263	120	117	114	114	115	113	108	118	118
1400	280	133	125	122	124	121	121	118	131	125
1500	296	143	134	127	134	138	131	124	140	133

Table 20

# of games	Total Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.08227	0.55815	1.27104	1.05591	0.70533	0.73352	0.47967	1.46209	0.73574	0.65395
200	0.2168	1.83969	1.65233	1.43626	1.35675	1.63769	1.7532	1.90813	1.177	1.93085
300	0.41948	2.48947	2.09202	2.08201	2.71156	2.08419	2.51553	2.48002	2.21092	2.67904
400	0.57537	3.03899	2.76039	2.63334	3.52349	3.20692	3.35073	3.22073	3.30999	3.44077
500	0.7399	3.76385	3.93479	3.55736	4.19862	4.1216	4.33455	4.02967	3.75436	4.15711
600	0.86196	4.89001	4.62954	4.22018	5.24678	4.41557	4.94537	4.92287	4.59046	5.15165
700	0.96295	5.96987	5.73229	4.96409	6.10724	5.29641	5.57137	5.28447	5.66082	6.00224
800	1.08181	6.90815	6.17194	6.09935	6.89696	5.91981	6.57098	5.83244	6.80157	6.64673
900	1.16102	8.03528	6.90199	7.09119	7.48035	7.0056	7.28668	6.44167	7.71524	7.18398
1000	1.22677	9.0494	7.53529	8.03523	8.03804	7.7961	8.02141	7.63322	8.27021	8.34094
1100	1.33619	9.6978	8.22963	8.99963	9.03629	8.65438	8.84384	8.44676	9.01027	9.32058
1200	1.40924	10.4462	9.42101	9.62364	9.57904	9.48842	9.69541	8.72344	9.80094	10.5199
1300	1.50682	11.0071	10.4675	10.2685	10.403	10.4653	10.3315	9.82239	10.8077	10.7927
1400	1.59586	12.2033	11.1363	11.017	11.3399	11.0212	11.0486	10.7472	12.026	11.4333
1500	1.68729	13.1132	11.9566	11.5053	12.2065	12.5871	11.9666	11.3013	12.8588	12.1364

Table 21

# of games	Average Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.00082	0.00558	0.01271	0.01056	0.00705	0.00734	0.0048	0.01462	0.00736	0.00654
200	0.00108	0.0092	0.00826	0.00718	0.00678	0.00819	0.00877	0.00954	0.00588	0.00965
300	0.0014	0.0083	0.00697	0.00694	0.00904	0.00695	0.00839	0.00827	0.00737	0.00893
400	0.00144	0.0076	0.0069	0.00658	0.00881	0.00802	0.00838	0.00805	0.00827	0.0086
500	0.00148	0.00753	0.00787	0.00711	0.0084	0.00824	0.00867	0.00806	0.00751	0.00831
600	0.00144	0.00815	0.00772	0.00703	0.00874	0.00736	0.00824	0.0082	0.00765	0.00859
700	0.00138	0.00853	0.00819	0.00709	0.00872	0.00757	0.00796	0.00755	0.00809	0.00857
800	0.00135	0.00864	0.00771	0.00762	0.00862	0.0074	0.00821	0.00729	0.0085	0.00831
900	0.00129	0.00893	0.00767	0.00788	0.00831	0.00778	0.0081	0.00716	0.00857	0.00798
1000	0.00123	0.00905	0.00754	0.00804	0.00804	0.0078	0.00802	0.00763	0.00827	0.00834
1100	0.00121	0.00882	0.00748	0.00818	0.00821	0.00787	0.00804	0.00768	0.00819	0.00847
1200	0.00117	0.00871	0.00785	0.00802	0.00798	0.00791	0.00808	0.00727	0.00817	0.00877
1300	0.00116	0.00847	0.00805	0.0079	0.008	0.00805	0.00795	0.00756	0.00831	0.0083
1400	0.00114	0.00872	0.00795	0.00787	0.0081	0.00787	0.00789	0.00768	0.00859	0.00817
1500	0.00112	0.00874	0.00797	0.00767	0.00814	0.00839	0.00798	0.00753	0.00857	0.00809

Table 22

Scenario 3. Bidders “1” and “2” underbid by 0.0825 while others used

equilibrium strategy.

# of games	Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.0069	0.00508	0.08713	0.09506	0.0893	0.09447	0.09273	0.09156	0.09101	0.08796
200	0.00631	0.00699	0.0894	0.09528	0.09013	0.0895	0.0925	0.09322	0.08887	0.09069
300	0.00566	0.00538	0.09056	0.0946	0.09155	0.08877	0.09193	0.09229	0.08911	0.09041
400	0.00619	0.00632	0.09124	0.09303	0.09134	0.08973	0.0912	0.0905	0.08962	0.09098
500	0.00644	0.00542	0.09155	0.09318	0.09039	0.08971	0.09106	0.09141	0.08811	0.08934
600	0.00635	0.00562	0.09071	0.09291	0.09099	0.08972	0.09021	0.09106	0.08837	0.0898
700	0.00551	0.00597	0.09085	0.09235	0.09108	0.08994	0.09001	0.09116	0.08899	0.09044
800	0.00524	0.00563	0.09123	0.09236	0.09148	0.08994	0.09014	0.08999	0.08914	0.0906
900	0.0054	0.00529	0.09094	0.09243	0.09121	0.08989	0.08983	0.08946	0.08906	0.0908
1000	0.00542	0.00549	0.09114	0.09246	0.09135	0.09019	0.08997	0.08945	0.08852	0.09066
1100	0.00544	0.00597	0.0909	0.09199	0.09119	0.09034	0.09029	0.08968	0.08875	0.09103
1200	0.00518	0.0059	0.09127	0.09199	0.09062	0.09063	0.09017	0.09004	0.08922	0.09116
1300	0.0054	0.00604	0.09133	0.09189	0.09079	0.09083	0.0904	0.09026	0.08912	0.09117
1400	0.0053	0.00587	0.09157	0.09138	0.09068	0.09114	0.09059	0.09005	0.08925	0.0914
1500	0.00553	0.00592	0.0914	0.09109	0.09081	0.09103	0.09052	0.09017	0.08919	0.09117

Table 23

# of games	Number of Games Won									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	14	11	13	9	9	11	11	10	4	8
200	29	24	19	19	14	25	21	17	12	20
300	52	38	31	29	21	33	27	25	19	25
400	62	60	39	40	31	43	34	33	23	35
500	80	72	50	51	37	52	44	44	30	40
600	99	93	54	58	42	67	50	50	36	51
700	121	110	62	65	48	74	61	52	47	60
800	134	133	71	73	53	80	70	62	54	70
900	154	154	74	78	62	85	83	71	63	76
1000	171	172	82	80	74	95	93	81	69	83
1100	193	189	92	92	80	100	99	90	74	91
1200	214	201	102	102	92	108	103	98	83	97
1300	229	224	112	107	98	117	115	105	91	102
1400	245	244	126	120	104	124	119	110	100	108
1500	264	259	135	130	114	133	128	117	109	111

Table 24

# of games	Total Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.09658	0.0559	1.13263	0.85553	0.80368	1.03914	1.02007	0.91562	0.36403	0.70368
200	0.18295	0.1677	1.69859	1.81037	1.26181	2.23741	1.94247	1.58467	1.06643	1.81385
300	0.29439	0.20431	2.80727	2.74341	1.92257	2.92945	2.48223	2.30737	1.69313	2.26036
400	0.38381	0.37924	3.55817	3.72134	2.83163	3.85857	3.10085	2.98647	2.06136	3.18438
500	0.51543	0.39029	4.57772	4.75219	3.34436	4.66516	4.00676	4.02223	2.64321	3.57366
600	0.62906	0.5229	4.89855	5.38859	3.82165	6.01145	4.5106	4.55298	3.18118	4.57999
700	0.66642	0.65634	5.63294	6.00272	4.37197	6.6556	5.49034	4.74017	4.18233	5.4264
800	0.70277	0.74877	6.47719	6.7423	4.84855	7.19507	6.30998	5.57959	4.81357	6.3419
900	0.83192	0.81442	6.72959	7.2098	5.65485	7.64059	7.45624	6.35173	5.61103	6.90056
1000	0.92757	0.94454	7.47348	7.39663	6.76023	8.56834	8.36761	7.24546	6.10775	7.52453
1100	1.04945	1.12836	8.3629	8.46345	7.29519	9.03396	8.93866	8.07154	6.56717	8.28415
1200	1.10751	1.18626	9.30985	9.38258	8.33709	9.7885	9.28735	8.8241	7.40553	8.84255
1300	1.23628	1.35282	10.2292	9.83216	8.8973	10.6271	10.396	9.47734	8.10997	9.2994
1400	1.29956	1.43238	11.538	10.9661	9.43076	11.3019	10.7808	9.90596	8.92511	9.87132
1500	1.4591	1.53433	12.3395	11.8423	10.3519	12.1077	11.5871	10.5495	9.72215	10.1197

Table 25

# of games	Average Expected profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.00097	0.00056	0.01133	0.00856	0.00804	0.01039	0.0102	0.00916	0.00364	0.00704
200	0.00091	0.00084	0.00849	0.00905	0.00631	0.01119	0.00971	0.00792	0.00533	0.00907
300	0.00098	0.00068	0.00936	0.00914	0.00641	0.00976	0.00827	0.00769	0.00564	0.00753
400	0.00096	0.00095	0.0089	0.0093	0.00708	0.00965	0.00775	0.00747	0.00515	0.00796
500	0.00103	0.00078	0.00916	0.0095	0.00669	0.00933	0.00801	0.00804	0.00529	0.00715
600	0.00105	0.00087	0.00816	0.00898	0.00637	0.01002	0.00752	0.00759	0.0053	0.00763
700	0.00095	0.00094	0.00805	0.00858	0.00625	0.00951	0.00784	0.00677	0.00597	0.00775
800	0.00088	0.00094	0.0081	0.00843	0.00606	0.00899	0.00789	0.00697	0.00602	0.00793
900	0.00092	0.0009	0.00748	0.00801	0.00628	0.00849	0.00828	0.00706	0.00623	0.00767
1000	0.00093	0.00094	0.00747	0.0074	0.00676	0.00857	0.00837	0.00725	0.00611	0.00752
1100	0.00095	0.00103	0.0076	0.00769	0.00663	0.00821	0.00813	0.00734	0.00597	0.00753
1200	0.00092	0.00099	0.00776	0.00782	0.00695	0.00816	0.00774	0.00735	0.00617	0.00737
1300	0.00095	0.00104	0.00787	0.00756	0.00684	0.00817	0.008	0.00729	0.00624	0.00715
1400	0.00093	0.00102	0.00824	0.00783	0.00674	0.00807	0.0077	0.00708	0.00638	0.00705
1500	0.00097	0.00102	0.00823	0.00789	0.0069	0.00807	0.00772	0.00703	0.00648	0.00675

Table 26

Scenario 4. Bidders “1”, “2”, “3” and “4” underbid by 0.0825, 0.075, 0.0675

and 0.06 respectively while others used equilibrium strategy.

# of games	Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.00438	0.01511	0.02549	0.03139	0.09044	0.09141	0.0904	0.09346	0.09334	0.09358
200	0.00531	0.01504	0.02509	0.03153	0.09143	0.09182	0.09034	0.09366	0.09419	0.09193
300	0.0057	0.01487	0.02396	0.0315	0.0899	0.09286	0.09107	0.09282	0.09311	0.09154
400	0.0058	0.01475	0.02309	0.03118	0.09076	0.09159	0.09056	0.09125	0.0918	0.09113
500	0.00641	0.01522	0.02346	0.03042	0.09005	0.09229	0.09167	0.09095	0.09156	0.09123
600	0.00659	0.0154	0.02348	0.0298	0.0902	0.09289	0.09134	0.09132	0.09094	0.09016
700	0.00722	0.01477	0.02328	0.02954	0.09041	0.09268	0.09115	0.09169	0.09064	0.09119
800	0.00735	0.01475	0.02315	0.0297	0.09024	0.09178	0.09099	0.09221	0.09063	0.09139
900	0.00724	0.01446	0.02297	0.02963	0.09068	0.09198	0.09114	0.09206	0.09042	0.09168
1000	0.00708	0.01478	0.02316	0.02964	0.09132	0.09212	0.09149	0.09242	0.09042	0.09152
1100	0.00658	0.01468	0.02282	0.02978	0.09141	0.09219	0.0916	0.09264	0.09059	0.0914
1200	0.00647	0.0147	0.02279	0.02967	0.09085	0.09215	0.09156	0.09277	0.09113	0.09135
1300	0.00641	0.0146	0.02274	0.02991	0.09093	0.09192	0.09112	0.09292	0.09064	0.09142
1400	0.00652	0.01461	0.02284	0.03008	0.09088	0.09189	0.09126	0.09278	0.09069	0.09157
1500	0.0063	0.0148	0.0229	0.03004	0.09083	0.09225	0.09116	0.09277	0.09064	0.09173

Table 27

# of games	Number of Games Won									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	17	13	19	11	6	4	7	8	7	8
200	34	34	30	24	11	13	14	14	10	16
300	52	50	48	32	14	23	22	20	16	23
400	72	66	60	43	20	29	30	28	21	31
500	91	87	75	50	28	36	36	34	29	34
600	106	98	90	67	37	47	45	36	34	40
700	119	117	104	82	44	55	48	42	40	49
800	133	133	113	99	51	63	56	49	46	57
900	146	147	127	115	57	68	62	57	57	64
1000	162	159	144	128	66	72	71	64	62	72
1100	183	174	159	139	68	75	81	69	71	81
1200	194	185	171	157	75	83	89	74	78	94
1300	206	199	189	170	81	88	103	81	82	101
1400	218	217	201	188	90	100	111	85	85	105
1500	233	237	215	200	97	109	121	93	88	107

Table 28

# of games	Total Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.07439	0.19648	0.48422	0.34525	0.54263	0.36564	0.6328	0.74772	0.65339	0.74866
200	0.18063	0.51132	0.7526	0.75671	1.00576	1.1937	1.26479	1.31117	0.94192	1.47084
300	0.29617	0.7434	1.1502	1.00801	1.25853	2.1358	2.00348	1.85648	1.4898	2.10546
400	0.41783	0.97346	1.38517	1.34073	1.81517	2.65622	2.71694	2.55498	1.92773	2.82496
500	0.58352	1.32379	1.75984	1.52112	2.52148	3.32242	3.30009	3.09222	2.65533	3.10198
600	0.69871	1.50964	2.11354	1.99679	3.33729	4.36588	4.11028	3.28768	3.09212	3.60628
700	0.85964	1.7284	2.42152	2.42201	3.97815	5.09724	4.37543	3.85107	3.62554	4.46824
800	0.97816	1.96132	2.61539	2.94001	4.60218	5.78207	5.09556	4.51851	4.16914	5.20947
900	1.05636	2.1251	2.91662	3.40688	5.16878	6.25492	5.65067	5.24715	5.15374	5.86763
1000	1.14669	2.35022	3.33472	3.79456	6.02686	6.63269	6.49585	5.91476	5.60589	6.58973
1100	1.20407	2.55394	3.62889	4.13939	6.21596	6.91459	7.41957	6.39214	6.43161	7.40303
1200	1.25503	2.7186	3.89789	4.65837	6.81354	7.64863	8.14909	6.86484	7.1079	8.58648
1300	1.32103	2.90502	4.29724	5.08494	7.36513	8.08872	9.38583	7.52654	7.43223	9.23342
1400	1.4216	3.17035	4.59082	5.65596	8.17886	9.18902	10.1296	7.88653	7.70834	9.61527
1500	1.46744	3.50842	4.92302	6.00835	8.81069	10.0552	11.0308	8.62794	7.97675	9.81493

Table 29

# of games	Average Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.00074	0.00196	0.00484	0.00345	0.00543	0.00366	0.00633	0.00748	0.00653	0.00749
200	0.0009	0.00256	0.00376	0.00378	0.00503	0.00597	0.00632	0.00656	0.00471	0.00735
300	0.00099	0.00248	0.00383	0.00336	0.0042	0.00712	0.00668	0.00619	0.00497	0.00702
400	0.00104	0.00243	0.00346	0.00335	0.00454	0.00664	0.00679	0.00639	0.00482	0.00706
500	0.00117	0.00265	0.00352	0.00304	0.00504	0.00664	0.0066	0.00618	0.00531	0.0062
600	0.00116	0.00252	0.00352	0.00333	0.00556	0.00728	0.00685	0.00548	0.00515	0.00601
700	0.00123	0.00247	0.00346	0.00346	0.00568	0.00728	0.00625	0.0055	0.00518	0.00638
800	0.00122	0.00245	0.00327	0.00368	0.00575	0.00723	0.00637	0.00565	0.00521	0.00651
900	0.00117	0.00236	0.00324	0.00379	0.00574	0.00695	0.00628	0.00583	0.00573	0.00652
1000	0.00115	0.00235	0.00333	0.00379	0.00603	0.00663	0.0065	0.00591	0.00561	0.00659
1100	0.00109	0.00232	0.0033	0.00376	0.00565	0.00629	0.00675	0.00581	0.00585	0.00673
1200	0.00105	0.00227	0.00325	0.00388	0.00568	0.00637	0.00679	0.00572	0.00592	0.00716
1300	0.00102	0.00223	0.00331	0.00391	0.00567	0.00622	0.00722	0.00579	0.00572	0.0071
1400	0.00102	0.00226	0.00328	0.00404	0.00584	0.00656	0.00724	0.00563	0.00551	0.00687
1500	0.00098	0.00234	0.00328	0.00401	0.00587	0.0067	0.00735	0.00575	0.00532	0.00654

Table 30

5. Fifteen-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

# of games	Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.06284	0.06461	0.06259	0.06113	0.06264	0.06279	0.06301	0.06448	0.06302	0.06396	0.06213	0.06176	0.06092	0.06144	0.06119
200	0.0631	0.06389	0.06326	0.06265	0.06219	0.06221	0.06379	0.06374	0.06412	0.06208	0.06129	0.06091	0.06227	0.06264	0.06152
300	0.06285	0.06281	0.06352	0.06211	0.0623	0.06167	0.06398	0.06345	0.06406	0.0629	0.06296	0.06153	0.06288	0.06329	0.06187
400	0.0632	0.06284	0.06365	0.06197	0.06173	0.06064	0.06346	0.06344	0.06415	0.0626	0.06336	0.06213	0.0628	0.06328	0.06201
500	0.06316	0.06256	0.0636	0.06228	0.06212	0.06097	0.06346	0.063	0.06433	0.06289	0.06332	0.06194	0.06238	0.06297	0.06193
600	0.06273	0.06249	0.0632	0.06209	0.06182	0.06121	0.06309	0.06313	0.06452	0.063	0.06337	0.06214	0.06268	0.06275	0.06213
700	0.06314	0.06259	0.06257	0.0616	0.06157	0.06128	0.06319	0.063	0.06415	0.06327	0.06312	0.06199	0.0626	0.06249	0.06223
800	0.06277	0.06244	0.06258	0.06158	0.06164	0.06175	0.06337	0.0632	0.06409	0.06325	0.06252	0.06215	0.06245	0.06268	0.06239
900	0.06282	0.06249	0.06244	0.06164	0.06182	0.06197	0.06334	0.06307	0.06401	0.06331	0.06252	0.0622	0.06243	0.0629	0.06258
1000	0.06278	0.06228	0.06253	0.0616	0.06203	0.06227	0.06322	0.06304	0.0639	0.06331	0.06236	0.06211	0.06236	0.06282	0.06243
1100	0.06287	0.06248	0.0625	0.06179	0.06223	0.06236	0.06323	0.06308	0.06371	0.06308	0.06232	0.06211	0.06217	0.06283	0.06219
1200	0.06282	0.06245	0.0624	0.06184	0.06217	0.06242	0.0631	0.06275	0.06371	0.06308	0.06245	0.06231	0.06199	0.06289	0.06215
1300	0.06266	0.06256	0.06245	0.06196	0.06227	0.06241	0.0629	0.06284	0.06366	0.06313	0.06264	0.06234	0.06174	0.063	0.06195
1400	0.06274	0.0626	0.06257	0.06209	0.06214	0.06242	0.06298	0.06281	0.06346	0.06303	0.06236	0.06248	0.06187	0.06294	0.06209
1500	0.06237	0.06256	0.06263	0.06216	0.06218	0.06244	0.0629	0.0629	0.0635	0.06306	0.06227	0.06234	0.06191	0.06289	0.06201

Table 31

# of games	Number of Games Won														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	6	7	7	7	8	7	10	6	5	5	5	8	5	8	6
200	11	13	16	15	16	12	17	12	14	12	7	12	13	17	13
300	18	19	25	20	23	18	24	18	21	17	16	17	20	25	19
400	21	25	30	27	28	31	33	26	34	22	22	22	28	28	23
500	30	27	40	36	35	38	35	35	39	32	25	30	34	34	30
600	38	38	46	45	40	45	41	38	48	42	28	36	41	42	32
700	46	45	55	54	47	53	48	45	57	48	32	41	42	51	36
800	55	48	67	59	54	64	53	50	60	54	41	47	46	58	44
900	62	58	78	63	61	69	63	58	66	57	48	52	51	64	50
1000	70	62	85	68	68	77	69	67	74	68	52	56	59	69	56
1100	74	67	96	74	72	86	78	72	81	73	59	60	68	76	64
1200	83	73	101	80	79	93	87	79	81	81	65	67	76	85	70
1300	91	79	106	83	87	97	96	91	86	92	71	70	86	89	76
1400	97	84	114	88	91	103	100	97	91	101	85	79	95	94	81
1500	104	94	121	95	103	112	105	100	95	104	90	86	102	101	88

Table 32

# of games	Total Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.37703	0.45225	0.43812	0.42792	0.50109	0.43952	0.6301	0.38886	0.31512	0.31978	0.31065	0.49405	0.30458	0.49152	0.36715
200	0.69407	0.83058	1.01209	0.93974	0.99503	0.74653	1.08441	0.76492	0.89771	0.745	0.42901	0.73093	0.80954	1.06492	0.79981
300	1.13136	1.19333	1.58801	1.24226	1.43281	1.11007	1.53554	1.14202	1.34521	1.06926	1.00742	1.04608	1.25766	1.58232	1.17545
400	1.32721	1.57107	1.90944	1.67328	1.72846	1.87993	2.09415	1.64945	2.18113	1.37709	1.39385	1.36688	1.7584	1.77185	1.42628
500	1.89478	1.68903	2.5439	2.24191	2.17417	2.31681	2.22093	2.20509	2.50893	2.01244	1.58303	1.85808	2.12081	2.14111	1.85785
600	2.38378	2.37453	2.907	2.79409	2.47261	2.75428	2.58676	2.39881	3.09708	2.64608	1.77434	2.23714	2.57007	2.63546	1.98803
700	2.90429	2.81674	3.44125	3.32642	2.89392	3.24791	3.03295	2.83513	3.65672	3.03719	2.01987	2.54168	2.62911	3.18719	2.24042
800	3.45222	2.99727	4.19261	3.63338	3.32857	3.95217	3.35846	3.15991	3.84515	3.41563	2.56338	2.92127	2.8728	3.63544	2.74501
900	3.89474	3.6242	4.86997	3.88348	3.77087	4.27593	3.99059	3.65806	4.22468	3.60882	3.00099	3.23463	3.18368	4.02576	3.12912
1000	4.3946	3.86141	5.31535	4.1887	4.21817	4.79498	4.36236	4.22399	4.72891	4.30503	3.24287	3.47815	3.67937	4.33462	3.49598
1100	4.6522	4.18596	5.99989	4.57264	4.48049	5.36307	4.93226	4.5416	5.16061	4.60504	3.67689	3.72639	4.22723	4.77542	3.98006
1200	5.21413	4.55886	6.30258	4.94759	4.91146	5.80519	5.49007	4.95751	5.16061	5.10963	4.05937	4.17469	4.71112	5.3453	4.35061
1300	5.70223	4.94192	6.6195	5.14262	5.41729	6.05381	6.0385	5.71844	5.47513	5.80761	4.44715	4.36371	5.30976	5.60678	4.70801
1400	6.08585	5.25851	7.13275	5.46391	5.65445	6.4295	6.29763	6.09267	5.77441	6.36573	5.30038	4.93585	5.87747	5.91681	5.02892
1500	6.48661	5.88043	7.57841	5.90476	6.40428	6.99315	6.60414	6.28976	6.03212	6.55817	5.60392	5.36106	6.31508	6.35177	5.45699

Table 33

# of games	Average Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.00377	0.00452	0.00438	0.00428	0.00501	0.0044	0.0063	0.00387	0.00315	0.0032	0.00311	0.00494	0.00305	0.00492	0.00367
200	0.00347	0.00415	0.00506	0.0047	0.00498	0.00373	0.00542	0.00382	0.00449	0.00373	0.00215	0.00365	0.00405	0.00532	0.004
300	0.00377	0.00398	0.00529	0.00414	0.00478	0.0037	0.00512	0.00381	0.00448	0.00356	0.00336	0.00349	0.00419	0.00527	0.00392
400	0.00332	0.00393	0.00477	0.00418	0.00432	0.0047	0.00524	0.00412	0.00545	0.00344	0.00348	0.00342	0.0044	0.00443	0.00357
500	0.00379	0.00338	0.00509	0.00448	0.00435	0.00463	0.00444	0.00441	0.00502	0.00402	0.00317	0.00372	0.00424	0.00428	0.00372
600	0.00397	0.00396	0.00485	0.00466	0.00412	0.00459	0.00431	0.004	0.00516	0.00441	0.00296	0.00373	0.00428	0.00439	0.00331
700	0.00415	0.00402	0.00492	0.00475	0.00413	0.00464	0.00433	0.00405	0.00522	0.00434	0.00289	0.00363	0.00376	0.00455	0.0032
800	0.00432	0.00375	0.00524	0.00454	0.00416	0.00494	0.0042	0.00395	0.00481	0.00427	0.0032	0.00365	0.00359	0.00454	0.00343
900	0.00433	0.00403	0.00541	0.00431	0.00419	0.00475	0.00443	0.00406	0.00469	0.00401	0.00333	0.00359	0.00354	0.00447	0.00348
1000	0.00439	0.00386	0.00532	0.00419	0.00422	0.00479	0.00436	0.00422	0.00473	0.00431	0.00324	0.00348	0.00368	0.00433	0.0035
1100	0.00423	0.00381	0.00545	0.00416	0.00407	0.00488	0.00448	0.00413	0.00469	0.00419	0.00334	0.00339	0.00384	0.00434	0.00362
1200	0.00435	0.0038	0.00525	0.00412	0.00409	0.00484	0.00458	0.00413	0.0043	0.00426	0.00338	0.00348	0.00393	0.00445	0.00363
1300	0.00439	0.0038	0.00509	0.00396	0.00417	0.00466	0.00465	0.0044	0.00421	0.00447	0.00342	0.00336	0.00408	0.00431	0.00362
1400	0.00435	0.00376	0.00509	0.0039	0.00404	0.00459	0.0045	0.00435	0.00412	0.00455	0.00379	0.00353	0.0042	0.00423	0.00359
1500	0.00432	0.00392	0.00505	0.00394	0.00427	0.00466	0.0044	0.00419	0.00402	0.00437	0.00374	0.00357	0.00421	0.00423	0.00364

Table 34

Scenario 2. Bidder1 underbid by 0.075 while others used equilibrium strategy.

# of games	Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.01134	0.06322	0.05986	0.06169	0.06163	0.06171	0.06277	0.06175	0.062	0.06271	0.06504	0.06405	0.06377	0.06418	0.06417
200	0.01178	0.06307	0.06182	0.06256	0.06107	0.06181	0.06322	0.06194	0.0617	0.06201	0.0649	0.06404	0.06303	0.06319	0.063
300	0.01187	0.0628	0.0627	0.06182	0.06165	0.06167	0.06287	0.0616	0.06144	0.06139	0.06355	0.06367	0.06359	0.06369	0.06253
400	0.01206	0.06302	0.06271	0.06236	0.0624	0.0611	0.06295	0.06136	0.06139	0.06191	0.06346	0.06392	0.06317	0.06236	0.06256
500	0.01127	0.06294	0.06326	0.06227	0.06255	0.06179	0.06308	0.06063	0.06159	0.06205	0.06333	0.06369	0.06243	0.06306	0.06284
600	0.01135	0.06268	0.06316	0.06239	0.06281	0.06224	0.06306	0.06092	0.06182	0.06142	0.06304	0.06332	0.06266	0.06319	0.0629
700	0.01165	0.0628	0.06335	0.06226	0.06301	0.06226	0.06286	0.06103	0.06183	0.06112	0.06265	0.06334	0.06275	0.06308	0.06274
800	0.01156	0.06242	0.06343	0.06218	0.06299	0.06218	0.06303	0.06137	0.06145	0.06163	0.06243	0.06344	0.06245	0.06327	0.06282
900	0.01151	0.06243	0.06319	0.06232	0.06305	0.06235	0.06304	0.06163	0.0614	0.06162	0.06229	0.06325	0.06233	0.06328	0.06289
1000	0.01166	0.06259	0.06323	0.06218	0.06302	0.06229	0.06303	0.06188	0.06146	0.06144	0.06233	0.0631	0.06214	0.06317	0.06246
1100	0.01163	0.06268	0.06314	0.06218	0.06298	0.0624	0.06308	0.06176	0.06134	0.06158	0.06224	0.06313	0.06219	0.06295	0.06254
1200	0.01161	0.06292	0.06324	0.06223	0.06296	0.06244	0.0631	0.06162	0.06155	0.06172	0.06219	0.06312	0.06237	0.06311	0.06263
1300	0.01164	0.06277	0.06321	0.06215	0.06298	0.06238	0.06318	0.06169	0.06152	0.06177	0.06217	0.06322	0.06247	0.06298	0.0627
1400	0.0117	0.06283	0.06308	0.06202	0.06308	0.06256	0.06301	0.06178	0.06154	0.06197	0.0623	0.0632	0.06234	0.06298	0.06279
1500	0.01161	0.06286	0.06309	0.06206	0.06304	0.06252	0.06295	0.06179	0.0616	0.06183	0.0624	0.06321	0.06242	0.06276	0.06288

Table 35

# of games	Number of Games Won														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	12	9	4	4	7	9	2	7	9	4	6	8	8	5	6
200	27	16	9	9	12	14	8	16	17	13	10	14	12	9	14
300	36	24	14	18	17	21	16	21	23	19	19	21	19	13	19
400	45	30	21	21	24	34	25	26	32	30	23	25	24	16	24
500	59	35	27	25	36	40	29	32	38	35	28	27	34	26	29
600	68	42	29	31	44	54	33	36	45	43	34	33	40	29	39
700	79	46	31	38	50	60	42	43	55	47	43	38	45	36	47
800	90	52	36	40	52	65	52	51	62	54	49	47	54	42	54
900	99	58	44	42	65	72	58	58	66	61	55	59	60	44	59
1000	109	64	49	47	73	80	63	68	70	68	63	63	70	48	65
1100	132	72	51	52	80	85	65	75	77	75	75	67	72	54	68
1200	145	78	61	55	84	91	70	81	83	83	81	69	83	60	76
1300	159	85	71	62	89	95	80	85	84	88	90	73	90	67	82
1400	168	89	79	65	95	104	83	87	89	97	94	80	99	79	92
1500	179	97	84	69	101	109	90	97	96	102	100	90	102	84	100

Table 36

# of games	Total Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.13613	0.56894	0.23944	0.24678	0.43139	0.55337	0.12534	0.43226	0.558	0.25086	0.39024	0.51244	0.51017	0.3209	0.38503
200	0.31808	1.00911	0.55641	0.56301	0.73285	0.8654	0.50576	0.99103	1.04888	0.80617	0.64903	0.89662	0.75634	0.56869	0.88199
300	0.42745	1.50721	0.87778	1.11282	1.04808	1.29502	1.00588	1.29357	1.41307	1.16642	1.2074	1.33714	1.20817	0.82798	1.18808
400	0.54252	1.8905	1.31697	1.30958	1.49765	2.07746	1.57381	1.59529	1.96464	1.85723	1.45966	1.59797	1.5161	0.99772	1.50145
500	0.66473	2.20296	1.70803	1.55669	2.2517	2.47173	1.8293	1.94026	2.34027	2.17187	1.77322	1.71955	2.12273	1.63959	1.82242
600	0.77171	2.63249	1.8317	1.93404	2.76347	3.36073	2.08114	2.19322	2.7818	2.6409	2.14337	2.08951	2.50652	1.83247	2.45294
700	0.92032	2.88883	1.96374	2.3657	3.15059	3.73587	2.64016	2.62448	3.40082	2.87259	2.6939	2.40677	2.82388	2.27079	2.94863
800	1.03998	3.24568	2.2834	2.48708	3.27536	4.04163	3.27739	3.13	3.81018	3.3279	3.0592	2.98164	3.37236	2.65743	3.39238
900	1.13991	3.62071	2.78032	2.61752	4.09849	4.48904	3.6564	3.57427	4.05257	3.75864	3.42617	3.73181	3.73956	2.78446	3.71063
1000	1.27136	4.00583	3.09808	2.92239	4.60046	4.98287	3.97085	4.20768	4.30187	4.17784	3.92702	3.97515	4.34982	3.03227	4.06007
1100	1.53512	4.51319	3.21999	3.23314	5.03838	5.30382	4.09997	4.63232	4.72319	4.61847	4.66782	4.22954	4.4774	3.39943	4.2525
1200	1.68365	4.90796	3.85757	3.42278	5.28836	5.68161	4.4172	4.99148	5.10874	5.12292	5.03755	4.35538	5.1766	3.78674	4.76006
1300	1.85138	5.33582	4.48814	3.85302	5.60555	5.92618	5.05404	5.24372	5.16788	5.43597	5.59522	4.61502	5.62271	4.21949	5.14156
1400	1.96511	5.59154	4.98324	4.03135	5.9924	6.50661	5.22945	5.37483	5.47737	6.01073	5.85617	5.05635	6.17201	4.97512	5.77687
1500	2.07798	6.09721	5.29944	4.28189	6.36677	6.81471	5.66514	5.99345	5.91407	6.30696	6.24025	5.68861	6.36726	5.27144	6.28848

Table 37

# of games	Average Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.00136	0.00569	0.00239	0.00247	0.00431	0.00555	0.00126	0.00432	0.00558	0.00251	0.0039	0.00512	0.0051	0.00321	0.00385
200	0.00159	0.00505	0.00278	0.00282	0.00366	0.00433	0.00253	0.00496	0.00524	0.00403	0.00325	0.00448	0.00378	0.00284	0.00441
300	0.00142	0.00502	0.00293	0.00371	0.00349	0.00432	0.00335	0.00431	0.00471	0.00389	0.00402	0.00446	0.00403	0.00276	0.00396
400	0.00136	0.00473	0.00329	0.00327	0.00374	0.00519	0.00393	0.00399	0.00491	0.00464	0.00365	0.00399	0.00379	0.00249	0.00375
500	0.00133	0.00441	0.00342	0.00311	0.0045	0.00494	0.00366	0.00388	0.00468	0.00434	0.00355	0.00344	0.00425	0.00328	0.00364
600	0.00129	0.00439	0.00305	0.00322	0.00461	0.0056	0.00347	0.00366	0.00464	0.0044	0.00357	0.00348	0.00418	0.00305	0.00409
700	0.00131	0.00413	0.00281	0.00338	0.0045	0.00534	0.00377	0.00375	0.00486	0.0041	0.00385	0.00344	0.00403	0.00324	0.00421
800	0.0013	0.00406	0.00285	0.00311	0.00409	0.00505	0.0041	0.00391	0.00476	0.00416	0.00382	0.00373	0.00422	0.00332	0.00424
900	0.00127	0.00402	0.00309	0.00291	0.00455	0.00499	0.00406	0.00397	0.0045	0.00418	0.00381	0.00415	0.00416	0.00309	0.00412
1000	0.00127	0.00401	0.0031	0.00292	0.0046	0.00498	0.00397	0.00421	0.0043	0.00418	0.00393	0.00398	0.00435	0.00303	0.00406
1100	0.0014	0.0041	0.00293	0.00294	0.00458	0.00482	0.00373	0.00421	0.00429	0.0042	0.00424	0.00385	0.00407	0.00309	0.00387
1200	0.0014	0.00409	0.00321	0.00285	0.00441	0.00473	0.00368	0.00416	0.00426	0.00427	0.0042	0.00363	0.00431	0.00316	0.00397
1300	0.00142	0.0041	0.00345	0.00296	0.00431	0.00456	0.00389	0.00403	0.00398	0.00418	0.0043	0.00355	0.00433	0.00325	0.00396
1400	0.0014	0.00399	0.00356	0.00288	0.00428	0.00465	0.00374	0.00384	0.00391	0.00429	0.00418	0.00361	0.00441	0.00355	0.00413
1500	0.00139	0.00406	0.00353	0.00285	0.00424	0.00454	0.00378	0.004	0.00394	0.0042	0.00416	0.00379	0.00424	0.00351	0.00419

Table 38

Scenario 3. Bidders “1”, “2” and “3” underbid by 0.055 while others used equilibrium strategy.

# of games	Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.0062	0.0076	0.0065	0.0656	0.0652	0.0629	0.0601	0.0606	0.0622	0.0602	0.0639	0.0603	0.0578	0.0621	0.0641
200	0.0058	0.0076	0.0058	0.0638	0.0648	0.0639	0.062	0.0626	0.0636	0.0613	0.0632	0.0606	0.0605	0.0623	0.0626
300	0.0055	0.0073	0.0065	0.0628	0.0647	0.0635	0.0625	0.062	0.0632	0.062	0.0638	0.0606	0.0614	0.0621	0.0631
400	0.0055	0.0074	0.0062	0.0628	0.0644	0.0641	0.0627	0.0623	0.0633	0.0619	0.064	0.0612	0.0614	0.0624	0.0631
500	0.0059	0.0071	0.0063	0.0625	0.0642	0.0642	0.0628	0.062	0.0637	0.0623	0.064	0.0606	0.0621	0.0623	0.0629
600	0.0058	0.0068	0.0063	0.0627	0.0635	0.0641	0.0632	0.0623	0.0639	0.0625	0.064	0.0607	0.0624	0.0625	0.0625
700	0.0059	0.0066	0.0064	0.0626	0.0637	0.0643	0.0634	0.0623	0.0638	0.0623	0.064	0.0606	0.0626	0.0621	0.0629
800	0.0057	0.0066	0.0067	0.0626	0.0636	0.0645	0.0629	0.062	0.0635	0.0623	0.0638	0.0609	0.0624	0.062	0.0628
900	0.0057	0.0066	0.0063	0.0627	0.0636	0.0641	0.0624	0.062	0.0631	0.0625	0.0639	0.0611	0.0624	0.062	0.0628
1000	0.0058	0.0067	0.0064	0.0625	0.0636	0.0641	0.0625	0.0623	0.0627	0.0623	0.0639	0.0612	0.0625	0.0624	0.063
1100	0.0057	0.0066	0.0064	0.0626	0.0634	0.064	0.0625	0.0624	0.0629	0.0624	0.0639	0.0612	0.0628	0.0625	0.0628
1200	0.0059	0.0068	0.0064	0.0625	0.0632	0.0641	0.0625	0.0621	0.063	0.0625	0.0638	0.0612	0.0627	0.0625	0.0628
1300	0.0058	0.0067	0.0065	0.0626	0.0632	0.064	0.0626	0.0618	0.0628	0.0626	0.0638	0.0612	0.0627	0.0624	0.0628
1400	0.0058	0.0066	0.0065	0.0625	0.0632	0.064	0.0626	0.062	0.0629	0.0626	0.0636	0.0613	0.0627	0.0625	0.0629
1500	0.0057	0.0066	0.0065	0.0625	0.0631	0.0638	0.0626	0.0617	0.063	0.0624	0.0635	0.0615	0.0629	0.0626	0.0629

Table 39

# of games	Number of Games Won														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	14	13	9	6	7	4	5	3	4	6	7	4	4	8	6
200	27	22	19	15	13	11	12	5	8	14	13	11	7	10	13
300	40	36	29	19	17	15	19	11	10	20	19	20	11	17	17
400	52	45	41	24	23	22	21	19	16	27	23	25	15	26	21
500	63	52	51	34	28	27	31	22	18	31	28	35	27	28	25
600	77	62	57	43	33	34	40	25	23	38	33	39	33	32	31
700	88	71	77	47	36	39	43	31	28	42	37	45	40	36	40
800	96	84	88	54	44	47	49	33	32	46	40	55	44	39	49
900	107	91	98	59	52	57	56	37	41	57	42	60	49	40	54
1000	119	103	109	65	56	63	59	43	51	62	46	62	54	49	59
1100	131	115	124	67	64	71	62	44	58	70	49	63	62	53	67
1200	144	125	136	82	71	74	65	48	62	76	54	63	68	56	76
1300	158	135	152	87	76	82	67	52	68	80	59	70	70	62	82
1400	167	154	160	93	81	84	69	60	72	85	68	78	71	71	87
1500	176	163	168	99	86	88	70	67	79	91	76	84	76	82	95

Table 40

# of games	Total Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.0873	0.0987	0.0589	0.3939	0.4562	0.2515	0.3007	0.1817	0.2489	0.3612	0.4476	0.2412	0.2312	0.4966	0.3846
200	0.1566	0.1664	0.1105	0.9569	0.8427	0.7033	0.7438	0.3129	0.5088	0.8579	0.8213	0.6668	0.4236	0.6233	0.8137
300	0.2185	0.2636	0.1895	1.1941	1.1006	0.9524	1.1873	0.6823	0.6324	1.2398	1.2128	1.212	0.6753	1.0561	1.0723
400	0.2874	0.3324	0.254	1.5064	1.481	1.4095	1.3169	1.1842	1.0133	1.6711	1.4711	1.5311	0.9214	1.6225	1.3243
500	0.3689	0.3694	0.3206	2.1257	1.7971	1.7341	1.947	1.3643	1.1463	1.9314	1.7912	2.1222	1.6768	1.744	1.5729
600	0.4502	0.419	0.3586	2.6967	2.0959	2.1793	2.5296	1.5571	1.4696	2.3738	2.112	2.3664	2.0596	2.0015	1.9362
700	0.5153	0.4672	0.4932	2.941	2.2923	2.5077	2.7256	1.9311	1.7861	2.617	2.3684	2.729	2.5036	2.2342	2.5148
800	0.5437	0.5523	0.5888	3.3806	2.7978	3.0329	3.0797	2.0451	2.0319	2.8674	2.5524	3.3499	2.7438	2.4172	3.0756
900	0.6103	0.6015	0.6216	3.7001	3.3048	3.6536	3.495	2.2947	2.5872	3.5605	2.6839	3.6637	3.0561	2.4806	3.3921
1000	0.6872	0.6928	0.7003	4.0638	3.5589	4.0414	3.6852	2.68	3.1979	3.8647	2.9412	3.7951	3.3763	3.0571	3.7148
1100	0.7461	0.7644	0.7915	4.1945	4.0602	4.5463	3.8773	2.7443	3.6468	4.3701	3.1287	3.8536	3.8907	3.3099	4.21
1200	0.8529	0.8467	0.8637	5.1255	4.487	4.7402	4.0649	2.9793	3.9071	4.7513	3.4453	3.8536	4.2628	3.5	4.7726
1300	0.9156	0.9026	0.9949	5.443	4.8001	5.2469	4.1935	3.2121	4.2733	5.0102	3.765	4.2855	4.3894	3.871	5.1477
1400	0.974	1.0135	1.0462	5.8137	5.1221	5.3775	4.3183	3.7184	4.529	5.3189	4.3262	4.7841	4.455	4.4365	5.471
1500	1.0074	1.0798	1.0914	6.1872	5.4229	5.6154	4.3818	4.1325	4.9732	5.6753	4.8255	5.1678	4.7795	5.1327	5.9767

Table 41

# of games	Average Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.0009	0.001	0.0006	0.0039	0.0046	0.0025	0.003	0.0018	0.0025	0.0036	0.0045	0.0024	0.0023	0.004966	0.003846
200	0.0008	0.0008	0.0006	0.0048	0.0042	0.0035	0.0037	0.0016	0.0025	0.0043	0.0041	0.0033	0.0021	0.003117	0.004068
300	0.0007	0.0009	0.0006	0.004	0.0037	0.0032	0.004	0.0023	0.0021	0.0041	0.004	0.004	0.0023	0.00352	0.003574
400	0.0007	0.0008	0.0006	0.0038	0.0037	0.0035	0.0033	0.003	0.0025	0.0042	0.0037	0.0038	0.0023	0.004056	0.003311
500	0.0007	0.0007	0.0006	0.0043	0.0036	0.0035	0.0039	0.0027	0.0023	0.0039	0.0036	0.0042	0.0034	0.003488	0.003146
600	0.0008	0.0007	0.0006	0.0045	0.0035	0.0036	0.0042	0.0026	0.0024	0.004	0.0035	0.0039	0.0034	0.003336	0.003227
700	0.0007	0.0007	0.0007	0.0042	0.0033	0.0036	0.0039	0.0028	0.0026	0.0037	0.0034	0.0039	0.0036	0.003192	0.003593
800	0.0007	0.0007	0.0007	0.0042	0.0035	0.0038	0.0038	0.0026	0.0025	0.0036	0.0032	0.0042	0.0034	0.003022	0.003844
900	0.0007	0.0007	0.0007	0.0041	0.0037	0.0041	0.0039	0.0025	0.0029	0.004	0.003	0.0041	0.0034	0.002756	0.003769
1000	0.0007	0.0007	0.0007	0.0041	0.0036	0.004	0.0037	0.0027	0.0032	0.0039	0.0029	0.0038	0.0034	0.003057	0.003715
1100	0.0007	0.0007	0.0007	0.0038	0.0037	0.0041	0.0035	0.0025	0.0033	0.004	0.0028	0.0035	0.0035	0.003009	0.003827
1200	0.0007	0.0007	0.0007	0.0043	0.0037	0.004	0.0034	0.0025	0.0033	0.004	0.0029	0.0032	0.0036	0.002917	0.003977
1300	0.0007	0.0007	0.0008	0.0042	0.0037	0.004	0.0032	0.0025	0.0033	0.0039	0.0029	0.0033	0.0034	0.002978	0.00396
1400	0.0007	0.0007	0.0007	0.0042	0.0037	0.0038	0.0031	0.0027	0.0032	0.0038	0.0031	0.0034	0.0032	0.003169	0.003908
1500	0.0007	0.0007	0.0007	0.0041	0.0036	0.0037	0.0029	0.0028	0.0033	0.0038	0.0032	0.0034	0.0032	0.003422	0.003984

Table 42

Scenario 4. Bidders “1”, “2”, “3” and “4” underbid by 0.055, 0.05, 0.045 and 0.04 respectively while others used equilibrium strategy.

# of games	Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.00361	0.01086	0.0181	0.02178	0.06058	0.06402	0.06454	0.06011	0.06285	0.06323	0.06263	0.06174	0.06112	0.06228	0.06248
200	0.00397	0.01142	0.01729	0.02226	0.06093	0.06357	0.0635	0.06209	0.06374	0.06401	0.06337	0.0619	0.0615	0.06138	0.06273
300	0.00465	0.01159	0.01633	0.02154	0.06182	0.06384	0.06311	0.06253	0.06356	0.06352	0.06319	0.06152	0.06161	0.06276	0.06234
400	0.00529	0.01164	0.01635	0.02192	0.06194	0.06327	0.06315	0.06272	0.06377	0.06201	0.06316	0.06213	0.06223	0.063	0.06224
500	0.00563	0.01173	0.01631	0.02201	0.06217	0.06339	0.06334	0.06289	0.06307	0.06232	0.06343	0.0618	0.06248	0.06269	0.06245
600	0.0056	0.01195	0.01634	0.02224	0.06236	0.0634	0.06343	0.06236	0.06291	0.06236	0.06343	0.06203	0.06224	0.06232	0.06255
700	0.00542	0.01144	0.01621	0.02191	0.06256	0.06329	0.06343	0.06258	0.06239	0.06244	0.06283	0.06189	0.06237	0.0616	0.06256
800	0.00577	0.01166	0.01648	0.02203	0.06244	0.06297	0.06344	0.06269	0.06261	0.06222	0.06289	0.06198	0.06211	0.06161	0.06265
900	0.00566	0.01172	0.01637	0.02211	0.06237	0.06303	0.06338	0.06274	0.0626	0.06234	0.06308	0.06233	0.0621	0.06157	0.06242
1000	0.00598	0.01144	0.01667	0.02228	0.06214	0.06296	0.06319	0.06281	0.06257	0.06212	0.0628	0.06225	0.06202	0.06167	0.06251
1100	0.00588	0.01139	0.01674	0.02217	0.06218	0.06281	0.0633	0.06299	0.06284	0.06217	0.063	0.06211	0.06196	0.06186	0.06243
1200	0.00584	0.01145	0.0168	0.02224	0.06221	0.06287	0.06327	0.06302	0.06284	0.06221	0.06277	0.06229	0.06199	0.06188	0.06238
1300	0.00596	0.01154	0.0168	0.02217	0.06239	0.06287	0.06329	0.06303	0.06271	0.06236	0.06271	0.06232	0.06197	0.06179	0.06241
1400	0.00604	0.01153	0.01695	0.02218	0.06254	0.0631	0.06332	0.06289	0.06269	0.06233	0.06278	0.06222	0.06191	0.06181	0.06268
1500	0.00605	0.01152	0.01689	0.02202	0.06255	0.06313	0.06331	0.06273	0.06275	0.06246	0.06282	0.06228	0.06192	0.06189	0.0627

Table 43

# of games	Number of Games Won														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	12	10	9	6	6	5	4	3	7	8	5	3	6	4	12
200	26	20	21	11	14	17	7	10	10	13	9	6	11	8	17
300	34	31	27	24	19	22	11	15	15	17	17	9	17	15	27
400	41	42	33	34	25	26	18	21	19	28	25	14	22	21	31
500	45	52	44	45	27	33	25	30	26	35	29	20	28	25	36
600	53	64	56	56	30	38	27	36	29	41	29	27	34	38	42
700	67	76	69	62	32	41	32	39	34	49	34	32	43	44	46
800	78	84	81	75	35	50	36	45	39	56	39	34	50	51	47
900	91	92	95	83	37	59	40	54	42	63	43	40	55	55	51
1000	105	100	107	96	43	64	45	59	47	69	48	45	60	58	54
1100	120	108	121	104	47	73	48	63	51	73	53	51	66	64	58
1200	127	118	132	114	58	77	53	70	57	77	59	57	70	68	63
1300	140	129	147	128	63	84	58	72	60	80	60	66	71	74	68
1400	149	142	155	144	67	93	64	77	61	82	63	72	75	77	79
1500	156	155	162	156	69	97	67	89	67	88	73	81	79	81	80

Table 44

# of games	Total Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.04337	0.1086	0.16288	0.13066	0.36349	0.32009	0.25816	0.18034	0.43992	0.50587	0.31314	0.18521	0.3667	0.24912	0.74979
200	0.10313	0.2285	0.36304	0.24483	0.85307	1.08067	0.44453	0.62092	0.63736	0.83218	0.57035	0.37142	0.67654	0.49101	1.06642
300	0.1582	0.35936	0.44098	0.51699	1.1745	1.40448	0.69419	0.93798	0.95334	1.07986	1.07421	0.5537	1.0474	0.94135	1.6833
400	0.21685	0.48905	0.5397	0.74519	1.54855	1.64507	1.13663	1.31705	1.2117	1.73625	1.57895	0.86982	1.36902	1.32309	1.92947
500	0.25352	0.61006	0.71761	0.99024	1.67851	2.09189	1.58358	1.88677	1.63979	2.18136	1.83958	1.23607	1.74942	1.56732	2.24815
600	0.29698	0.76495	0.91523	1.24551	1.87071	2.40901	1.71255	2.24505	1.82434	2.55695	1.83958	1.67482	2.1162	2.36821	2.6269
700	0.36298	0.86912	1.11847	1.3585	2.00201	2.59498	2.02986	2.44063	2.12124	3.05967	2.13624	1.98059	2.68195	2.71038	2.87795
800	0.44995	0.9793	1.33501	1.6521	2.18529	3.14864	2.28383	2.82094	2.44168	3.48418	2.45273	2.10716	3.1057	3.1419	2.94435
900	0.5152	1.07807	1.55537	1.83532	2.30757	3.71902	2.53507	3.38788	2.62899	3.92737	2.71248	2.49303	3.41534	3.38658	3.18341
1000	0.62826	1.14357	1.78413	2.13883	2.6722	4.02925	2.84364	3.70584	2.94078	4.28657	3.01453	2.80146	3.72103	3.57709	3.37536
1100	0.70508	1.2299	2.0252	2.30615	2.92264	4.58548	3.03829	3.96822	3.20479	4.53864	3.33885	3.1677	4.08907	3.95894	3.62073
1200	0.74141	1.35052	2.21726	2.53562	3.60802	4.84126	3.35357	4.41143	3.5816	4.7903	3.70352	3.55068	4.33919	4.20751	3.92977
1300	0.83375	1.48862	2.46987	2.83771	3.93032	5.28112	3.67109	4.53836	3.76238	4.98851	3.76276	4.11292	4.40018	4.57261	4.24376
1400	0.90024	1.63771	2.62772	3.19336	4.19018	5.86839	4.05236	4.84216	3.82436	5.11139	3.95526	4.48012	4.64309	4.75944	4.95161
1500	0.9441	1.78621	2.73697	3.43539	4.31565	6.12357	4.24164	5.58314	4.20418	5.49658	4.58613	5.04455	4.89204	5.01345	5.01629

Table 45

# of games	Average Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.00043	0.00109	0.00163	0.00131	0.00363	0.0032	0.00258	0.0018	0.0044	0.00506	0.00313	0.00185	0.00367	0.00249	0.00075
200	0.00052	0.00114	0.00182	0.00122	0.00427	0.0054	0.00222	0.0031	0.00319	0.00416	0.00285	0.00186	0.00338	0.00246	0.00533
300	0.00053	0.0012	0.00147	0.00172	0.00391	0.00468	0.00231	0.00313	0.00318	0.0036	0.00358	0.00185	0.00349	0.00314	0.00561
400	0.00054	0.00122	0.00135	0.00186	0.00387	0.00411	0.00284	0.00329	0.00303	0.00434	0.00395	0.00217	0.00342	0.00331	0.00482
500	0.00051	0.00122	0.00144	0.00198	0.00336	0.00418	0.00317	0.00377	0.00328	0.00436	0.00368	0.00247	0.0035	0.00313	0.0045
600	0.00049	0.00127	0.00153	0.00208	0.00312	0.00402	0.00285	0.00374	0.00304	0.00426	0.00307	0.00279	0.00353	0.00395	0.00438
700	0.00052	0.00124	0.0016	0.00194	0.00286	0.00371	0.0029	0.00349	0.00303	0.00437	0.00305	0.00283	0.00383	0.00387	0.00411
800	0.00056	0.00122	0.00167	0.00207	0.00273	0.00394	0.00285	0.00353	0.00305	0.00436	0.00307	0.00263	0.00388	0.00393	0.00368
900	0.00057	0.0012	0.00173	0.00204	0.00256	0.00413	0.00282	0.00376	0.00292	0.00436	0.00301	0.00277	0.00379	0.00376	0.00354
1000	0.00063	0.00114	0.00178	0.00214	0.00267	0.00403	0.00284	0.00371	0.00294	0.00429	0.00301	0.0028	0.00372	0.00358	0.00338
1100	0.00064	0.00112	0.00184	0.0021	0.00266	0.00417	0.00276	0.00361	0.00291	0.00413	0.00304	0.00288	0.00372	0.0036	0.00329
1200	0.00062	0.00113	0.00185	0.00211	0.00301	0.00403	0.00279	0.00368	0.00298	0.00399	0.00309	0.00296	0.00362	0.00351	0.00327
1300	0.00064	0.00115	0.0019	0.00218	0.00302	0.00406	0.00282	0.00349	0.00289	0.00384	0.00289	0.00316	0.00338	0.00352	0.00326
1400	0.00064	0.00117	0.00188	0.00228	0.00299	0.00419	0.00289	0.00346	0.00273	0.00365	0.00283	0.0032	0.00332	0.0034	0.00354
1500	0.00063	0.00119	0.00182	0.00229	0.00288	0.00408	0.00283	0.00372	0.0028	0.00366	0.00306	0.00336	0.00326	0.00334	0.00334

Table 46

B. TRIANGULAR COST DISTRIBUTION

1. Two-Bidder Bidding Game

Scenario 1. Both bidders used equilibrium strategy.

# of games	Expected Profit		# of Games Won		Total Profit		Ave. Exp. Profit	
	Bidder1	Bidder2	Bidder1	Bidder2	Bidder1	Bidder2	Bidder1	Bidder2
100	0.206219	0.238089	49	51	10.10471	12.14254	0.101047	0.121425
200	0.214211	0.238282	92	108	19.70744	25.73446	0.098537	0.128672
300	0.221497	0.228089	131	169	29.01608	38.5471	0.09672	0.12849
400	0.226884	0.221775	179	221	40.61228	49.01231	0.101531	0.122531
500	0.223304	0.221598	224	276	50.02017	61.16102	0.10004	0.122322
600	0.223888	0.220807	278	322	62.2409	71.09995	0.103735	0.1185
700	0.224465	0.222031	336	364	75.42034	80.81943	0.107743	0.115456
800	0.224813	0.220863	378	422	84.97923	93.20432	0.106224	0.116505
900	0.223285	0.221015	429	471	95.78906	104.0983	0.106432	0.115665
1000	0.223027	0.221912	479	521	106.8301	115.6162	0.10683	0.115616
1100	0.221668	0.222563	535	565	118.5924	125.7482	0.107811	0.114317
1200	0.221597	0.221951	575	625	127.4181	138.7192	0.106182	0.115599
1300	0.221923	0.22262	621	679	137.8139	151.1589	0.106011	0.116276
1400	0.222089	0.223333	669	731	148.5776	163.2562	0.106127	0.116612
1500	0.220204	0.223635	725	775	159.6482	173.3172	0.106432	0.115545
1600	0.220878	0.223417	782	818	172.7268	182.7552	0.107954	0.114222
1700	0.220718	0.222405	827	873	182.5334	194.1595	0.107373	0.114211
1800	0.221144	0.223524	871	929	192.6167	207.6536	0.107009	0.115363
1900	0.221703	0.22439	931	969	206.4058	217.4339	0.108635	0.114439
2000	0.221375	0.224752	976	1024	216.0625	230.1461	0.108031	0.115073
2100	0.221022	0.224139	1023	1077	226.1053	241.3976	0.107669	0.114951
2200	0.221547	0.224441	1074	1126	237.9419	252.7204	0.108155	0.114873
2300	0.222452	0.224167	1130	1170	251.3706	262.2757	0.109292	0.114033
2400	0.222262	0.224398	1178	1222	261.8242	274.2143	0.109093	0.114256
2500	0.222308	0.223598	1231	1269	273.6609	283.7462	0.109464	0.113498

Table 47

Scenario 2. Bidder1 underbid by 0.15 while bidder2 used equilibrium strategy.

# of games	Expected Profit		# of Games Won		Total Profit		Ave. Exp. Profit	
	Bidder1	Bidder2	Bidder1	Bidder2	Bidder1	Bidder2	Bidder1	Bidder2
100	0.062809	0.256952	68	32	4.271004	8.222466	0.04271	0.082225
200	0.045651	0.252716	143	57	6.528056	14.40484	0.03264	0.072024
300	0.042422	0.251165	209	91	8.866159	22.85599	0.029554	0.076187
400	0.043888	0.248662	288	112	12.63972	27.85014	0.031599	0.069625
500	0.046166	0.245824	359	141	16.57369	34.66119	0.033147	0.069322
600	0.048138	0.245034	433	167	20.84361	40.92069	0.034739	0.068201
700	0.045984	0.245149	498	202	22.90004	49.52015	0.032714	0.070743
800	0.046905	0.244969	572	228	26.82973	55.85291	0.033537	0.069816
900	0.046433	0.244144	646	254	29.99544	62.01265	0.033328	0.068903
1000	0.04572	0.24415	707	293	32.32439	71.53586	0.032324	0.071536
1100	0.045926	0.244085	778	322	35.73045	78.59541	0.032482	0.07145
1200	0.046346	0.24425	849	351	39.34792	85.73181	0.03279	0.071443
1300	0.048115	0.244711	924	376	44.45839	92.01122	0.034199	0.070778
1400	0.048381	0.244242	992	408	47.99432	99.65085	0.034282	0.071179
1500	0.048138	0.243496	1068	432	51.41143	105.1904	0.034274	0.070127

Table 48

Scenario 3. Both bidders did not use equilibrium strategy and underbid by 0.1.

# of games	Expected Profit		# of Games Won		Total Profit		Ave.Exp. Profit	
	Bidder1	Bidder2	Bidder1	Bidder2	Bidder1	Bidder2	Bidder1	Bidder2
100	0.113678	0.112292	47	53	5.342864	5.951497	0.053429	0.059515
200	0.12326	0.121317	98	102	12.07945	12.37434	0.060397	0.061872
300	0.129273	0.120065	154	146	19.90808	17.52956	0.06636	0.058432
400	0.126543	0.122567	202	198	25.56173	24.26822	0.063904	0.060671
500	0.125876	0.118776	255	245	32.09846	29.10015	0.064197	0.0582
600	0.127644	0.121315	299	301	38.16551	36.5157	0.063609	0.06086
700	0.127994	0.120436	351	349	44.92607	42.03215	0.06418	0.060046
800	0.125206	0.120689	393	407	49.20592	49.12057	0.061507	0.061401
900	0.125597	0.121691	451	449	56.64446	54.63906	0.062938	0.06071
1000	0.123275	0.120888	501	499	61.76094	60.32298	0.061761	0.060323
1100	0.122718	0.120584	547	553	67.12702	66.68284	0.061025	0.060621
1200	0.122694	0.121073	597	603	73.24847	73.00698	0.06104	0.060839
1300	0.12317	0.121394	654	646	80.55314	78.42025	0.061964	0.060323
1400	0.121684	0.122553	705	695	85.78711	85.17436	0.061277	0.060839
1500	0.121015	0.122933	755	745	91.36622	91.58526	0.060911	0.061057
1600	0.121948	0.121706	808	792	98.53395	96.39112	0.061584	0.060244
1700	0.122473	0.122582	860	840	105.3271	102.9691	0.061957	0.06057
1800	0.121631	0.122916	914	886	111.1707	108.9035	0.061762	0.060502
1900	0.121133	0.122171	968	932	117.2568	113.8637	0.061714	0.059928
2000	0.121426	0.122314	1019	981	123.7332	119.9902	0.061867	0.059995
2100	0.120975	0.122718	1072	1028	129.6853	126.1544	0.061755	0.060074
2200	0.121425	0.123081	1122	1078	136.2389	132.6812	0.061927	0.06031
2300	0.120909	0.123714	1160	1140	140.255	141.0338	0.06098	0.061319
2400	0.120884	0.123577	1210	1190	146.2696	147.0561	0.060946	0.061273
2500	0.121266	0.123949	1259	1241	152.6738	153.8211	0.06107	0.061528

Table 49

2. Three-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

# of games	Expected Profit			Number of Games Won			Total Profit			Average Expected Profit		
	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3
100	0.14	0.1468	0.1514	35	30	35	4.9012	4.4051	5.299	0.049	0.0441	0.053
200	0.1491	0.1463	0.1554	74	61	65	11.032	8.9265	10.099	0.0552	0.0446	0.0505
300	0.1469	0.1485	0.1535	113	93	94	16.595	13.807	14.433	0.0553	0.046	0.0481
400	0.1452	0.1479	0.15	146	128	126	21.205	18.936	18.901	0.053	0.0473	0.0473
500	0.1465	0.147	0.1471	172	170	158	25.206	24.993	23.237	0.0504	0.05	0.0465
600	0.1486	0.1478	0.1477	215	200	185	31.958	29.551	27.321	0.0533	0.0493	0.0455
700	0.1486	0.147	0.1485	241	231	228	35.825	33.954	33.865	0.0512	0.0485	0.0484
800	0.1495	0.1481	0.1497	280	256	264	41.862	37.917	39.527	0.0523	0.0474	0.0494
900	0.1504	0.1488	0.1501	310	287	303	46.618	42.698	45.493	0.0518	0.0474	0.0505
1000	0.1499	0.1491	0.1506	339	325	336	50.822	48.467	50.613	0.0508	0.0485	0.0506
1100	0.1499	0.1488	0.1508	380	353	367	56.955	52.519	55.351	0.0518	0.0477	0.0503
1200	0.1486	0.1495	0.1504	414	374	412	61.525	55.924	61.975	0.0513	0.0466	0.0516
1300	0.1472	0.1492	0.1501	449	407	444	66.084	60.724	66.64	0.0508	0.0467	0.0513
1400	0.1479	0.1501	0.15	479	445	476	70.853	66.777	71.378	0.0506	0.0477	0.051
1500	0.1473	0.1495	0.15	510	479	511	75.134	71.621	76.653	0.0501	0.0477	0.0511
1600	0.1479	0.1495	0.1498	551	509	540	81.481	76.085	80.877	0.0509	0.0476	0.0505
1700	0.1483	0.149	0.1501	581	537	582	86.177	80.022	87.363	0.0507	0.0471	0.0514
1800	0.1484	0.1493	0.1505	618	561	621	91.697	83.757	93.472	0.0509	0.0465	0.0519
1900	0.1491	0.1496	0.1508	642	598	660	95.72	89.475	99.503	0.0504	0.0471	0.0524
2000	0.1488	0.1496	0.1511	675	635	690	100.41	94.983	104.24	0.0502	0.0475	0.0521

Table 50

Scenario 2. Bidder1 underbid by 0.1 while others used equilibrium strategy.

# of games	Expected Profit			Number of Games Won			Total Profit			Average Expected Profit		
	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3
100	0.0454	0.1467	0.1594	47	29	24	2.1341	4.2538	3.8264	0.0213	0.0425	0.0383
200	0.0428	0.1491	0.1548	92	48	60	3.9393	7.1574	9.2883	0.0197	0.0358	0.0464
300	0.0452	0.1541	0.1508	136	78	86	6.1407	12.017	12.967	0.0205	0.0401	0.0432
400	0.0424	0.1519	0.151	177	106	117	7.4977	16.1	17.672	0.0187	0.0403	0.0442
500	0.0414	0.1534	0.1512	229	132	139	9.4821	20.25	21.02	0.019	0.0405	0.042
600	0.0407	0.1521	0.1521	273	161	166	11.098	24.485	25.254	0.0185	0.0408	0.0421
700	0.0426	0.1512	0.1534	322	187	191	13.703	28.274	29.295	0.0196	0.0404	0.0419
800	0.0413	0.153	0.1533	380	206	214	15.696	31.527	32.813	0.0196	0.0394	0.041
900	0.0402	0.1524	0.1528	428	233	239	17.184	35.505	36.511	0.0191	0.0395	0.0406
1000	0.0399	0.1526	0.151	467	268	265	18.626	40.904	40.021	0.0186	0.0409	0.04
1100	0.0407	0.1526	0.152	520	296	284	21.161	45.175	43.166	0.0192	0.0411	0.0392
1200	0.0405	0.1516	0.153	559	323	318	22.638	48.961	48.669	0.0189	0.0408	0.0406
1300	0.0414	0.1522	0.1534	607	345	348	25.13	52.521	53.38	0.0193	0.0404	0.0411
1400	0.0419	0.1526	0.1536	646	373	381	27.054	56.934	58.53	0.0193	0.0407	0.0418
1500	0.0416	0.1529	0.1541	687	405	408	28.554	61.925	62.879	0.019	0.0413	0.0419

Table 51

Scenario 3. Bidders “1” and “2” underbid by 0.1 and 0.12 respectively.

# of games	Expected Profit			Number of Games Won			Total Profit			Average Expected Profit		
	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3	Bidder1	Bidder2	Bidder3
100	0.0465	0.0272	0.1513	27	47	26	1.2558	1.2775	3.9342	0.0126	0.0128	0.0393
200	0.0514	0.0226	0.1559	61	88	51	3.1362	1.9853	7.9525	0.0157	0.0099	0.0398
300	0.0527	0.02	0.1581	98	120	82	5.1683	2.3989	12.968	0.0172	0.008	0.0432
400	0.0516	0.0229	0.1587	134	158	108	6.9093	3.6177	17.139	0.0173	0.009	0.0428
500	0.0488	0.0244	0.1587	169	201	130	8.2412	4.9023	20.635	0.0165	0.0098	0.0413
600	0.0496	0.0223	0.1581	207	245	148	10.276	5.4653	23.401	0.0171	0.0091	0.039
700	0.049	0.024	0.1583	238	294	168	11.667	7.0605	26.591	0.0167	0.0101	0.038
800	0.048	0.0228	0.1573	285	331	184	13.679	7.5342	28.945	0.0171	0.0094	0.0362
900	0.048	0.0237	0.1579	320	378	202	15.369	8.9754	31.902	0.0171	0.01	0.0354
1000	0.0481	0.0241	0.1575	357	414	229	17.156	9.9641	36.077	0.0172	0.01	0.0361
1100	0.0471	0.0229	0.1567	397	456	247	18.712	10.462	38.703	0.017	0.0095	0.0352
1200	0.0463	0.0233	0.1582	429	502	269	19.867	11.687	42.562	0.0166	0.0097	0.0355
1300	0.0466	0.0236	0.1575	466	543	291	21.709	12.84	45.841	0.0167	0.0099	0.0353
1400	0.0469	0.0237	0.1577	505	591	304	23.705	14.011	47.947	0.0169	0.01	0.0342
1500	0.0469	0.0237	0.1578	535	639	326	25.093	15.152	51.438	0.0167	0.0101	0.0343
1600	0.0461	0.0237	0.1573	573	681	346	26.395	16.116	54.424	0.0165	0.0101	0.034
1700	0.0456	0.0234	0.1575	611	716	373	27.877	16.759	58.73	0.0164	0.0099	0.0345
1800	0.0461	0.0236	0.1574	643	763	394	29.635	17.99	62.018	0.0165	0.01	0.0345
1900	0.0467	0.0237	0.1569	681	804	415	31.798	19.074	65.118	0.0167	0.01	0.0343
2000	0.0469	0.0239	0.1573	720	851	429	33.771	20.352	67.483	0.0169	0.0102	0.0337

Table 52

3. Five-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

# of games	Expected Profit					Number of Games Won				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	0.08749	0.09988	0.08972	0.08912	0.08875	15	15	23	25	22
200	0.09069	0.09436	0.09066	0.08947	0.09082	33	42	40	36	49
300	0.09114	0.09304	0.08912	0.09072	0.09182	54	60	64	53	69
400	0.09091	0.09187	0.08984	0.09145	0.09122	75	79	80	80	86
500	0.09193	0.09233	0.09091	0.0921	0.0911	103	96	95	102	104
600	0.09242	0.09207	0.09159	0.09272	0.0919	121	119	117	123	120
700	0.09259	0.09162	0.09119	0.09293	0.09108	140	142	136	150	132
800	0.093	0.09141	0.09153	0.09219	0.09193	158	161	153	171	157
900	0.09308	0.09191	0.09097	0.09184	0.09189	174	193	172	186	175
1000	0.09271	0.0922	0.09133	0.0914	0.09166	187	217	185	211	200
1100	0.0922	0.09201	0.09114	0.09156	0.09124	207	232	205	235	221
1200	0.09179	0.09208	0.09112	0.09221	0.09101	223	248	228	254	247
1300	0.09218	0.09218	0.0908	0.0927	0.09128	241	264	253	275	267
1400	0.09201	0.09252	0.09098	0.09276	0.0916	260	288	276	291	285
1500	0.09229	0.09242	0.09119	0.09274	0.09183	279	310	297	311	303

Table 53

# of games	Total Profit					Average Expected Profit				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	1.31239	1.4982	2.06353	2.22795	1.9524	0.01312	0.01498	0.02064	0.02228	0.01952
200	2.99263	3.96305	3.62641	3.22092	4.45015	0.01496	0.01982	0.01813	0.0161	0.02225
300	4.92172	5.58251	5.70388	4.80836	6.33535	0.01641	0.01861	0.01901	0.01603	0.02112
400	6.81839	7.25786	7.18723	7.31595	7.84471	0.01705	0.01814	0.01797	0.01829	0.01961
500	9.46874	8.86324	8.63624	9.394	9.47491	0.01894	0.01773	0.01727	0.01879	0.01895
600	11.1831	10.9568	10.7161	11.4043	11.0286	0.01864	0.01826	0.01786	0.01901	0.01838
700	12.962	13.0094	12.4024	13.9388	12.0229	0.01852	0.01858	0.01772	0.01991	0.01718
800	14.6941	14.7163	14.0046	15.7645	14.4335	0.01837	0.0184	0.01751	0.01971	0.01804
900	16.1953	17.738	15.6474	17.0824	16.081	0.01799	0.01971	0.01739	0.01898	0.01787
1000	17.3369	20.0068	16.8958	19.2861	18.3316	0.01734	0.02001	0.0169	0.01929	0.01833
1100	19.0863	21.3458	18.6837	21.5154	20.1631	0.01735	0.01941	0.01699	0.01956	0.01833
1200	20.4695	22.8358	20.7747	23.4201	22.4783	0.01706	0.01903	0.01731	0.01952	0.01873
1300	22.2152	24.3365	22.9722	25.4933	24.3709	0.01709	0.01872	0.01767	0.01961	0.01875
1400	23.9232	26.6467	25.1104	26.9925	26.107	0.01709	0.01903	0.01794	0.01928	0.01865
1500	25.749	28.6512	27.083	28.8432	27.8236	0.01717	0.0191	0.01806	0.01923	0.01855

Table 54

Scenario 2. Bidder1 underbid by 0.05 while others used equilibrium strategy.

# of games	Expected Profit					Number of Games Won				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	0.04421	0.09189	0.09409	0.09356	0.09471	30	10	19	16	25
200	0.0402	0.09197	0.09405	0.09558	0.09412	60	32	32	32	44
300	0.03931	0.091	0.09468	0.0967	0.09561	91	43	59	48	59
400	0.03921	0.09139	0.09439	0.09708	0.09486	121	56	80	70	73
500	0.03991	0.0924	0.09421	0.09613	0.09473	153	68	99	83	97
600	0.04001	0.09305	0.0939	0.09602	0.09366	178	92	117	100	113
700	0.03954	0.09377	0.09422	0.09533	0.09413	202	110	135	117	136
800	0.03992	0.09342	0.09442	0.09456	0.09438	227	129	153	133	158
900	0.03993	0.09268	0.09455	0.09397	0.09399	248	155	166	153	178
1000	0.04028	0.09278	0.09415	0.09391	0.09371	276	178	178	176	192
1100	0.04022	0.09247	0.09395	0.09392	0.09334	309	195	190	195	211
1200	0.04026	0.09232	0.09388	0.09381	0.0937	331	209	215	213	232
1300	0.04046	0.09232	0.0939	0.09372	0.09322	359	228	228	230	255
1400	0.04062	0.09252	0.09393	0.09332	0.09326	383	250	250	244	273
1500	0.04068	0.09272	0.09401	0.09315	0.09358	402	273	269	270	286

Table 55

# of games	Total Profit					Average Expected Profit				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	1.32642	0.91892	1.78778	1.49689	2.36768	0.01326	0.00919	0.01788	0.01497	0.02368
200	2.41209	2.94303	3.00973	3.0584	4.1414	0.01206	0.01472	0.01505	0.01529	0.02071
300	3.57749	3.91316	5.58639	4.64137	5.64104	0.01192	0.01304	0.01862	0.01547	0.0188
400	4.74467	5.11773	7.55149	6.79533	6.92446	0.01186	0.01279	0.01888	0.01699	0.01731
500	6.10665	6.28286	9.32679	7.97881	9.1889	0.01221	0.01257	0.01865	0.01596	0.01838
600	7.12091	8.56039	10.9868	9.60236	10.5832	0.01187	0.01427	0.01831	0.016	0.01764
700	7.98719	10.3145	12.7194	11.1532	12.8018	0.01141	0.01474	0.01817	0.01593	0.01829
800	9.06094	12.0515	14.4462	12.5765	14.912	0.01133	0.01506	0.01806	0.01572	0.01864
900	9.90244	14.3655	15.6957	14.377	16.7295	0.011	0.01596	0.01744	0.01597	0.01859
1000	11.1185	16.5152	16.7589	16.528	17.9917	0.01112	0.01652	0.01676	0.01653	0.01799
1100	12.4287	18.0318	17.8513	18.3153	19.6943	0.0113	0.01639	0.01623	0.01665	0.0179
1200	13.3257	19.2948	20.1846	19.9812	21.739	0.0111	0.01608	0.01682	0.01665	0.01812
1300	14.5249	21.0483	21.4086	21.5563	23.7717	0.01117	0.01619	0.01647	0.01658	0.01829
1400	15.5561	23.1292	23.4823	22.7698	25.4607	0.01111	0.01652	0.01677	0.01626	0.01819
1500	16.3539	25.3116	25.2892	25.1509	26.7629	0.0109	0.01687	0.01686	0.01677	0.01784

Table 56

Scenario 3. Bidders “1” and “2” underbid by 0.05 while others used equilibrium strategy.

# of games	Expected Profit					Number of Games Won				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	0.04182	0.04036	0.088	0.09277	0.09275	27	29	20	11	13
200	0.03974	0.04107	0.08954	0.09311	0.09657	49	53	33	29	36
300	0.03946	0.04074	0.09098	0.09213	0.09617	78	80	49	42	51
400	0.03927	0.04122	0.09188	0.09232	0.09596	106	104	69	59	62
500	0.03935	0.04016	0.09139	0.09209	0.09636	127	134	83	79	77
600	0.03903	0.03931	0.0911	0.0908	0.09563	155	153	98	94	100
700	0.03934	0.03983	0.09116	0.09162	0.09517	175	168	120	114	123
800	0.03969	0.04073	0.0923	0.09217	0.09458	192	195	141	135	137
900	0.0403	0.04085	0.09292	0.09221	0.09454	213	216	161	155	155
1000	0.0405	0.04059	0.09299	0.09215	0.09436	244	241	179	168	168
1100	0.04056	0.04073	0.09256	0.09218	0.09368	270	258	196	186	190
1200	0.04049	0.04052	0.09215	0.09219	0.09389	285	279	224	202	210
1300	0.04055	0.04099	0.09218	0.09242	0.09421	305	312	239	212	232
1400	0.04099	0.04085	0.0922	0.09223	0.09391	327	338	255	226	254
1500	0.04058	0.04106	0.09227	0.09219	0.09371	351	359	272	242	276

Table 57

# of games	Total Profit					Average Expected Profit				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	1.12904	1.1705	1.76	1.02048	1.20575	0.01129	0.0117	0.0176	0.0102	0.01206
200	1.94749	2.1767	2.95472	2.7002	3.47652	0.00974	0.01088	0.01477	0.0135	0.01738
300	3.0778	3.25927	4.45789	3.86953	4.90488	0.01026	0.01086	0.01486	0.0129	0.01635
400	4.16245	4.28717	6.34	5.44716	5.94929	0.01041	0.01072	0.01585	0.01362	0.01487
500	4.99722	5.38145	7.58527	7.27489	7.42006	0.00999	0.01076	0.01517	0.01455	0.01484
600	6.05031	6.01496	8.9278	8.5356	9.56324	0.01008	0.01002	0.01488	0.01423	0.01594
700	6.88429	6.69195	10.9387	10.4452	11.7063	0.00983	0.00956	0.01563	0.01492	0.01672
800	7.62075	7.9431	13.014	12.4435	12.9572	0.00953	0.00993	0.01627	0.01555	0.0162
900	8.58449	8.82439	14.9603	14.2931	14.653	0.00954	0.0098	0.01662	0.01588	0.01628
1000	9.88266	9.78181	16.6443	15.4819	15.853	0.00988	0.00978	0.01664	0.01548	0.01585
1100	10.9506	10.5072	18.1409	17.1449	17.7996	0.00996	0.00955	0.01649	0.01559	0.01618
1200	11.5403	11.304	20.6423	18.6217	19.7171	0.00962	0.00942	0.0172	0.01552	0.01643
1300	12.3685	12.7889	22.0318	19.5933	21.8564	0.00951	0.00984	0.01695	0.01507	0.01681
1400	13.4044	13.8081	23.5105	20.8441	23.8539	0.00957	0.00986	0.01679	0.01489	0.01704
1500	14.2425	14.7406	25.0985	22.3112	25.8637	0.0095	0.00983	0.01673	0.01487	0.01724

Table 58

Scenario 4. Bidders “1” and “2” underbid by 0.05 and 0.075 respectively while others used equilibrium strategy.

# of games	Expected Profit					Number of Games Won				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	0.04674	0.01812	0.09244	0.09663	0.09607	23	21	18	18	20
200	0.04745	0.01615	0.09205	0.09375	0.09433	47	45	35	34	39
300	0.04605	0.01695	0.09205	0.09358	0.09387	69	74	50	52	55
400	0.04549	0.01678	0.09322	0.09323	0.0948	98	103	64	69	66
500	0.04366	0.0169	0.09281	0.09297	0.09354	115	128	83	90	84
600	0.04315	0.01636	0.09278	0.09343	0.09352	135	162	96	99	108
700	0.04304	0.01631	0.09118	0.09336	0.09353	162	184	114	114	126
800	0.04235	0.01569	0.0917	0.09339	0.09326	186	210	128	138	138
900	0.04258	0.01595	0.09214	0.09333	0.09312	220	235	139	154	152
1000	0.04268	0.01605	0.09241	0.09372	0.0934	240	262	160	172	166
1100	0.04271	0.01578	0.0922	0.09374	0.09267	259	292	175	199	175
1200	0.04299	0.01542	0.09233	0.09339	0.09318	280	320	187	215	198
1300	0.04292	0.01548	0.09201	0.09318	0.09315	301	359	203	224	213
1400	0.04268	0.01557	0.0922	0.09289	0.09282	316	390	214	242	238
1500	0.0423	0.01542	0.09206	0.09289	0.09245	341	410	231	266	252
1600	0.04209	0.01544	0.09215	0.09321	0.09213	359	431	251	281	278
1700	0.04221	0.01556	0.09185	0.09359	0.09224	385	448	273	302	292
1800	0.04209	0.0158	0.09205	0.09362	0.09236	406	480	293	316	305
1900	0.04228	0.0158	0.09213	0.09362	0.09244	431	506	310	333	320
2000	0.04225	0.01572	0.09188	0.09362	0.09246	458	538	328	343	333
2100	0.04202	0.01575	0.09189	0.0936	0.09287	480	564	341	361	354
2200	0.04184	0.01574	0.09194	0.09347	0.09296	507	594	356	374	369
2300	0.04177	0.0157	0.09206	0.09346	0.09302	534	621	375	390	380
2400	0.0417	0.01549	0.09195	0.09336	0.09321	555	652	388	410	395
2500	0.04151	0.01513	0.09196	0.09316	0.09321	582	674	402	424	418

Table 59

# of games	Total Profit					Average Expected Profit				
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5
100	1.075	0.3805	1.664	1.7394	1.9214	0.0107	0.0038	0.0166	0.0174	0.0192
200	2.23	0.7269	3.2216	3.1875	3.6791	0.0111	0.0036	0.0161	0.0159	0.0184
300	3.1777	1.2542	4.6025	4.866	5.1628	0.0106	0.0042	0.0153	0.0162	0.0172
400	4.4579	1.7284	5.9661	6.4331	6.2567	0.0111	0.0043	0.0149	0.0161	0.0156
500	5.0208	2.1638	7.7036	8.3677	7.857	0.01	0.0043	0.0154	0.0167	0.0157
600	5.8259	2.6507	8.9067	9.2495	10.1	0.0097	0.0044	0.0148	0.0154	0.0168
700	6.973	3.0006	10.395	10.643	11.785	0.01	0.0043	0.0148	0.0152	0.0168
800	7.8774	3.2954	11.738	12.888	12.869	0.0098	0.0041	0.0147	0.0161	0.0161
900	9.3682	3.7474	12.807	14.372	14.154	0.0104	0.0042	0.0142	0.016	0.0157
1000	10.243	4.2043	14.785	16.12	15.505	0.0102	0.0042	0.0148	0.0161	0.0155
1100	11.062	4.6091	16.135	18.655	16.218	0.0101	0.0042	0.0147	0.017	0.0147
1200	12.037	4.9337	17.266	20.078	18.45	0.01	0.0041	0.0144	0.0167	0.0154
1300	12.919	5.5556	18.677	20.872	19.84	0.0099	0.0043	0.0144	0.0161	0.0153
1400	13.488	6.073	19.73	22.48	22.091	0.0096	0.0043	0.0141	0.0161	0.0158
1500	14.424	6.3237	21.267	24.71	23.297	0.0096	0.0042	0.0142	0.0165	0.0155
1600	15.111	6.6525	23.131	26.192	25.612	0.0094	0.0042	0.0145	0.0164	0.016
1700	16.253	6.9694	25.076	28.265	26.934	0.0096	0.0041	0.0148	0.0166	0.0158
1800	17.091	7.5816	26.971	29.584	28.17	0.0095	0.0042	0.015	0.0164	0.0156
1900	18.222	7.9954	28.559	31.175	29.582	0.0096	0.0042	0.015	0.0164	0.0156
2000	19.351	8.4585	30.138	32.11	30.788	0.0097	0.0042	0.0151	0.0161	0.0154
2100	20.169	8.882	31.333	33.79	32.874	0.0096	0.0042	0.0149	0.0161	0.0157
2200	21.213	9.3467	32.73	34.96	34.303	0.0096	0.0042	0.0149	0.0159	0.0156
2300	22.307	9.749	34.521	36.45	35.347	0.0097	0.0042	0.015	0.0158	0.0154
2400	23.142	10.101	35.676	38.278	36.818	0.0096	0.0042	0.0149	0.0159	0.0153
2500	24.156	10.194	36.97	39.499	38.961	0.0097	0.0041	0.0148	0.0158	0.0156

Table 60

4. Ten-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

# of games	Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.05026	0.04687	0.04906	0.04925	0.04982	0.04808	0.04671	0.04911	0.04737	0.04993
200	0.04791	0.04597	0.04851	0.0481	0.04933	0.04775	0.04745	0.04825	0.04707	0.04938
300	0.04808	0.04694	0.04849	0.04783	0.04937	0.04743	0.04838	0.04786	0.04694	0.04899
400	0.04804	0.0471	0.04762	0.04784	0.04914	0.04709	0.04823	0.0478	0.04762	0.04851
500	0.04847	0.04723	0.04772	0.04777	0.04895	0.04723	0.04824	0.04806	0.04763	0.0482
600	0.04827	0.04735	0.04794	0.04781	0.0489	0.0474	0.04805	0.04825	0.04723	0.04842
700	0.04818	0.04775	0.04799	0.04817	0.04894	0.04773	0.04798	0.04817	0.0475	0.0486
800	0.04776	0.0478	0.04767	0.0481	0.04876	0.04778	0.04811	0.04817	0.04758	0.0485
900	0.0478	0.04747	0.04763	0.0482	0.04882	0.04794	0.04779	0.04802	0.04773	0.04866
1000	0.04775	0.04758	0.0473	0.04825	0.04868	0.04752	0.04799	0.04804	0.04776	0.04871
1100	0.04769	0.04772	0.04731	0.04844	0.04869	0.04764	0.04809	0.04808	0.04782	0.04864
1200	0.04776	0.04784	0.04738	0.04835	0.04865	0.0477	0.04788	0.04818	0.04795	0.04861
1300	0.04773	0.04783	0.04759	0.04809	0.04851	0.04774	0.04779	0.04807	0.04798	0.04821
1400	0.04758	0.04787	0.04772	0.04814	0.0485	0.04778	0.0478	0.0478	0.04778	0.04824
1500	0.04758	0.04792	0.04777	0.04816	0.04843	0.04784	0.04788	0.04783	0.04792	0.04814

Table 61

# of games	Number of Games Won									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	3	10	13	11	8	9	11	10	11	14
200	14	22	23	19	17	22	22	18	19	24
300	25	30	34	28	24	35	30	32	26	36
400	33	40	47	34	31	46	40	44	40	45
500	44	48	62	45	38	54	53	52	51	53
600	61	60	68	53	46	66	61	63	57	65
700	72	69	84	63	51	81	70	71	66	73
800	84	78	94	67	65	94	79	81	76	82
900	94	87	104	75	76	108	89	88	86	93
1000	107	100	113	85	79	122	99	103	93	99
1100	117	113	122	97	87	132	107	116	97	112
1200	126	128	130	110	97	139	117	126	106	121
1300	136	136	139	121	105	153	128	137	113	132
1400	153	145	150	132	115	161	137	143	120	144
1500	161	153	155	145	130	171	144	152	133	156

Table 62

# of games	Total Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.15079	0.4687	0.63781	0.54174	0.39859	0.43274	0.51377	0.4911	0.52107	0.69908
200	0.67075	1.01141	1.11579	0.91394	0.83867	1.05043	1.04391	0.86841	0.89432	1.18501
300	1.20193	1.40814	1.64852	1.33925	1.18478	1.65995	1.45142	1.53138	1.22045	1.76347
400	1.58536	1.88405	2.23817	1.62651	1.5232	2.16622	1.92932	2.10319	1.90461	2.18315
500	2.13288	2.26682	2.95892	2.14976	1.86006	2.55046	2.55657	2.49924	2.42891	2.55475
600	2.94422	2.84113	3.25965	2.53367	2.24959	3.12811	2.9312	3.03944	2.69183	3.14708
700	3.46909	3.29466	4.03112	3.03458	2.49581	3.86584	3.35857	3.42031	3.13471	3.54814
800	4.01204	3.72815	4.48109	3.22297	3.16916	4.49151	3.80043	3.902	3.61583	3.97693
900	4.49303	4.12965	4.95399	3.61473	3.71014	5.17737	4.25317	4.22602	4.10521	4.52576
1000	5.10881	4.75789	5.34487	4.10115	3.84593	5.79802	4.75128	4.94776	4.44183	4.8218
1100	5.58027	5.39242	5.77149	4.69851	4.23619	6.28816	5.14514	5.57671	4.63856	5.44733
1200	6.01733	6.12402	6.15977	5.31874	4.71887	6.63022	5.60246	6.07045	5.08234	5.88167
1300	6.49161	6.50554	6.615	5.81876	5.09331	7.30392	6.1175	6.5852	5.42129	6.36337
1400	7.28042	6.94089	7.15868	6.35497	5.57765	7.69201	6.5487	6.83476	5.73393	6.94706
1500	7.66067	7.3325	7.4039	6.98389	6.29654	8.17982	6.89477	7.27088	6.37304	7.51016

Table 63

# of games	Average Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.00151	0.00469	0.00638	0.00542	0.00399	0.00433	0.00514	0.00491	0.00521	0.00699
200	0.00335	0.00506	0.00558	0.00457	0.00419	0.00525	0.00522	0.00434	0.00447	0.00593
300	0.00401	0.00469	0.0055	0.00446	0.00395	0.00553	0.00484	0.0051	0.00407	0.00588
400	0.00396	0.00471	0.0056	0.00407	0.00381	0.00542	0.00482	0.00526	0.00476	0.00546
500	0.00427	0.00453	0.00592	0.0043	0.00372	0.0051	0.00511	0.005	0.00486	0.00511
600	0.00491	0.00474	0.00543	0.00422	0.00375	0.00521	0.00489	0.00507	0.00449	0.00525
700	0.00496	0.00471	0.00576	0.00434	0.00357	0.00552	0.0048	0.00489	0.00448	0.00507
800	0.00502	0.00466	0.0056	0.00403	0.00396	0.00561	0.00475	0.00488	0.00452	0.00497
900	0.00499	0.00459	0.0055	0.00402	0.00412	0.00575	0.00473	0.0047	0.00456	0.00503
1000	0.00511	0.00476	0.00534	0.0041	0.00385	0.0058	0.00475	0.00495	0.00444	0.00482
1100	0.00507	0.0049	0.00525	0.00427	0.00385	0.00572	0.00468	0.00507	0.00422	0.00495
1200	0.00501	0.0051	0.00513	0.00443	0.00393	0.00553	0.00467	0.00506	0.00424	0.0049
1300	0.00499	0.005	0.00509	0.00448	0.00392	0.00562	0.00471	0.00507	0.00417	0.00489
1400	0.0052	0.00496	0.00511	0.00454	0.00398	0.00549	0.00468	0.00488	0.0041	0.00496
1500	0.00511	0.00489	0.00494	0.00466	0.0042	0.00545	0.0046	0.00485	0.00425	0.00501

Table 64

Scenario 2. Bidder1 underbid by 0.025 while others used equilibrium strategy.

# of games	Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.0217	0.0507	0.0494	0.0483	0.0481	0.047	0.0479	0.0472	0.0484	0.049
200	0.0217	0.0489	0.0486	0.0486	0.0466	0.0473	0.0479	0.0474	0.0486	0.0483
300	0.0212	0.0492	0.0478	0.0479	0.0475	0.0465	0.0478	0.0469	0.0487	0.0469
400	0.022	0.0488	0.0479	0.0483	0.0475	0.047	0.0477	0.0474	0.0485	0.0472
500	0.022	0.0488	0.0479	0.0482	0.0473	0.0476	0.0481	0.0476	0.0483	0.0477
600	0.0221	0.0488	0.0478	0.0483	0.0473	0.0471	0.0478	0.0477	0.0484	0.048
700	0.0221	0.0487	0.0475	0.0486	0.0473	0.0471	0.0478	0.0477	0.048	0.0478
800	0.0223	0.0484	0.0474	0.0486	0.047	0.0472	0.048	0.0473	0.0481	0.0477
900	0.0223	0.0483	0.0474	0.0485	0.0468	0.0474	0.048	0.0476	0.0483	0.0479
1000	0.0225	0.0483	0.0477	0.0484	0.0467	0.0476	0.0479	0.0475	0.0482	0.0478
1100	0.0225	0.0484	0.0476	0.0483	0.0468	0.0477	0.048	0.0475	0.0482	0.0477
1200	0.0223	0.0485	0.0476	0.0483	0.0469	0.0477	0.0481	0.0476	0.0483	0.0478
1300	0.0224	0.0484	0.0477	0.048	0.0469	0.0476	0.048	0.0476	0.0479	0.0477
1400	0.0225	0.0485	0.0478	0.0481	0.0472	0.0476	0.048	0.0477	0.0478	0.0477
1500	0.0225	0.0485	0.0479	0.0483	0.0472	0.0476	0.048	0.0477	0.0478	0.0476
1600	0.0224	0.0483	0.048	0.0482	0.0473	0.0477	0.048	0.0476	0.0478	0.0476
1700	0.0225	0.0483	0.048	0.0483	0.0472	0.0476	0.0477	0.0477	0.0478	0.0476
1800	0.0226	0.0483	0.0479	0.0483	0.0472	0.0476	0.0476	0.0477	0.0479	0.0477
1900	0.0225	0.0484	0.0478	0.0483	0.0472	0.0477	0.0477	0.0478	0.0479	0.0476
2000	0.0224	0.0484	0.0477	0.0484	0.0472	0.0477	0.0477	0.0479	0.048	0.0477
2100	0.0225	0.0483	0.0477	0.0485	0.0473	0.0477	0.0477	0.0479	0.0479	0.0477
2200	0.0226	0.0484	0.0479	0.0485	0.0474	0.0476	0.0475	0.048	0.048	0.0476
2300	0.0225	0.0484	0.0479	0.0484	0.0474	0.0476	0.0477	0.048	0.048	0.0476
2400	0.0225	0.0484	0.048	0.0484	0.0474	0.0476	0.0478	0.048	0.0479	0.0477
2500	0.0225	0.0484	0.0481	0.0483	0.0474	0.0477	0.0478	0.048	0.048	0.0477

Table 65

# of games	Number of Games Won									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	15	14	7	10	6	10	11	8	10	9
200	25	28	13	16	16	22	22	18	21	19
300	36	41	22	25	35	35	29	22	27	28
400	45	51	38	39	42	42	39	30	39	35
500	53	66	47	54	58	50	46	32	48	46
600	64	75	55	64	64	70	57	40	55	56
700	75	84	71	77	70	80	65	50	64	64
800	88	93	85	87	77	88	69	63	76	74
900	101	107	93	97	84	99	79	74	82	84
1000	112	116	105	108	94	109	85	85	94	92
1100	120	124	113	123	101	124	97	93	104	101
1200	130	133	118	132	113	135	111	100	116	112
1300	144	138	126	145	119	145	122	114	127	120
1400	159	151	136	151	133	155	127	123	134	131
1500	178	160	146	162	140	169	131	132	141	141
1600	195	165	153	170	149	182	138	139	157	152
1700	206	172	163	177	165	194	148	146	168	161
1800	218	178	179	189	172	206	157	155	174	172
1900	227	188	192	197	189	216	164	166	182	179
2000	235	196	204	211	197	225	174	175	197	186
2100	245	208	212	226	210	229	180	184	205	201
2200	257	213	222	235	216	238	193	198	220	208
2300	265	228	235	245	223	248	205	207	229	215
2400	276	238	244	249	233	259	219	218	238	226
2500	289	246	257	255	244	270	229	226	246	238

Table 66

# of games	Total Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.3252	0.7101	0.3456	0.4828	0.2889	0.47	0.5274	0.3779	0.484	0.4409
200	0.5418	1.3688	0.6314	0.7774	0.7451	1.0417	1.0531	0.8524	1.0197	0.917
300	0.7647	2.0162	1.0508	1.1968	1.664	1.6284	1.385	1.0314	1.3143	1.3142
400	0.9903	2.4872	1.8188	1.8827	1.9937	1.976	1.8622	1.4217	1.8899	1.6521
500	1.1665	3.2196	2.2519	2.6004	2.7447	2.3804	2.2107	1.5224	2.3201	2.1941
600	1.4152	3.6569	2.6297	3.0931	3.0268	3.3003	2.7218	1.9075	2.6632	2.6859
700	1.6568	4.0934	3.376	3.7438	3.3123	3.7692	3.1079	2.3841	3.0716	3.0565
800	1.9617	4.4979	4.0272	4.2296	3.6161	4.1525	3.3095	2.9784	3.6586	3.5308
900	2.2565	5.1641	4.4109	4.7027	3.9282	4.693	3.79	3.5209	3.9626	4.0257
1000	2.5205	5.6029	5.0042	5.2256	4.3928	5.1862	4.0703	4.0382	4.5313	4.401
1100	2.6953	5.9997	5.382	5.936	4.7301	5.9183	4.6528	4.4207	5.0124	4.821
1200	2.9012	6.4525	5.615	6.3691	5.2963	6.4429	5.3356	4.7587	5.6063	5.3497
1300	3.2297	6.6856	6.0123	6.9659	5.577	6.9054	5.8584	5.4236	6.0865	5.7283
1400	3.5791	7.3168	6.5073	7.258	6.2721	7.3733	6.1007	5.8692	6.4063	6.2494
1500	4.0011	7.7628	6.9986	7.8166	6.6119	8.0409	6.293	6.2929	6.7457	6.7108
1600	4.372	7.9771	7.3455	8.2025	7.0534	8.678	6.6245	6.6109	7.5113	7.2307
1700	4.6287	8.3098	7.8207	8.5468	7.7884	9.2265	7.0657	6.9613	8.0355	7.663
1800	4.9233	8.5942	8.5704	9.1217	8.1113	9.8048	7.4765	7.3946	8.33	8.1996
1900	5.1088	9.0943	9.1807	9.5111	8.9199	10.3	7.819	7.9405	8.7265	8.5292
2000	5.2698	9.4777	9.7404	10.217	9.2893	10.736	8.297	8.3835	9.4467	8.8703
2100	5.514	10.057	10.122	10.956	9.9334	10.925	8.5882	8.8097	9.829	9.5834
2200	5.8054	10.307	10.629	11.393	10.235	11.329	9.1767	9.5034	10.566	9.9025
2300	5.9633	11.04	11.259	11.856	10.573	11.815	9.7797	9.9395	10.988	10.24
2400	6.2043	11.518	11.716	12.044	11.042	12.341	10.463	10.473	11.397	10.779
2500	6.5026	11.9	12.349	12.322	11.557	12.876	10.943	10.859	11.798	11.356

Table 67

# of games	Average Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.0033	0.0071	0.0035	0.0048	0.0029	0.0047	0.0053	0.0038	0.0048	0.0044
200	0.0027	0.0068	0.0032	0.0039	0.0037	0.0052	0.0053	0.0043	0.0051	0.0046
300	0.0025	0.0067	0.0035	0.004	0.0055	0.0054	0.0046	0.0034	0.0044	0.0044
400	0.0025	0.0062	0.0045	0.0047	0.005	0.0049	0.0047	0.0036	0.0047	0.0041
500	0.0023	0.0064	0.0045	0.0052	0.0055	0.0048	0.0044	0.003	0.0046	0.0044
600	0.0024	0.0061	0.0044	0.0052	0.005	0.0055	0.0045	0.0032	0.0044	0.0045
700	0.0024	0.0058	0.0048	0.0053	0.0047	0.0054	0.0044	0.0034	0.0044	0.0044
800	0.0025	0.0056	0.005	0.0053	0.0045	0.0052	0.0041	0.0037	0.0046	0.0044
900	0.0025	0.0057	0.0049	0.0052	0.0044	0.0052	0.0042	0.0039	0.0044	0.0045
1000	0.0025	0.0056	0.005	0.0052	0.0044	0.0052	0.0041	0.004	0.0045	0.0044
1100	0.0025	0.0055	0.0049	0.0054	0.0043	0.0054	0.0042	0.004	0.0046	0.0044
1200	0.0024	0.0054	0.0047	0.0053	0.0044	0.0054	0.0044	0.004	0.0047	0.0045
1300	0.0025	0.0051	0.0046	0.0054	0.0043	0.0053	0.0045	0.0042	0.0047	0.0044
1400	0.0026	0.0052	0.0046	0.0052	0.0045	0.0053	0.0044	0.0042	0.0046	0.0045
1500	0.0027	0.0052	0.0047	0.0052	0.0044	0.0054	0.0042	0.0042	0.0045	0.0045
1600	0.0027	0.005	0.0046	0.0051	0.0044	0.0054	0.0041	0.0041	0.0047	0.0045
1700	0.0027	0.0049	0.0046	0.005	0.0046	0.0054	0.0042	0.0041	0.0047	0.0045
1800	0.0027	0.0048	0.0048	0.0051	0.0045	0.0054	0.0042	0.0041	0.0046	0.0046
1900	0.0027	0.0048	0.0048	0.005	0.0047	0.0054	0.0041	0.0042	0.0046	0.0045
2000	0.0026	0.0047	0.0049	0.0051	0.0046	0.0054	0.0041	0.0042	0.0047	0.0044
2100	0.0026	0.0048	0.0048	0.0052	0.0047	0.0052	0.0041	0.0042	0.0047	0.0046
2200	0.0026	0.0047	0.0048	0.0052	0.0047	0.0051	0.0042	0.0043	0.0048	0.0045
2300	0.0026	0.0048	0.0049	0.0052	0.0046	0.0051	0.0043	0.0043	0.0048	0.0045
2400	0.0026	0.0048	0.0049	0.005	0.0046	0.0051	0.0044	0.0044	0.0047	0.0045
2500	0.0026	0.0048	0.0049	0.0049	0.0046	0.0052	0.0044	0.0043	0.0047	0.0045

Table 68

Scenario 3. Bidders “1” and “2” underbid by 0.025 while others used

equilibrium strategy.

# of games	Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.02212	0.02373	0.0487	0.04898	0.04677	0.04714	0.04748	0.04981	0.04578	0.04711
200	0.02207	0.02381	0.04921	0.04882	0.04797	0.04861	0.04696	0.04608	0.04735	0.04866
300	0.02203	0.02328	0.04903	0.04859	0.04752	0.04817	0.04762	0.04588	0.04626	0.04945
400	0.02199	0.02298	0.04873	0.04901	0.04763	0.04793	0.04801	0.04686	0.04673	0.04955
500	0.02202	0.02291	0.04886	0.04915	0.04775	0.048	0.04788	0.04694	0.04698	0.04901
600	0.02221	0.02302	0.04853	0.04893	0.04787	0.04827	0.04792	0.04724	0.04711	0.04875
700	0.02252	0.02285	0.04854	0.04893	0.0474	0.04813	0.04795	0.04763	0.04707	0.04831
800	0.0222	0.02259	0.0484	0.04866	0.0473	0.04784	0.04791	0.0479	0.0475	0.04763
900	0.02237	0.02287	0.04826	0.04872	0.04741	0.04787	0.04804	0.0478	0.04752	0.04761
1000	0.02255	0.02265	0.04837	0.04875	0.0472	0.0478	0.04805	0.04791	0.04768	0.04773
1100	0.02244	0.02273	0.04801	0.0488	0.04731	0.04779	0.04809	0.04796	0.04773	0.04773
1200	0.02234	0.023	0.04818	0.04861	0.04743	0.04798	0.04811	0.04813	0.04788	0.04792
1300	0.02246	0.02301	0.04827	0.04866	0.04725	0.04801	0.04806	0.0479	0.04758	0.04791
1400	0.02247	0.02297	0.04822	0.04879	0.04738	0.048	0.04798	0.04787	0.04757	0.04771
1500	0.02258	0.02307	0.04816	0.04889	0.04735	0.048	0.04802	0.04801	0.04759	0.0478

Table 69

# of games	Number of Games Won									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	11	15	8	14	11	6	13	7	8	7
200	22	23	18	21	24	19	18	23	14	18
300	34	31	28	36	35	28	31	29	21	27
400	44	39	35	45	46	40	43	40	35	33
500	57	49	47	55	51	48	51	47	48	47
600	72	56	58	68	61	55	59	63	56	52
700	84	64	67	80	71	66	67	75	61	65
800	100	73	73	93	73	78	72	85	74	79
900	112	92	83	99	86	87	78	95	80	88
1000	122	103	93	106	98	93	94	108	89	94
1100	139	117	110	114	105	95	102	120	95	103
1200	147	133	119	123	111	102	110	139	106	110
1300	157	143	132	131	117	114	121	148	116	121
1400	169	150	144	142	128	126	128	157	123	133
1500	184	161	152	149	132	136	141	170	131	144

Table 70

# of games	Total Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.24331	0.35598	0.3896	0.68573	0.51442	0.28283	0.61725	0.34869	0.36623	0.32979
200	0.4856	0.54767	0.88579	1.02517	1.15136	0.92353	0.84526	1.05989	0.66284	0.87587
300	0.74908	0.72159	1.37281	1.74907	1.66318	1.34871	1.47631	1.33042	0.97153	1.33509
400	0.96776	0.89614	1.70571	2.20533	2.19088	1.91717	2.06425	1.87429	1.63549	1.63505
500	1.25502	1.12266	2.29643	2.70325	2.43527	2.304	2.44196	2.20628	2.25522	2.30336
600	1.59923	1.28921	2.81496	3.32723	2.91996	2.65465	2.82752	2.97594	2.63821	2.53521
700	1.89195	1.46243	3.25206	3.91403	3.36545	3.17633	3.21252	3.57201	2.87118	3.14025
800	2.21981	1.64926	3.53285	4.52538	3.45279	3.73164	3.44953	4.07183	3.51474	3.76287
900	2.50595	2.10389	4.00517	4.82356	4.07686	4.1649	3.74683	4.54129	3.80147	4.18985
1000	2.75127	2.33287	4.49838	5.167	4.62512	4.44522	4.51633	5.17454	4.24391	4.48682
1100	3.11969	2.65933	5.28077	5.56285	4.96761	4.53987	4.90521	5.75562	4.53427	4.91599
1200	3.28444	3.05956	5.73341	5.97865	5.26508	4.89431	5.29241	6.68969	5.07568	5.27066
1300	3.5257	3.29036	6.37126	6.37396	5.52846	5.4737	5.81544	7.08965	5.51965	5.79725
1400	3.79716	3.44557	6.94326	6.92853	6.06497	6.04738	6.14142	7.5158	5.85118	6.34563
1500	4.1538	3.71366	7.31958	7.28469	6.24978	6.52806	6.77046	8.16089	6.234	6.88334

Table 71

# of games	Average Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.00243	0.00356	0.0039	0.00686	0.00514	0.00283	0.00617	0.00349	0.00366	0.0033
200	0.00243	0.00274	0.00443	0.00513	0.00576	0.00462	0.00423	0.0053	0.00331	0.00438
300	0.0025	0.00241	0.00458	0.00583	0.00554	0.0045	0.00492	0.00443	0.00324	0.00445
400	0.00242	0.00224	0.00426	0.00551	0.00548	0.00479	0.00516	0.00469	0.00409	0.00409
500	0.00251	0.00225	0.00459	0.00541	0.00487	0.00461	0.00488	0.00441	0.00451	0.00461
600	0.00267	0.00215	0.00469	0.00555	0.00487	0.00442	0.00471	0.00496	0.0044	0.00423
700	0.0027	0.00209	0.00465	0.00559	0.00481	0.00454	0.00459	0.0051	0.0041	0.00449
800	0.00277	0.00206	0.00442	0.00566	0.00432	0.00466	0.00431	0.00509	0.00439	0.0047
900	0.00278	0.00234	0.00445	0.00536	0.00453	0.00463	0.00416	0.00505	0.00422	0.00466
1000	0.00275	0.00233	0.0045	0.00517	0.00463	0.00445	0.00452	0.00517	0.00424	0.00449
1100	0.00284	0.00242	0.0048	0.00506	0.00452	0.00413	0.00446	0.00523	0.00412	0.00447
1200	0.00274	0.00255	0.00478	0.00498	0.00439	0.00408	0.00441	0.00557	0.00423	0.00439
1300	0.00271	0.00253	0.0049	0.0049	0.00425	0.00421	0.00447	0.00545	0.00425	0.00446
1400	0.00271	0.00246	0.00496	0.00495	0.00433	0.00432	0.00439	0.00537	0.00418	0.00453
1500	0.00277	0.00248	0.00488	0.00486	0.00417	0.00435	0.00451	0.00544	0.00416	0.00459

Table 72

Scenario 4. Bidders “1”, “2”, “3” and “4” underbid by 0.03, 0.025, 0.02 and 0.015 respectively while others used equilibrium strategy.

# of games	Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.0082	0.0149	0.0152	0.022	0.0484	0.0499	0.0496	0.0493	0.0475	0.0477
200	0.0082	0.0136	0.016	0.0228	0.0478	0.0472	0.0496	0.0492	0.0481	0.0456
300	0.0076	0.0133	0.0173	0.0231	0.0483	0.047	0.0472	0.049	0.048	0.0466
400	0.0079	0.0128	0.0172	0.023	0.0484	0.047	0.0481	0.0487	0.0484	0.0471
500	0.0079	0.0128	0.0169	0.0231	0.0484	0.047	0.0477	0.0486	0.0482	0.0476
600	0.0078	0.0131	0.0169	0.0227	0.0484	0.0472	0.048	0.0489	0.0482	0.047
700	0.0079	0.0129	0.0171	0.0226	0.0485	0.0477	0.0483	0.0488	0.0482	0.0473
800	0.0078	0.0129	0.017	0.0228	0.0487	0.0477	0.0483	0.0489	0.0484	0.0473
900	0.0077	0.013	0.017	0.0225	0.0484	0.0479	0.0485	0.0489	0.0483	0.0475
1000	0.0077	0.0128	0.0169	0.0226	0.048	0.0478	0.0483	0.0488	0.0482	0.0474
1100	0.0078	0.0129	0.017	0.0225	0.0479	0.0479	0.0484	0.0487	0.0482	0.0475
1200	0.0078	0.0128	0.0171	0.0225	0.0479	0.048	0.0483	0.0487	0.0482	0.0475
1300	0.0077	0.013	0.0172	0.0224	0.0479	0.0481	0.0482	0.0486	0.0482	0.0475
1400	0.0075	0.013	0.0173	0.0224	0.048	0.0481	0.0482	0.0486	0.0481	0.0474
1500	0.0074	0.013	0.0174	0.0224	0.0481	0.0482	0.0479	0.0487	0.0481	0.0474
1600	0.0073	0.0131	0.0174	0.0225	0.0481	0.0483	0.048	0.0486	0.0481	0.0473
1700	0.0074	0.013	0.0174	0.0226	0.0481	0.0482	0.0478	0.0487	0.0482	0.0473
1800	0.0074	0.0129	0.0174	0.0224	0.0479	0.048	0.0477	0.0485	0.0481	0.0471
1900	0.0074	0.013	0.0175	0.0225	0.0477	0.048	0.0478	0.0486	0.0482	0.0473
2000	0.0074	0.0129	0.0174	0.0225	0.0477	0.0481	0.0479	0.0484	0.0482	0.0473

Table 73

# of games	Number of Games Won									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	7	11	15	11	10	10	6	15	5	10
200	22	25	23	24	16	18	14	23	16	19
300	32	43	36	35	25	27	21	29	26	26
400	40	57	47	41	34	34	31	42	36	38
500	52	68	64	50	38	44	42	52	47	43
600	68	80	79	60	42	52	53	59	55	52
700	77	93	92	68	49	61	67	67	69	57
800	93	106	105	77	55	76	73	72	78	65
900	110	118	117	84	71	85	84	78	86	67
1000	125	137	129	95	83	92	88	82	93	76
1100	139	149	143	109	90	104	94	94	98	80
1200	151	161	155	126	95	109	102	104	109	88
1300	161	174	168	134	107	118	109	117	115	97
1400	181	181	179	144	121	128	116	121	126	103
1500	201	195	185	156	131	137	120	127	133	115
1600	215	210	202	169	135	144	124	135	142	124
1700	225	224	213	176	142	154	139	145	152	130
1800	239	235	232	185	145	165	147	154	160	138
1900	250	255	235	200	159	176	154	161	166	144
2000	265	273	244	206	166	181	165	168	180	152

Table 74

# of games	Total Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.0573	0.1639	0.2282	0.2416	0.4837	0.4988	0.2973	0.7399	0.2376	0.4774
200	0.1795	0.3398	0.3688	0.5468	0.7651	0.8502	0.6947	1.1314	0.7699	0.8655
300	0.2446	0.5729	0.6244	0.8086	1.208	1.2691	0.9914	1.4207	1.249	1.2108
400	0.3148	0.7275	0.8063	0.945	1.6456	1.5979	1.4897	2.0459	1.7411	1.7914
500	0.411	0.8681	1.0833	1.1554	1.8378	2.0672	2.0049	2.5295	2.2653	2.0449
600	0.5273	1.0449	1.3385	1.3649	2.0328	2.4532	2.5422	2.8828	2.6517	2.4442
700	0.606	1.1976	1.5771	1.536	2.3743	2.9091	3.2364	3.2723	3.3269	2.6967
800	0.7282	1.3654	1.7808	1.7523	2.6761	3.6278	3.5295	3.5211	3.7738	3.0774
900	0.8452	1.5333	1.9847	1.8923	3.4358	4.068	4.0741	3.8108	4.1546	3.1794
1000	0.9587	1.7545	2.1758	2.1503	3.9857	4.4017	4.2535	4.002	4.4862	3.6047
1100	1.0906	1.9154	2.436	2.457	4.3113	4.977	4.5508	4.5732	4.7201	3.7971
1200	1.1722	2.0667	2.6498	2.8404	4.5538	5.2305	4.9311	5.0645	5.2541	4.1835
1300	1.2348	2.2678	2.8897	2.9959	5.1273	5.6795	5.2534	5.6898	5.5403	4.6117
1400	1.3578	2.3604	3.0952	3.2268	5.8035	6.1622	5.5878	5.8856	6.0622	4.8814
1500	1.4857	2.5406	3.2198	3.4992	6.3051	6.6027	5.7517	6.18	6.3968	5.451
1600	1.5725	2.7434	3.5155	3.8005	6.4965	6.951	5.949	6.5631	6.8297	5.8683
1700	1.6575	2.9125	3.6983	3.9721	6.8242	7.4179	6.6387	7.062	7.3214	6.1503
1800	1.7591	3.0343	4.0452	4.1413	6.9431	7.9245	7.0067	7.4717	7.696	6.5066
1900	1.8505	3.3144	4.1048	4.5068	7.5811	8.4559	7.3549	7.8192	7.9965	6.8099
2000	1.9694	3.5223	4.2502	4.6414	7.9129	8.7005	7.8983	8.1379	8.682	7.1935

Table 75

# of games	Average Expected Profit									
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10
100	0.0006	0.0016	0.0023	0.0024	0.0048	0.005	0.003	0.0074	0.0024	0.0048
200	0.0009	0.0017	0.0018	0.0027	0.0038	0.0043	0.0035	0.0057	0.0038	0.0043
300	0.0008	0.0019	0.0021	0.0027	0.004	0.0042	0.0033	0.0047	0.0042	0.004
400	0.0008	0.0018	0.002	0.0024	0.0041	0.004	0.0037	0.0051	0.0044	0.0045
500	0.0008	0.0017	0.0022	0.0023	0.0037	0.0041	0.004	0.0051	0.0045	0.0041
600	0.0009	0.0017	0.0022	0.0023	0.0034	0.0041	0.0042	0.0048	0.0044	0.0041
700	0.0009	0.0017	0.0023	0.0022	0.0034	0.0042	0.0046	0.0047	0.0048	0.0039
800	0.0009	0.0017	0.0022	0.0022	0.0033	0.0045	0.0044	0.0044	0.0047	0.0038
900	0.0009	0.0017	0.0022	0.0021	0.0038	0.0045	0.0045	0.0042	0.0046	0.0035
1000	0.001	0.0018	0.0022	0.0022	0.004	0.0044	0.0043	0.004	0.0045	0.0036
1100	0.001	0.0017	0.0022	0.0022	0.0039	0.0045	0.0041	0.0042	0.0043	0.0035
1200	0.001	0.0017	0.0022	0.0024	0.0038	0.0044	0.0041	0.0042	0.0044	0.0035
1300	0.0009	0.0017	0.0022	0.0023	0.0039	0.0044	0.004	0.0044	0.0043	0.0035
1400	0.001	0.0017	0.0022	0.0023	0.0041	0.0044	0.004	0.0042	0.0043	0.0035
1500	0.001	0.0017	0.0021	0.0023	0.0042	0.0044	0.0038	0.0041	0.0043	0.0036
1600	0.001	0.0017	0.0022	0.0024	0.0041	0.0043	0.0037	0.0041	0.0043	0.0037
1700	0.001	0.0017	0.0022	0.0023	0.004	0.0044	0.0039	0.0042	0.0043	0.0036
1800	0.001	0.0017	0.0022	0.0023	0.0039	0.0044	0.0039	0.0042	0.0043	0.0036
1900	0.001	0.0017	0.0022	0.0024	0.004	0.0045	0.0039	0.0041	0.0042	0.0036
2000	0.001	0.0018	0.0021	0.0023	0.004	0.0044	0.0039	0.0041	0.0043	0.0036

Table 76

5. Fifteen-Bidder Bidding Game

Scenario 1. All bidders used equilibrium strategy.

# of games	Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.0335	0.0318	0.0321	0.0328	0.0326	0.0327	0.0329	0.0331	0.0304	0.0326	0.0329	0.0338	0.0323	0.0319	0.033
200	0.0326	0.0323	0.0321	0.0322	0.0323	0.0329	0.0327	0.0303	0.0299	0.0323	0.0333	0.033	0.0326	0.0319	0.0322
300	0.0327	0.0321	0.0325	0.0322	0.0318	0.0328	0.0323	0.0316	0.0304	0.0326	0.0331	0.0329	0.0322	0.032	0.0323
400	0.0328	0.0319	0.0327	0.0321	0.0319	0.0328	0.0324	0.032	0.0306	0.0328	0.0328	0.0327	0.032	0.032	0.0325
500	0.0327	0.0319	0.0324	0.0321	0.0318	0.0325	0.0322	0.0321	0.0309	0.0328	0.0327	0.0325	0.0325	0.0321	0.0323
600	0.0325	0.0319	0.0324	0.0323	0.0319	0.0326	0.0323	0.0322	0.0313	0.0328	0.0326	0.0326	0.0326	0.0318	0.0323
700	0.0326	0.0318	0.0321	0.0323	0.0321	0.0325	0.0323	0.0322	0.0313	0.0325	0.0326	0.0325	0.0325	0.0319	0.0322
800	0.0326	0.0318	0.0322	0.0324	0.0322	0.0325	0.0323	0.0321	0.0316	0.0325	0.0326	0.0326	0.0326	0.0319	0.0322
900	0.0327	0.0319	0.0322	0.0325	0.0323	0.0324	0.0325	0.0321	0.0318	0.0326	0.0326	0.0325	0.0325	0.032	0.0322
1000	0.0325	0.0319	0.0321	0.0325	0.0323	0.0324	0.0326	0.0321	0.0319	0.0327	0.0326	0.0325	0.0324	0.032	0.0323
1100	0.0325	0.0319	0.0322	0.0326	0.0323	0.0324	0.0325	0.0323	0.0319	0.0327	0.0325	0.0325	0.0322	0.032	0.0324
1200	0.0325	0.0319	0.0322	0.0325	0.0324	0.0322	0.0325	0.0323	0.032	0.0327	0.0324	0.0323	0.0323	0.0321	0.0324
1300	0.0325	0.0319	0.0323	0.0326	0.0323	0.0323	0.0326	0.0323	0.0321	0.0328	0.0324	0.0324	0.0324	0.0321	0.0324
1400	0.0326	0.0319	0.0323	0.0325	0.0323	0.0322	0.0326	0.0322	0.0321	0.0325	0.0323	0.0323	0.0324	0.032	0.0324
1500	0.0326	0.0319	0.0323	0.0324	0.0324	0.0323	0.0326	0.0322	0.0321	0.0325	0.0322	0.0323	0.0324	0.032	0.0324

Table 77

# of games	Number of Games Won														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	6	15	5	7	5	7	6	3	9	6	5	4	6	7	9
200	11	22	13	16	9	9	12	6	15	13	12	13	13	15	21
300	15	38	19	23	15	20	18	14	20	19	20	18	17	20	24
400	21	47	28	27	22	23	25	22	22	24	29	26	23	27	34
500	25	56	32	34	34	31	28	26	29	32	36	32	30	32	43
600	30	59	44	42	40	37	37	37	37	38	41	36	33	40	49
700	37	66	61	46	48	41	41	44	42	47	46	41	37	47	56
800	40	75	66	51	56	47	47	49	51	54	51	49	46	53	65
900	44	79	72	57	60	55	54	59	56	58	60	55	54	62	75
1000	52	88	82	64	64	63	60	67	63	70	66	57	59	67	78
1100	55	97	86	73	73	69	67	75	64	80	74	65	64	72	86
1200	63	103	91	83	76	74	73	81	73	89	82	72	70	76	94
1300	67	114	98	90	87	84	78	89	81	92	88	76	78	80	98
1400	77	122	103	93	88	92	84	93	92	103	94	80	83	86	110
1500	82	130	108	102	96	101	90	99	97	111	100	82	90	89	123

Table 78

# of games	Total Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.2009	0.4774	0.1605	0.2297	0.163	0.2291	0.1974	0.0992	0.2734	0.1956	0.1646	0.1352	0.1935	0.2233	0.2972
200	0.3587	0.7114	0.4167	0.5144	0.2907	0.2961	0.392	0.1819	0.4481	0.4204	0.4001	0.4293	0.4239	0.479	0.6764
300	0.4901	1.2192	0.6174	0.7398	0.4772	0.6554	0.582	0.4419	0.6076	0.6197	0.6626	0.5921	0.5467	0.6409	0.7742
400	0.6896	1.497	0.9142	0.8655	0.701	0.7536	0.8088	0.7033	0.6734	0.7873	0.9514	0.8494	0.7371	0.8645	1.1067
500	0.817	1.7859	1.0376	1.0929	1.0812	1.0071	0.901	0.834	0.8975	1.0487	1.1781	1.0412	0.9735	1.0257	1.3887
600	0.9761	1.8794	1.4236	1.3562	1.276	1.2049	1.1943	1.1927	1.1569	1.246	1.3368	1.1721	1.0744	1.2716	1.5846
700	1.206	2.1009	1.9589	1.4877	1.5403	1.3331	1.3255	1.4154	1.3164	1.5266	1.4979	1.3328	1.2023	1.5004	1.8042
800	1.3057	2.3847	2.1256	1.6518	1.8015	1.5264	1.5182	1.5726	1.6122	1.7561	1.6609	1.5959	1.5	1.6927	2.0933
900	1.4397	2.5171	2.3171	1.8525	1.9355	1.7809	1.7528	1.8931	1.7791	1.8916	1.956	1.7869	1.7557	1.9843	2.4174
1000	1.6893	2.8055	2.6331	2.0789	2.0674	2.0435	1.9543	2.1531	2.0071	2.2889	2.1502	1.8546	1.91	2.1448	2.5162
1100	1.7891	3.0973	2.7661	2.3762	2.3607	2.2354	2.1797	2.4189	2.0415	2.6138	2.4018	2.1119	2.0633	2.307	2.7825
1200	2.0491	3.2853	2.9293	2.7009	2.4595	2.3852	2.3757	2.6177	2.3359	2.913	2.653	2.3272	2.261	2.4363	3.0416
1300	2.1772	3.6393	3.1608	2.9304	2.8063	2.7139	2.541	2.8725	2.599	3.0132	2.8498	2.4617	2.5303	2.5649	3.1719
1400	2.5075	3.888	3.3238	3.018	2.8406	2.966	2.738	2.9961	2.9554	3.3499	3.0349	2.5855	2.6892	2.754	3.5652
1500	2.6708	4.1422	3.4835	3.3099	3.1072	3.2633	2.9335	3.1886	3.1143	3.6105	3.2205	2.6471	2.9164	2.8502	3.9867

Table 79

# of games	Average Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.002	0.0048	0.0016	0.0023	0.0016	0.0023	0.002	0.001	0.0027	0.002	0.0016	0.0014	0.0019	0.0022	0.003
200	0.0018	0.0036	0.0021	0.0026	0.0015	0.0015	0.002	0.0009	0.0022	0.0021	0.002	0.0021	0.0021	0.0024	0.0034
300	0.0016	0.0041	0.0021	0.0025	0.0016	0.0022	0.0019	0.0015	0.002	0.0021	0.0022	0.002	0.0018	0.0021	0.0026
400	0.0017	0.0037	0.0023	0.0022	0.0018	0.0019	0.002	0.0018	0.0017	0.002	0.0024	0.0021	0.0018	0.0022	0.0028
500	0.0016	0.0036	0.0021	0.0022	0.0022	0.002	0.0018	0.0017	0.0018	0.0021	0.0024	0.0021	0.0019	0.0021	0.0028
600	0.0016	0.0031	0.0024	0.0023	0.0021	0.002	0.002	0.002	0.0019	0.0021	0.0022	0.002	0.0018	0.0021	0.0026
700	0.0017	0.003	0.0028	0.0021	0.0022	0.0019	0.0019	0.002	0.0019	0.0022	0.0021	0.0019	0.0017	0.0021	0.0026
800	0.0016	0.003	0.0027	0.0021	0.0023	0.0019	0.0019	0.002	0.002	0.0022	0.0021	0.002	0.0019	0.0021	0.0026
900	0.0016	0.0028	0.0026	0.0021	0.0022	0.002	0.0019	0.0021	0.002	0.0021	0.0022	0.002	0.002	0.0022	0.0027
1000	0.0017	0.0028	0.0026	0.0021	0.0021	0.002	0.002	0.0022	0.002	0.0023	0.0022	0.0019	0.0019	0.0021	0.0025
1100	0.0016	0.0028	0.0025	0.0022	0.0021	0.002	0.002	0.0022	0.0019	0.0024	0.0022	0.0019	0.0019	0.0021	0.0025
1200	0.0017	0.0027	0.0024	0.0023	0.002	0.002	0.002	0.0022	0.0019	0.0024	0.0022	0.0019	0.0019	0.002	0.0025
1300	0.0017	0.0028	0.0024	0.0023	0.0022	0.0021	0.002	0.0022	0.002	0.0023	0.0022	0.0019	0.0019	0.002	0.0024
1400	0.0018	0.0028	0.0024	0.0022	0.002	0.0021	0.002	0.0021	0.0021	0.0024	0.0022	0.0018	0.0019	0.002	0.0025
1500	0.0018	0.0028	0.0023	0.0022	0.0021	0.0022	0.002	0.0021	0.0021	0.0024	0.0021	0.0018	0.0019	0.0019	0.0027

Table 80

Scenario 2. Bidder1 underbid by 0.015 while others used equilibrium strategy.

# of games	Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.01851	0.03183	0.03209	0.03269	0.03259	0.03273	0.03291	0.03307	0.03037	0.03261	0.03292	0.03381	0.03189	0.0319	0.03293
200	0.01774	0.03234	0.03206	0.03206	0.0323	0.0329	0.03267	0.03032	0.02987	0.03234	0.03334	0.03302	0.03248	0.03193	0.03213
300	0.01777	0.03208	0.03252	0.0321	0.03182	0.03277	0.03233	0.03157	0.03038	0.03262	0.03313	0.0329	0.03204	0.03205	0.03219
400	0.01788	0.03185	0.03268	0.032	0.03186	0.03276	0.03235	0.03197	0.03061	0.0328	0.03281	0.03267	0.03196	0.03202	0.03251
500	0.01773	0.03189	0.03244	0.0321	0.0318	0.03249	0.03218	0.03208	0.03095	0.03277	0.03272	0.03254	0.03239	0.03203	0.03226
600	0.01745	0.03185	0.03242	0.0323	0.0319	0.03256	0.03228	0.03224	0.03127	0.03279	0.03261	0.03256	0.03251	0.03176	0.03231
700	0.01754	0.03183	0.03215	0.03236	0.03205	0.03249	0.03233	0.03217	0.03134	0.03248	0.03256	0.03251	0.03245	0.0319	0.03219
800	0.01749	0.03184	0.03224	0.0324	0.03214	0.03246	0.0323	0.03209	0.03161	0.03252	0.03257	0.03257	0.03258	0.03192	0.03218
900	0.01757	0.03191	0.03221	0.03252	0.03223	0.03236	0.03246	0.03209	0.03177	0.03261	0.0326	0.03249	0.03248	0.03199	0.03221
1000	0.01739	0.03192	0.03213	0.0325	0.03228	0.03242	0.03257	0.03214	0.03186	0.0327	0.03258	0.03254	0.03234	0.032	0.03224
1100	0.01742	0.03197	0.03219	0.03257	0.03232	0.03228	0.03253	0.03225	0.0319	0.03268	0.03246	0.03249	0.03221	0.03203	0.03234
1200	0.01737	0.03193	0.03221	0.03255	0.03234	0.03222	0.03254	0.03232	0.03205	0.03274	0.03235	0.03232	0.03227	0.03205	0.03234
1300	0.01737	0.03195	0.03228	0.03257	0.03224	0.03229	0.03258	0.03228	0.03212	0.03276	0.03238	0.03238	0.03242	0.03205	0.03235
1400	0.01733	0.03193	0.0323	0.03246	0.03226	0.03223	0.03259	0.03228	0.03215	0.03253	0.03229	0.03231	0.03238	0.03201	0.0324
1500	0.01735	0.03192	0.03228	0.03246	0.03235	0.0323	0.03259	0.03226	0.03213	0.03253	0.0322	0.03227	0.03239	0.03201	0.0324

Table 81

# of games	Number of Games Won														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	9	15	5	6	5	7	6	3	9	6	5	4	5	7	8
200	15	22	12	15	9	9	12	6	15	13	12	13	12	15	20
300	19	38	18	22	15	20	18	14	20	19	20	18	16	20	23
400	25	47	27	26	22	23	25	22	22	24	29	26	22	27	33
500	30	56	31	33	34	31	28	26	29	32	36	32	29	31	42
600	38	59	41	40	40	37	37	37	37	38	41	36	32	39	48
700	47	66	58	44	47	40	41	44	42	47	46	41	36	46	55
800	51	74	63	49	55	46	47	49	51	54	51	49	45	52	64
900	55	78	69	55	59	54	54	59	56	58	60	55	53	61	74
1000	64	87	78	62	63	62	60	67	63	70	66	57	58	66	77
1100	68	96	82	71	72	68	67	75	64	79	74	65	63	71	85
1200	77	102	87	81	75	73	73	81	72	88	82	72	69	75	93
1300	83	113	94	88	86	83	78	89	79	91	88	75	77	79	97
1400	95	120	99	91	87	91	84	92	90	102	94	79	82	85	109
1500	100	128	104	100	95	100	90	98	95	110	100	81	89	88	122

Table 82

# of games	Total Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.16655	0.47743	0.16047	0.19616	0.16295	0.2291	0.19743	0.09921	0.27336	0.19563	0.16461	0.13524	0.15946	0.22328	0.26348
200	0.2661	0.71141	0.38472	0.48086	0.29069	0.29611	0.39202	0.18194	0.44807	0.4204	0.40013	0.42927	0.38978	0.479	0.64267
300	0.33755	1.21919	0.58539	0.70623	0.47723	0.65539	0.58197	0.44192	0.60759	0.61973	0.66256	0.59213	0.51264	0.64094	0.7404
400	0.44702	1.497	0.88226	0.83193	0.70101	0.75357	0.80876	0.70332	0.67342	0.78726	0.95139	0.84943	0.70301	0.86447	1.07293
500	0.53181	1.78589	1.00566	1.05928	1.0812	1.00713	0.90102	0.83398	0.89754	1.04872	1.17808	1.04117	0.93943	0.99278	1.35492
600	0.66299	1.87942	1.32928	1.29212	1.27601	1.20488	1.19427	1.19271	1.15692	1.24603	1.33685	1.17213	1.0403	1.23868	1.55082
700	0.82456	2.10087	1.86454	1.4237	1.50639	1.29965	1.32555	1.41539	1.31645	1.52659	1.49786	1.33282	1.16823	1.46746	1.77049
800	0.89222	2.35633	2.03131	1.58776	1.76756	1.49293	1.51816	1.57261	1.61225	1.7561	1.66091	1.59589	1.46588	1.65979	2.05957
900	0.96617	2.48877	2.22277	1.78846	1.9016	1.74741	1.7528	1.89305	1.77914	1.89164	1.956	1.78691	1.72164	1.95138	2.3836
1000	1.11269	2.77711	2.50646	2.01491	2.03342	2.01003	1.95434	2.15305	2.00714	2.28888	2.15018	1.8546	1.87587	2.11191	2.48245
1100	1.18424	3.0689	2.6395	2.31218	2.32674	2.20192	2.17965	2.41893	2.04146	2.58179	2.40183	2.11194	2.0292	2.27409	2.7487
1200	1.33749	3.2569	2.80265	2.63688	2.42561	2.35175	2.37571	2.6177	2.30724	2.88102	2.65296	2.32718	2.22692	2.40339	3.00787
1300	1.44182	3.6109	3.03414	2.86639	2.77236	2.68043	2.54102	2.87253	2.53709	2.98123	2.84979	2.42835	2.49622	2.53201	3.13812
1400	1.64671	3.83127	3.19721	2.95397	2.80667	2.93251	2.73797	2.9695	2.89348	3.31794	3.03492	2.55216	2.65508	2.72105	3.5314
1500	1.735	4.08555	3.35686	3.24584	3.07331	3.22983	2.93347	3.16197	3.05242	3.57853	3.22046	2.61378	2.88232	2.81727	3.95297

Table 83

# of games	Average Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.00167	0.00477	0.0016	0.00196	0.00163	0.00229	0.00197	0.00099	0.00273	0.00196	0.00165	0.00135	0.00159	0.00223	0.00263
200	0.00133	0.00356	0.00192	0.0024	0.00145	0.00148	0.00196	0.00091	0.00224	0.0021	0.002	0.00215	0.00195	0.0024	0.00321
300	0.00113	0.00406	0.00195	0.00235	0.00159	0.00218	0.00194	0.00147	0.00203	0.00207	0.00221	0.00197	0.00171	0.00214	0.00247
400	0.00112	0.00374	0.00221	0.00208	0.00175	0.00188	0.00202	0.00176	0.00168	0.00197	0.00238	0.00212	0.00176	0.00216	0.00268
500	0.00106	0.00357	0.00201	0.00212	0.00216	0.00201	0.0018	0.00167	0.0018	0.0021	0.00236	0.00208	0.00188	0.00199	0.00271
600	0.0011	0.00313	0.00222	0.00215	0.00213	0.00201	0.00199	0.00199	0.00193	0.00208	0.00223	0.00195	0.00173	0.00206	0.00258
700	0.00118	0.003	0.00266	0.00203	0.00215	0.00186	0.00189	0.00202	0.00188	0.00218	0.00214	0.0019	0.00167	0.0021	0.00253
800	0.00112	0.00295	0.00254	0.00198	0.00221	0.00187	0.0019	0.00197	0.00202	0.0022	0.00208	0.00199	0.00183	0.00207	0.00257
900	0.00107	0.00277	0.00247	0.00199	0.00211	0.00194	0.00195	0.0021	0.00198	0.0021	0.00217	0.00199	0.00191	0.00217	0.00265
1000	0.00111	0.00278	0.00251	0.00201	0.00203	0.00201	0.00195	0.00215	0.00201	0.00229	0.00215	0.00185	0.00188	0.00211	0.00248
1100	0.00108	0.00279	0.0024	0.0021	0.00212	0.002	0.00198	0.0022	0.00186	0.00235	0.00218	0.00192	0.00184	0.00207	0.0025
1200	0.00111	0.00271	0.00234	0.0022	0.00202	0.00196	0.00198	0.00218	0.00192	0.0024	0.00221	0.00194	0.00186	0.002	0.00251
1300	0.00111	0.00278	0.00233	0.0022	0.00213	0.00206	0.00195	0.00221	0.00195	0.00229	0.00219	0.00187	0.00192	0.00195	0.00241
1400	0.00118	0.00274	0.00228	0.00211	0.002	0.00209	0.00196	0.00212	0.00207	0.00237	0.00217	0.00182	0.0019	0.00194	0.00252
1500	0.00116	0.00272	0.00224	0.00216	0.00205	0.00215	0.00196	0.00211	0.00203	0.00239	0.00215	0.00174	0.00192	0.00188	0.00264

Table 84

Scenario 3. Bidders “1” “2” and “3” underbid by 0.02 while others used equilibrium strategy.

# of games	Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.0124	0.0125	0.0106	0.0317	0.0318	0.0331	0.0333	0.0324	0.0319	0.0323	0.0322	0.033	0.0307	0.0329	0.0317
200	0.0124	0.0125	0.0118	0.0329	0.0322	0.0319	0.0334	0.032	0.0324	0.0332	0.0322	0.0331	0.0323	0.0331	0.032
300	0.0125	0.0125	0.0117	0.0329	0.0319	0.0317	0.0328	0.0324	0.0327	0.0327	0.0321	0.0329	0.0324	0.033	0.0322
400	0.0124	0.0124	0.0118	0.0329	0.032	0.0319	0.0328	0.0325	0.0328	0.0329	0.0324	0.0328	0.0326	0.0328	0.0321
500	0.0123	0.0123	0.012	0.033	0.032	0.0316	0.0324	0.0326	0.0329	0.033	0.0322	0.0325	0.0324	0.0326	0.0322
600	0.0123	0.0123	0.0119	0.0329	0.032	0.0318	0.0324	0.0325	0.0328	0.033	0.0321	0.0326	0.0324	0.0327	0.0323
700	0.0123	0.0122	0.0119	0.0327	0.0321	0.032	0.0325	0.0327	0.0327	0.033	0.032	0.0326	0.0324	0.0327	0.0322
800	0.0123	0.0121	0.012	0.0328	0.0321	0.032	0.0324	0.0327	0.0324	0.0329	0.0319	0.0323	0.0324	0.0325	0.0322
900	0.0123	0.0121	0.0118	0.0327	0.0321	0.032	0.0324	0.0328	0.0326	0.0329	0.0321	0.0325	0.0323	0.0325	0.0323
1000	0.0123	0.0121	0.0119	0.0327	0.0322	0.0321	0.0325	0.0327	0.0325	0.0328	0.0321	0.0325	0.0321	0.0325	0.0322
1100	0.0123	0.0121	0.012	0.0328	0.0322	0.0322	0.0325	0.0326	0.0324	0.0329	0.032	0.0325	0.0322	0.0325	0.0322
1200	0.0124	0.0121	0.0119	0.0328	0.0322	0.0321	0.0325	0.0325	0.0325	0.0328	0.032	0.0325	0.0321	0.0324	0.0322
1300	0.0124	0.0122	0.012	0.0327	0.0323	0.0322	0.0324	0.0326	0.0324	0.0327	0.032	0.0326	0.0322	0.0324	0.0322
1400	0.0124	0.0123	0.012	0.0326	0.0323	0.0323	0.0325	0.0326	0.0324	0.0326	0.0321	0.0325	0.0322	0.0324	0.0322
1500	0.0125	0.0122	0.012	0.0326	0.0323	0.0322	0.0325	0.0326	0.0324	0.0326	0.032	0.0322	0.0322	0.0325	0.0322
1600	0.0125	0.0123	0.0121	0.0325	0.0324	0.0322	0.0325	0.0325	0.0324	0.0327	0.032	0.0322	0.0322	0.0325	0.0323
1700	0.0125	0.0122	0.012	0.0325	0.0324	0.0323	0.0326	0.0325	0.0325	0.0326	0.0321	0.0322	0.0322	0.0324	0.0323
1800	0.0125	0.0122	0.0121	0.0325	0.0324	0.0324	0.0326	0.0324	0.0325	0.0327	0.0321	0.0322	0.0322	0.0324	0.0324
1900	0.0125	0.0122	0.0121	0.0325	0.0324	0.0324	0.0325	0.0324	0.0326	0.0327	0.0321	0.0323	0.0322	0.0323	0.0324
2000	0.0125	0.0122	0.0121	0.0325	0.0325	0.0323	0.0325	0.0324	0.0325	0.0326	0.0321	0.0323	0.0323	0.0323	0.0324

Table 85

# of games	Number of Games Won														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	8	11	6	5	7	7	7	6	4	2	5	5	4	8	15
200	17	18	12	15	11	17	13	14	11	7	9	10	9	17	20
300	23	25	22	24	17	19	20	20	19	12	17	16	18	25	23
400	31	35	34	26	23	23	27	25	22	18	23	22	23	34	34
500	42	42	44	31	30	29	35	29	27	23	29	28	29	38	44
600	54	48	55	39	33	34	38	35	37	30	35	36	38	41	47
700	62	55	60	47	41	41	44	42	47	36	41	42	45	44	53
800	70	63	71	53	41	50	48	49	56	41	42	47	52	53	64
900	78	72	79	57	43	54	55	57	62	45	51	54	60	63	70
1000	87	79	87	59	51	63	63	62	65	46	61	63	64	70	80
1100	95	86	98	65	55	70	72	71	70	50	68	65	70	77	88
1200	100	96	107	72	61	74	77	80	77	60	76	69	76	83	92
1300	112	107	117	78	66	81	83	85	84	73	77	73	79	89	96
1400	125	120	124	82	70	86	92	89	91	78	81	82	86	94	100
1500	133	133	130	86	76	88	96	97	96	85	88	91	94	102	105
1600	143	137	141	89	84	96	99	108	106	92	91	99	96	107	112
1700	152	141	148	97	91	106	106	116	113	97	95	105	100	114	119
1800	165	147	160	104	95	113	110	123	120	106	101	109	109	118	120
1900	175	157	164	110	100	119	116	127	128	111	107	113	119	125	129
2000	182	165	173	119	108	121	123	132	135	119	111	125	121	133	133

Table 86

# of games	Total Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.0995	0.1379	0.0638	0.1586	0.2226	0.2314	0.2332	0.1946	0.1276	0.0645	0.1608	0.1652	0.1228	0.2635	0.4757
200	0.2116	0.2243	0.1418	0.4928	0.3547	0.5416	0.4341	0.4478	0.3561	0.2326	0.2895	0.331	0.2909	0.5629	0.6397
300	0.288	0.3126	0.2583	0.7899	0.5427	0.6028	0.657	0.6476	0.6216	0.3923	0.5461	0.526	0.5839	0.8261	0.7397
400	0.3848	0.433	0.4024	0.8566	0.737	0.7331	0.8861	0.8113	0.7214	0.5922	0.7446	0.721	0.7499	1.1142	1.0915
500	0.5157	0.5146	0.5262	1.0228	0.9596	0.9163	1.1354	0.9452	0.8891	0.7585	0.9326	0.9098	0.9387	1.2401	1.4163
600	0.6644	0.5884	0.6562	1.2835	1.0563	1.0805	1.2301	1.137	1.2141	0.9891	1.1242	1.1737	1.2317	1.3407	1.5165
700	0.7612	0.6689	0.7137	1.5391	1.3163	1.3101	1.4301	1.3714	1.5386	1.1874	1.3118	1.3692	1.4586	1.4396	1.7064
800	0.858	0.7642	0.8532	1.7374	1.3163	1.5981	1.5558	1.6031	1.8161	1.3493	1.3404	1.5179	1.6833	1.7246	2.0622
900	0.9562	0.8684	0.9349	1.8636	1.3795	1.7289	1.7842	1.867	2.0185	1.4798	1.6375	1.7531	1.9385	2.0496	2.263
1000	1.0716	0.9547	1.039	1.9315	1.641	2.0215	2.0445	2.0252	2.1101	1.5095	1.9596	2.0499	2.0558	2.2737	2.5747
1100	1.1732	1.0429	1.1733	2.1347	1.7734	2.2532	2.3419	2.3116	2.2701	1.6431	2.1774	2.1143	2.2518	2.5019	2.8332
1200	1.2359	1.1663	1.2774	2.3631	1.9646	2.3736	2.5005	2.5984	2.4993	1.9706	2.4342	2.2455	2.4422	2.6873	2.9621
1300	1.3844	1.3072	1.4051	2.5493	2.1295	2.6071	2.6913	2.7687	2.7212	2.39	2.4657	2.3783	2.5417	2.8821	3.0941
1400	1.5541	1.4705	1.4833	2.6757	2.2602	2.7756	2.9875	2.9004	2.9462	2.5452	2.5965	2.6677	2.7679	3.0472	3.2244
1500	1.6599	1.6251	1.5607	2.802	2.4533	2.8371	3.1195	3.1577	3.1081	2.7749	2.8185	2.9326	3.0234	3.3114	3.3842
1600	1.7859	1.6803	1.6995	2.894	2.7201	3.091	3.2205	3.5066	3.436	3.0055	2.9165	3.1854	3.087	3.4745	3.6159
1700	1.9007	1.7191	1.7796	3.149	2.9488	3.4222	3.4516	3.7681	3.6724	3.1654	3.0482	3.3832	3.2172	3.6978	3.8478
1800	2.0572	1.7907	1.938	3.3794	3.0811	3.6574	3.5815	3.9904	3.9049	3.4664	3.243	3.5148	3.5138	3.8193	3.8823
1900	2.1861	1.9137	1.9906	3.5745	3.2425	3.8506	3.7722	4.1118	4.1681	3.6247	3.4362	3.6501	3.8373	4.0333	4.1809
2000	2.277	2.0129	2.0946	3.8675	3.5074	3.9058	4.0014	4.2785	4.3863	3.8763	3.5663	4.0318	3.9047	4.2906	4.3072

Table 87

# of games	Average Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.001	0.0014	0.0006	0.0016	0.0022	0.0023	0.0023	0.0019	0.0013	0.0006	0.0016	0.0017	0.0012	0.0026	0.0048
200	0.0011	0.0011	0.0007	0.0025	0.0018	0.0027	0.0022	0.0022	0.0018	0.0012	0.0014	0.0017	0.0015	0.0028	0.0032
300	0.001	0.001	0.0009	0.0026	0.0018	0.002	0.0022	0.0022	0.0021	0.0013	0.0018	0.0018	0.0019	0.0028	0.0025
400	0.001	0.0011	0.001	0.0021	0.0018	0.0018	0.0022	0.002	0.0018	0.0015	0.0019	0.0018	0.0019	0.0028	0.0027
500	0.001	0.001	0.0011	0.002	0.0019	0.0018	0.0023	0.0019	0.0018	0.0015	0.0019	0.0018	0.0019	0.0025	0.0028
600	0.0011	0.001	0.0011	0.0021	0.0018	0.0018	0.0021	0.0019	0.002	0.0016	0.0019	0.002	0.0021	0.0022	0.0025
700	0.0011	0.001	0.001	0.0022	0.0019	0.0019	0.002	0.002	0.0022	0.0017	0.0019	0.002	0.0021	0.0021	0.0024
800	0.0011	0.001	0.0011	0.0022	0.0016	0.002	0.0019	0.002	0.0023	0.0017	0.0017	0.0019	0.0021	0.0022	0.0026
900	0.0011	0.001	0.001	0.0021	0.0015	0.0019	0.002	0.0021	0.0022	0.0016	0.0018	0.0019	0.0022	0.0023	0.0025
1000	0.0011	0.001	0.001	0.0019	0.0016	0.002	0.002	0.002	0.0021	0.0015	0.002	0.002	0.0021	0.0023	0.0026
1100	0.0011	0.0009	0.0011	0.0019	0.0016	0.002	0.0021	0.0021	0.0021	0.0015	0.002	0.0019	0.002	0.0023	0.0026
1200	0.001	0.001	0.0011	0.002	0.0016	0.002	0.0021	0.0022	0.0021	0.0016	0.002	0.0019	0.002	0.0022	0.0025
1300	0.0011	0.001	0.0011	0.002	0.0016	0.002	0.0021	0.0021	0.0021	0.0018	0.0019	0.0018	0.002	0.0022	0.0024
1400	0.0011	0.0011	0.0011	0.0019	0.0016	0.002	0.0021	0.0021	0.0021	0.0018	0.0019	0.0019	0.002	0.0022	0.0023
1500	0.0011	0.0011	0.001	0.0019	0.0016	0.0019	0.0021	0.0021	0.0021	0.0018	0.0019	0.002	0.002	0.0022	0.0023
1600	0.0011	0.0011	0.0011	0.0018	0.0017	0.0019	0.002	0.0022	0.0021	0.0019	0.0018	0.002	0.0019	0.0022	0.0023
1700	0.0011	0.001	0.001	0.0019	0.0017	0.002	0.002	0.0022	0.0022	0.0019	0.0018	0.002	0.0019	0.0022	0.0023
1800	0.0011	0.001	0.0011	0.0019	0.0017	0.002	0.002	0.0022	0.0022	0.0019	0.0018	0.002	0.002	0.0021	0.0022
1900	0.0012	0.001	0.001	0.0019	0.0017	0.002	0.002	0.0022	0.0022	0.0019	0.0018	0.0019	0.002	0.0021	0.0022
2000	0.0011	0.001	0.001	0.0019	0.0018	0.002	0.002	0.0021	0.0022	0.0019	0.0018	0.002	0.002	0.0021	0.0022

Table 88

Scenario 4. Bidders “1”, “2”, “3”, “4” and “5” underbid by 0.025, 0.0225, 0.02, 0.0175 and 0.015 respectively while others used equilibrium strategy.

# of games	Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.0072	0.0101	0.0112	0.0145	0.0136	0.032	0.0334	0.0317	0.0313	0.0309	0.0326	0.0325	0.0336	0.0312	0.0332
200	0.0075	0.0097	0.012	0.0148	0.0166	0.0322	0.033	0.0318	0.0319	0.0315	0.0326	0.0327	0.0326	0.0324	0.0333
300	0.0077	0.0101	0.0118	0.0147	0.0169	0.0323	0.033	0.0321	0.0322	0.0314	0.0323	0.0325	0.033	0.0321	0.0331
400	0.0077	0.0098	0.0119	0.0145	0.0172	0.0324	0.0329	0.0321	0.0323	0.0318	0.0324	0.0326	0.0319	0.0322	0.0332
500	0.0077	0.0098	0.0119	0.0144	0.0173	0.0326	0.0327	0.0323	0.0323	0.0319	0.0325	0.0326	0.032	0.0323	0.0331
600	0.0075	0.0097	0.012	0.0146	0.0173	0.0327	0.0327	0.0323	0.0321	0.0321	0.0326	0.0326	0.0323	0.0325	0.0329
700	0.0076	0.0097	0.0119	0.0146	0.0173	0.0324	0.0326	0.0322	0.0319	0.0321	0.0326	0.0325	0.0323	0.0324	0.033
800	0.0075	0.0098	0.012	0.0145	0.0173	0.0325	0.0326	0.0322	0.0321	0.0321	0.0327	0.0325	0.0323	0.0324	0.0329
900	0.0074	0.0097	0.0118	0.0146	0.0175	0.0324	0.0326	0.0321	0.0321	0.0322	0.0327	0.0326	0.0322	0.0324	0.0327
1000	0.0074	0.0097	0.0117	0.0145	0.0175	0.0324	0.0327	0.0322	0.0322	0.0323	0.0325	0.0325	0.0323	0.0325	0.0325
1100	0.0075	0.0097	0.0117	0.0146	0.0175	0.0324	0.0327	0.0323	0.032	0.0323	0.0325	0.0326	0.0322	0.0325	0.0325
1200	0.0075	0.0098	0.0118	0.0145	0.0175	0.0324	0.0326	0.0321	0.0318	0.0323	0.0326	0.0326	0.0323	0.0324	0.0324
1300	0.0075	0.0096	0.0119	0.0145	0.0173	0.0325	0.0326	0.0321	0.0318	0.0323	0.0327	0.0326	0.0323	0.0324	0.0324
1400	0.0075	0.0095	0.012	0.0145	0.0172	0.0325	0.0325	0.0322	0.0318	0.0324	0.0326	0.0326	0.0322	0.0324	0.0325
1500	0.0075	0.0096	0.012	0.0145	0.0172	0.0324	0.0325	0.0322	0.0318	0.0324	0.0326	0.0325	0.0323	0.0324	0.0324
1600	0.0075	0.0095	0.012	0.0146	0.0172	0.0325	0.0325	0.0322	0.0319	0.0324	0.0326	0.0325	0.0324	0.0323	0.0324
1700	0.0074	0.0095	0.0119	0.0146	0.0173	0.0324	0.0325	0.0322	0.0319	0.0323	0.0326	0.0325	0.0324	0.0323	0.0323
1800	0.0074	0.0095	0.012	0.0146	0.0173	0.0324	0.0325	0.0322	0.032	0.0324	0.0326	0.0325	0.0324	0.0323	0.0323
1900	0.0075	0.0095	0.012	0.0147	0.0173	0.0324	0.0325	0.0322	0.032	0.0324	0.0326	0.0325	0.0324	0.0323	0.0323
2000	0.0074	0.0095	0.012	0.0147	0.0173	0.0324	0.0326	0.0322	0.0321	0.0324	0.0326	0.0325	0.0323	0.0323	0.0323

Table 89

# of games	Number of Games Won														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidder5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	9	9	3	7	2	4	4	9	4	7	13	7	5	6	11
200	17	16	12	14	8	8	13	16	9	16	15	17	10	13	16
300	28	24	14	18	17	10	18	25	14	20	21	25	15	23	28
400	36	39	20	26	21	12	26	30	22	25	26	37	22	26	32
500	47	47	26	37	23	17	32	34	29	31	29	45	32	34	37
600	54	52	32	43	28	25	38	40	37	37	37	51	43	39	44
700	62	62	38	47	34	33	45	46	44	47	42	57	47	47	49
800	70	68	47	51	41	40	54	51	50	54	49	61	56	54	54
900	78	78	56	55	48	42	59	57	60	58	55	70	62	59	63
1000	85	85	61	67	54	47	64	67	68	62	61	75	65	67	72
1100	95	100	68	74	59	54	69	70	77	68	66	79	69	73	79
1200	101	108	76	83	64	56	74	73	83	74	73	87	76	81	91
1300	105	126	82	94	74	67	77	79	87	80	79	91	79	86	94
1400	111	135	92	100	82	70	82	87	89	84	88	99	85	92	104
1500	116	146	96	109	94	77	85	91	96	92	94	104	92	95	113
1600	122	151	111	122	100	83	94	99	103	97	97	108	94	101	118
1700	131	159	128	126	108	86	98	106	107	106	102	113	98	108	124
1800	135	169	133	132	117	92	104	109	116	113	107	119	107	114	133
1900	140	174	141	138	122	99	111	119	120	117	115	125	114	123	142
2000	149	186	153	146	127	105	116	126	127	126	120	127	119	128	145

Table 90

# of games	Total Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidders5	Bidder6	Bidder7	Bidder8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.0652	0.0913	0.0337	0.1014	0.0272	0.128	0.1337	0.2857	0.125	0.2165	0.4233	0.2273	0.1682	0.1873	0.3657
200	0.1276	0.156	0.1441	0.2071	0.1331	0.2575	0.4287	0.5092	0.287	0.5045	0.4896	0.5555	0.3264	0.4206	0.532
300	0.2161	0.2417	0.1655	0.2653	0.2873	0.323	0.594	0.8033	0.4508	0.629	0.6783	0.8132	0.4943	0.7373	0.9262
400	0.2779	0.3818	0.238	0.3776	0.3609	0.389	0.8548	0.9642	0.7112	0.795	0.8413	1.2055	0.702	0.8371	1.0611
500	0.361	0.4585	0.3101	0.5311	0.3988	0.5547	1.0461	1.0982	0.9354	0.9898	0.9429	1.4687	1.0239	1.0991	1.2255
600	0.4047	0.5054	0.3827	0.6262	0.4845	0.8169	1.2412	1.293	1.1873	1.1889	1.2058	1.6614	1.3874	1.2669	1.4494
700	0.4714	0.5988	0.452	0.6865	0.5891	1.0708	1.4651	1.4789	1.4054	1.5095	1.3707	1.8543	1.516	1.5245	1.6148
800	0.5263	0.6633	0.5626	0.7409	0.7108	1.2984	1.758	1.6397	1.6027	1.7333	1.6033	1.9844	1.8115	1.7522	1.7741
900	0.5789	0.7564	0.6607	0.8032	0.8378	1.3609	1.9236	1.8274	1.9248	1.8661	1.7981	2.2801	1.9975	1.9114	2.0597
1000	0.6326	0.827	0.7123	0.9724	0.9425	1.5207	2.0902	2.1586	2.1873	2.0002	1.9798	2.4389	2.097	2.1803	2.3371
1100	0.7155	0.9669	0.7979	1.0796	1.0304	1.7498	2.2533	2.2578	2.4607	2.1958	2.1445	2.5742	2.2246	2.3718	2.5666
1200	0.7551	1.054	0.8954	1.2068	1.117	1.8168	2.4144	2.3405	2.6354	2.3866	2.38	2.834	2.4549	2.6275	2.9458
1300	0.7865	1.2112	0.976	1.3674	1.2773	2.1761	2.5086	2.5358	2.7666	2.5859	2.5817	2.9661	2.5484	2.7895	3.0436
1400	0.836	1.2865	1.1065	1.4485	1.4144	2.2743	2.6677	2.7972	2.8324	2.7208	2.8705	3.2234	2.7388	2.985	3.3761
1500	0.8708	1.3946	1.1499	1.5835	1.6146	2.4964	2.76	2.9278	3.0565	2.9823	3.0631	3.3815	2.9752	3.0794	3.6581
1600	0.9096	1.4396	1.3312	1.7784	1.7194	2.6942	3.0532	3.19	3.2843	3.1465	3.1596	3.5124	3.0431	3.2639	3.8201
1700	0.9754	1.5103	1.5265	1.84	1.8637	2.7889	3.1845	3.4127	3.4155	3.4271	3.3206	3.6731	3.171	3.4927	4.0099
1800	1.0049	1.6011	1.5933	1.9315	2.0238	2.9822	3.3771	3.5063	3.7113	3.6566	3.4837	3.8697	3.4687	3.685	4.2989
1900	1.0464	1.655	1.6898	2.0272	2.1114	3.2045	3.6118	3.8267	3.8441	3.7921	3.7498	4.0607	3.6911	3.9766	4.5896
2000	1.1038	1.7695	1.8318	2.1479	2.2021	3.4011	3.7798	4.0537	4.0721	4.0867	3.9122	4.1284	3.8453	4.1372	4.6884

Table 9i

# of games	Average Expected Profit														
	Bidder1	Bidder2	Bidder3	Bidder4	Bidders5	Bidder6	Bidder7	Bidders8	Bidder9	Bidder10	Bidder11	Bidder12	Bidder13	Bidder14	Bidder15
100	0.0007	0.0009	0.0003	0.001	0.0003	0.0013	0.0013	0.0029	0.0013	0.0022	0.0042	0.0023	0.0017	0.0019	0.0037
200	0.0006	0.0008	0.0007	0.001	0.0007	0.0013	0.0021	0.0025	0.0014	0.0025	0.0024	0.0028	0.0016	0.0021	0.0027
300	0.0007	0.0008	0.0006	0.0009	0.001	0.0011	0.002	0.0027	0.0015	0.0021	0.0023	0.0027	0.0016	0.0025	0.0031
400	0.0007	0.001	0.0006	0.0009	0.0009	0.001	0.0021	0.0024	0.0018	0.002	0.0021	0.003	0.0018	0.0021	0.0027
500	0.0007	0.0009	0.0006	0.0011	0.0008	0.0011	0.0021	0.0022	0.0019	0.002	0.0019	0.0029	0.002	0.0022	0.0025
600	0.0007	0.0008	0.0006	0.001	0.0008	0.0014	0.0021	0.0022	0.002	0.002	0.002	0.0028	0.0023	0.0021	0.0024
700	0.0007	0.0009	0.0006	0.001	0.0008	0.0015	0.0021	0.0021	0.002	0.0022	0.002	0.0026	0.0022	0.0022	0.0023
800	0.0007	0.0008	0.0007	0.0009	0.0009	0.0016	0.0022	0.002	0.002	0.0022	0.002	0.0025	0.0023	0.0022	0.0022
900	0.0006	0.0008	0.0007	0.0009	0.0009	0.0015	0.0021	0.002	0.0021	0.0021	0.002	0.0025	0.0022	0.0021	0.0023
1000	0.0006	0.0008	0.0007	0.001	0.0009	0.0015	0.0021	0.0022	0.0022	0.002	0.002	0.0024	0.0021	0.0022	0.0023
1100	0.0007	0.0009	0.0007	0.001	0.0009	0.0016	0.002	0.0021	0.0022	0.002	0.0019	0.0023	0.002	0.0022	0.0023
1200	0.0006	0.0009	0.0007	0.001	0.0009	0.0015	0.002	0.002	0.0022	0.002	0.002	0.0024	0.002	0.0022	0.0025
1300	0.0006	0.0009	0.0008	0.0011	0.001	0.0017	0.0019	0.002	0.0021	0.002	0.002	0.0023	0.002	0.0021	0.0023
1400	0.0006	0.0009	0.0008	0.001	0.001	0.0016	0.0019	0.002	0.002	0.0019	0.0021	0.0023	0.002	0.0021	0.0024
1500	0.0006	0.0009	0.0008	0.0011	0.0011	0.0017	0.0018	0.002	0.002	0.002	0.002	0.0023	0.002	0.0021	0.0024
1600	0.0006	0.0009	0.0008	0.0011	0.0011	0.0017	0.0019	0.002	0.0021	0.002	0.002	0.0022	0.0019	0.002	0.0024
1700	0.0006	0.0009	0.0009	0.0011	0.0011	0.0016	0.0019	0.002	0.002	0.002	0.002	0.0022	0.0019	0.0021	0.0024
1800	0.0006	0.0009	0.0009	0.0011	0.0011	0.0017	0.0019	0.0019	0.0021	0.002	0.0019	0.0021	0.0019	0.002	0.0024
1900	0.0006	0.0009	0.0009	0.0011	0.0011	0.0017	0.0019	0.002	0.002	0.002	0.002	0.0021	0.0019	0.0021	0.0024
2000	0.0006	0.0009	0.0009	0.0011	0.0011	0.0017	0.0019	0.002	0.002	0.002	0.002	0.0021	0.0019	0.0021	0.0023

Table 92

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